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ON ONE METHOD OF RESEARCH OF MULTIPERIODIC SOLUTION OF BLOCK-MATRIX TYPE SYSTEM WITH VARIOUS DIFFERENTIATION OPERATORS

Abstract. There is researched the problem of existence and integral representation of a unique multiperiodic solution in all independent variables of a linear system with constant coefficients and with various differentiation operators in the direction of a vector field. Based on the Cauchy characteristics method, a methodology is developed for constructing solutions of initial problem for a linear system with constant coefficients and various special differentiation operators along two straight lines of the independent variables space, where integration characteristics are determined using a projector. It is given a methodology for constructing a matrix of homogeneous block-triangular system, as well as a matricant of a homogeneous linear system in the general case when a Jordan block is split into the sum of two sub-blocks. The Cauchy problems for linear homogeneous and nonhomogeneous systems with integral representation are solved using this methodology. At the same time, the introduced projectors for determining characteristics were of significant importance. Along with the construction of general solutions of linear systems with two differentiation operators, a theorem on the conditions of multiperiodicity of their solutions is proved. On their basis, in noncritical case, the theorem on existence and uniqueness of a multiperiodic solution of linear nonhomogeneous system is proved and its integral representation is given. The developed methodology has the perspective of extending the results obtained to the quasilinear case of system under consideration, as well as to the cases of a system with n various differentiation operators and multiperiodic matrices with partial derivatives of the desired vector function.

Key words: multiperiodic solution, method of characteristics, projection operators, differentiation operators by vector fields, integral representation.

Introduction. In solving many problems of modern science and technology, we often have to deal with oscillatory processes that are described by partial differential equations. Thusfore, research of oscillatory processes described by single and multifrequency periodic solutions of differential equations systems has important theoretical and applied value. It is known that basis of oscillatory solutions theory of differential equations originates from classical works of A.Lyapunov, A.Poincare, N.N.Bogolyubov, N.M.Krylov, Yu.A.Mitropolsky, A.M.Samoilenko, A.N.Kolmogorov, V.I.Arnold, Yu.Moser and et al. Methods for integrating systems of quasilinear differential equations with identical main part are described in fundamental works [1-6]. It is known that basis of the theory of almost periodic and multiperiodic solutions of partial differential equations systems is laid down in the works [4-10]. The works of many authors has been devoted to finding effective signs of solvability and constructing constructive methods for researching problems for systems of differential equations. we note only [11,12]. The research of periodic, both time and space variables of the wave motion of the particle flow, non-stationary flows of compressible liquids and gases [2], and linear gas whose molecules have different velocity values that change each other during collisions, described by a system of partial differential equations [13], is of considerable interest in the continuum mechanics theory. Note that the integration of quasilinear differential equations systems with different main parts is one of the little-studied problems in the partial differential equations theory. Therefore, the development of methods for solving multiperiodic solutions problems of such systems is at the initial stage of its development. Some ideas of these works methods, based on researches [14-17], are extended in [18,19] to study multiperiodic solutions problems of quasilinear equations systems with various differentiation operators along their characteristics. The questions of multiperiodic solutions of quasilinear equations systems with various differentiation operators are studied in [5] in terms of the matricant when the matrix of coefficients is block-diagonal. In [20] in the case of a triangular matrix and in [18] the block-matrix method is used to construct the matricant, and in [19] the problems of multiperiodic solutions are researched by introducing projection operators [21]. Numerous applications of problems for differential equations with various differentiation operators and the necessity to expand the class of multiperiodic functions solvable in space, the creation of new approaches to solving such problems represents a new scientific interest. The article proposes a method for researching a multiperiodic solution of system with various differentiation operators, constructing matricant of a linear system in general case, when splitting a Jordan block into the sum of two sub-blocks, introducing projectors to determine characteristics, and establishing conditions for the existence of (θ,ω)-periodic solutions of the system under consideration and their integral representations.

1. Problem statement. We consider linear system

$$Dx = Ax + f(\tau,t), \tag{1.1}$$

where $x=(x_1,x_2)$ is unknown vector function; x_i are n_i-vector, $n_1+n_2=n$, $D=(D_1,D_2)$ is differentiation operator with various components

$$D_1 = \frac{\partial}{\partial\tau} + \left\langle a_1, \frac{\partial}{\partial t}\right\rangle, \tag{1.1'}$$

$$D_2 = \frac{\partial}{\partial\tau} + \left\langle a_2, \frac{\partial}{\partial t}\right\rangle, \tag{1.1''}$$

$Dx=(D_1x_1, D_2x_2)$, $\left\langle a_i, \frac{\partial}{\partial t}\right\rangle$ is scalar product of vectors a_i and $\frac{\partial}{\partial t}$; $a_1 \neq a_2$ are constant m-vectors; $\frac{\partial}{\partial t} = \left(\frac{\partial}{\partial t_1},\ldots,\frac{\partial}{\partial t_m}\right)$, $A=\left[A_{ij}\right]_{i,j=1,2}$ is constant block $n\times n$-matrix with blocks A_{ij} of dimension $n_i \times n_j$:

$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}, \tag{1.2}$$

$f(\tau,t)=(f_1(\tau,t), f_2(\tau,t))$ is given n-vector function with vector components $f_i(\tau,t)$ of dimension n_i, $i=\overline{1,2}$, (τ,t) are independent variables, $\tau \in (-\infty,+\infty)=R$, $t=(t_1,\ldots,t_m)\in R^m$.

We set the problem of developing a methodology for integration, establishing the conditions for the existence of (θ,ω)-periodic solutions of system (1.1) and their integral representations. In connection with introduction of operator with various components (1.1′)-(1.1″), system (1.1) is represented by blocks of matrix and functions of input data, which require a new approach to the issue of its integration.

2. Methodology for constructing matricant of a homogeneous block-triangular system

For this purpose, we consider homogeneous system

$$Dx = Ax, \tag{2.1}$$

corresponding to the system (1.1), which we describe in the section of differentiation operators

$$\begin{aligned} D_1x_1 &= A_{11}x_1 + A_{12}x_2, \\ D_2x_2 &= A_{21}x_1 + A_{22}x_2. \end{aligned} \tag{2.2}$$

The problem of constructing a matricant $X(\tau)$ of system (2.1), or (2.2), in accordance with the division into blocks of $A=\left[A_{ij}\right]_{i,j=1,2}$. At first, we consider the block-triangular case when $A_{12}=O_{12}$.

Therefore,

$$A=\begin{pmatrix} A_{11} & O_{12} \\ A_{21} & A_{22} \end{pmatrix}. \qquad (2.3)$$

In this case, we write the system (2.2) as

$$D_1x_1 = A_{11}x_1, \qquad (2.4_I)$$

$$D_2x_2 = A_{21}x_1 + A_{22}x_2 . \ (2.4_{II}) \qquad (2.4)$$

Using [4-6, 8], we construct a matricant $X_{11}(\tau)$ of system (2.4_I) with condition $X_{11}(0)=E_1$, based on

$$X_{11}(\tau)=E_1+\int_0^\tau A_{11}X_{11}(s)ds, \qquad (2.5_I)$$

and then we define a solution $X_{22}=X_{22}(\tau)$ with initial condition $X_{22}(0)=E_2$ from the matrix equation (2.4_{II}), where E_1 and E_2 are identity matrices, $E=diag[E_1,E_2]$ is n-matrix.

It is obvious that such initial problem is equivalent to integral matrix equation

$$X_{22}(\tau)=E_2+\int_0^\tau X_{22}(\tau-s)A_{21}X_{11}(s)ds . \ (2.5_{II})$$

This solution $X_{22}(\tau)$ can be built using the same methodology as the matrix $X_{11}(\tau)$ is built.

Thus, we have a matricant $X(\tau)$ of system (2.4) in the form

$$X(\tau)=diag[X_{11}(\tau),X_{22}(\tau)]. \qquad (2.5)$$

Lemma. *If the matrix A has form* (2.3), *then the matricant $X(\tau)$ of system* (2.4) *is represented as* (2.5), *where the diagonal blocks are defined by the integral equations* (2.5_I) *and* (2.5_{II}).

3. Construction of matricant of a homogeneous linear system in the general case

By replacement

$$x=By \qquad (3.1)$$

with a nondegenerate constant n-matrix B, we bring the system (2.1) to the form

$$Dy=Jy, \ J=diag(J_1,...,J_k), \qquad (3.2)$$

J_j are blocks of Jordan matrix J an order l_j with subdiagonal units, $j=\overline{1,k}$, $l_1+...+l_k=n_1+n_2=n$, according to components D_1, D_2 of operator D, the unknown function y has coordinates y_1, y_2.

In connection with equality $l_1+...+l_k=n_1+n_2$, two cases should be distinguished:

I. $l_1+...+l_{k_1}=n_1$, $l_{k_1+1}+...+l_{k_1+k_2}=n_2$, where $k_1+k_2=k$.

II. $l_1+...+l_{k_1-1}+l_{k_1'}=n_1$, $l_{k_1''}+l_{k_1+1}+...+l_{k_1+k_2}=n_2$, where $l_{k_1'}+l_{k_1''}=l_{k_1}$, $l_{k_1'}>0$, $l_{k_1''}>0$, $k_1+k_2=k$.

In the case I, the system (3.2) has the form

$$D_1y_1=J'y_1, \ \ J'=diag\left(J_1,...,J_{k_1}\right), \qquad (3.3_I')$$

$$D_2y_2=J''y_2, \ \ J''=diag\left(l_{k_1+1}...+l_{k_1+k_2}\right). \ (3.3_I'') \qquad (3.3_I)$$

Then its matricant is defined in the diagonal form

$$Y(\tau)=diag(Y_1(\tau),Y_2(\tau)) \qquad (3.4)$$

with n_i-blocks $Y_i(\tau)$ that are constructed using known methodology for constructing matricant.

In the case II, the Jordan block J_{k_1} is split into the sum of two sub-blocks J_{k_1}', J_{k_1}'' and the connecting sub-block J_{k_1}''' an orders l_{k_1}', l_{k_1}'', $l_{k_1}'+l_{k_1}''=l_{k_1}$ and the system (3.2) is represented as

$$D_1y_1=J'y_1, \qquad (3.3_{II}')$$

$$D_1 y'_{k_1} = J'_{k_1} y'_{k_1}, \; D_2 y''_{k_1} = J'''_{k_1} y'_{k_1} + J''_{k_1} y''_{k_1}, \qquad (3.3''_{II})\,(3.3_{II})$$

$$D_2 y_2 = J'' y_2, \qquad (3.3'''_{II})$$

where $\left(y'_{k_1}, y''_{k_1}\right) = y_{k_1}$, $J' = diag\left(J_1,...,J_{k_1-1}\right)$, $J_k = diag\left(J'_{k_1}, J''_{k_1}\right)$, $J'' = diag\left(J_{k_1+1},...,J_{k_1+k_2}\right)$, $k_1 + k_2 = k$.

The matricants $Y_1(\tau)$ and $Y_3(\tau)$ of systems ($3.3'_{II}$) and ($3.3'''_{II}$) are constructed using the known methodology for constructing matricants of systems with single differentiation operator. In order to determine the matricant of system ($3.3''_{II}$), corresponding to the block J_{k_1}, what is represented in sub-blocks J'_{k_1}, J''_{k_1} and J'''_{k_1}, we write in an easy-to-understand form

$$\begin{pmatrix} D_1 u \\ D_2 v \end{pmatrix} = \begin{pmatrix} J'_{k_1} & 0 \\ J'''_{k_1} & J''_{k_1} \end{pmatrix} \begin{pmatrix} w \\ v \end{pmatrix}, \; w = y'_{k_1}, \; v = y''_{k_1}, \qquad (3.5)$$

$$J'_{k_1} = \begin{pmatrix} \mu & 0 & 0 & ... & 0 & 0 \\ 1 & \mu & 0 & ... & 0 & 0 \\ ... & ... & ... & ... & ... & ... \\ 0 & 0 & 0 & ... & \mu & 0 \\ 0 & 0 & 0 & ... & 1 & \mu \end{pmatrix}, \; J''_{k_1} = \begin{pmatrix} \mu & 0 & 0 & ... & 0 & 0 \\ 1 & \mu & 0 & ... & 0 & 0 \\ ... & ... & ... & ... & ... & ... \\ 0 & 0 & 0 & ... & \mu & 0 \\ 0 & 0 & 0 & ... & 1 & \mu \end{pmatrix}, \; J'''_{k_1} = \begin{pmatrix} 0 & ... & 0 & 1 \\ 0 & ... & 0 & 0 \\ ... & ... & ... & ... \\ 0 & ... & 0 & 0 \end{pmatrix}.$$

Assuming $\lambda_k = \mu$ write down the system (3.5) in coordinate form:

$$\begin{array}{ll} D_1 w_1 = \mu\, w_1 & D_2 v_1 = w_{k'_1} + \mu\, v_1 \\ .... & .... \\ D_1 w_{k'_1} = w_{k'_1-1} + \mu\, w_{k'_1} & D_2 v_{k'_1+k''_1} = v_{k'_1+k''-1} + \mu\, v_{k'_1+k''} \end{array} \qquad (3.6)$$

Since the system (3.5) and, therefore, the system (3.6) has a block-triangular form, we construct its matricant $Y_2(\tau)$ based on the proved lemma. So, in case II, we have a matricant of the form

$$Y(\tau) = diag\left[Y_1(\tau), Y_2(\tau), Y_3(\tau)\right]. \qquad (3.7)$$

Therefore, from the transformation (3.1) we have the matricant of the system (2.1)

$$X(\tau) = BY(\tau)B^{-1}, \qquad (3.8)$$

where $Y(\tau)$ is determined by the relations (3.4) and (3.7). Thus, in this section it specifies a peculiar approach to constructing a matrix of a homogeneous linear block-matrix system (2.2) and, consequently, a system (2.1). Let's formalize the result in the form of the following theorem.

Theorem 1. *The matricant of linear homogeneous system* (2.1) *with block constant matrix* (1.2) *has the form* (3.8), *where the matrix* $Y(\tau)$ *has form* (3.4) *or* (3.7).

4. Solutions of linear systems with various operators and their integral representations

From the characteristic system

$$\frac{dt}{d\tau} = a_i, \; i = 1,2 \qquad (4.1)$$

we have solutions $t = t^0 + a_i\left(\tau - \tau^0\right) \equiv h_i\left(\tau, \tau^0, t^0\right)$, $i = 1,2$ with arbitrary initial data $\left(\tau^0, t^0\right) \in R \times R^m$. Let's assume that characteristic variable $h = h\left(\tau, \tau^0, t^0\right)$ changes on set of above defined characteristics $H = \left\{h_1\left(\tau, \tau^0, t^0\right), h_2\left(\tau, \tau^0, t^0\right)\right\}$. Note that $h_i\left(\tau^0, \tau, t\right) = t^0$, $i = 1,2$ are the first integrals of (4.1). Functions $u\left(h_i\left(\tau^0, \tau, t\right)\right)$ are zeros of operators D_i, respectively; where $u(t) \in C_t^{(e)}\left(R^m\right)$, $e = (1,...,1)$ is m-vector.

Next, the matricant $X(\tau)$ of system (2.1) is represented using block matrices $X_{ij}(\tau)$ in the form

$$X(\tau)=\begin{pmatrix} X_{11}(\tau) & X_{12}(\tau) \\ X_{21}(\tau) & X_{22}(\tau) \end{pmatrix},\ X_{ij}(\tau) \text{ are } n_i \times n_j\text{-matrices.} \quad (4.2)$$

Let the operators P_i act on function $u(t)$ defined on one of two characteristics $t=h_i(\tau,\tau^0,t^0)$ as follows

$$P_i u(h(\tau,\tau^0,t^0))=u(h_i(\tau,\tau^0,t^0)),\ i=1,2,\ h(\tau,\tau^0,t^0)\in H. \quad (4.3)$$

Operators P_i can be called projectors that define a function on the corresponding characteristic.

We introduce the operator P associated with projectors P_1 and P_2 acting on matricant $X(\tau)$ on the right by following relation $X(\tau)P=\lfloor X_{ij}(\tau)P_i\rfloor$, $i=1,2$, where the blocks $X_{ij}(\tau)$ and projectors P_i are defined by the formulas (4.2) and (4.3). We set the problem of constructing a solution x of the system (2.1) with initial condition $x|_{\tau=\tau^0}=u(t)$, $u(t)\in C_t^{(e)}(R^m)$, $e=(1,...,1)$ is m-vector.

By checking directly, you can make sure that the problem has a unique solution of the form

$$x(\tau,t)=X(\tau-\tau^0)Pu(h(\tau^0,\tau,t)),\ h(\tau,\tau^0,t^0)\in H. \quad (4.4)$$

Here $X(\tau)$ is matricant of system (2.1), P is projector. Thus, the following theorem is proved.

Theorem 2. *The unique solution of linear homogeneous system* (2.1) *with various differentiation operators D_1 and D_2, satisfying initial condition $x|_{\tau=\tau^0}=u(t)$ is determined by the relation* (4.4).

Let the vector functions $f(\tau,t)$ have smoothness property of the order $(0,e)=(0,1,...,1)$:

$$f(\tau,t)\in C_{\tau,t}^{(0,e)}(R\times R^m). \quad (4.5)$$

Theorem 3. *Under condition* (4.5), *the unique solution x of linear nonhomogeneous system* (1.1) *that satisfies initial condition $x|_{\tau=\tau^0}=u(t)$, $u(t)\in C_t^{(e)}(R^m)$ is determined by the relation*

$$x(\tau,t)=X(\tau-\tau^0)Pu(h(\tau^0,\tau,t))+\int_{\tau^0}^{\tau}X(\tau-s)Pf(s,h(s,\tau,t))ds. \quad (4.6)$$

Proof. It is obvious that the second term of equality (4.6) under condition (4.5) is a solution of a nonhomogeneous system (1.1), and the first term, in accordance with theorem 2, is a solution of a homogeneous system (2.1) that satisfies the given initial condition. Therefore, the relation (4.6) represents the solution of the system (1.1). Uniqueness follows from theorem 2. Q.E.D.

5. Multiperiodic solutions of systems with various operators and their integral representations

Let the vector functions $f_i(\tau,t)$, $i=1,2$ have (θ,ω)-periodicity and the smoothness property

$$f_i(\tau+\theta,t+q\omega)=f_i(\tau,t)\in C_{\tau,t}^{(0,e)}(R\times R^m),\ q\in Z^m, \quad (5.1)$$

where $(\theta,\omega)=(\theta,\omega_1,...,\omega_m)$ is a period with rationally incommensurable coordinates $\omega_0=\theta,\omega_1,...,\omega_m$.

Theorem 4. *Under condition* (5.1) *for (θ,ω)-periodicity of solution $x(\tau,t)=\varphi(\tau,t,u(h(0,\tau,t)))$ of system* (1.1) *with initial function $u(t)$, it is necessary and sufficient that the functional-difference system*

$$u(t)=\varphi(\theta,t,u(t+\Delta(t))). \quad (5.2)$$

was solvable in the class ω-periodic smooth functions $u(t)=u(t+q\omega)\in C_t^{(e)}(R^m)$, $q\in Z^m$.

Proof. Note that if $x=x(\tau,t)$ is solution of system (1.1), then $y(\tau,t)=x(\tau+\theta,t+q\omega)$, also satisfies the system (1.1), and $z(\tau,t)=y(\tau,t)-x(\tau,t)$ is solution of (2.1). It is obvious that if the initial function

$u(t)$ of solution $x(\tau,t)$ of (1.1) has property $u(t)=u(t+q\omega)\in C_t^{(e)}(R^m)$, $q\in Z^m$, then $x(\tau,t)$ is ω-periodic by $t\in R^m$ and vice versa. In the future, we will assume that this condition always satisfied.

If the initial conditions for these solutions $x(\tau,t)$ and $y(\tau,t)$ for the same

$$x(0,t)=x(\theta,t), \tag{5.3}$$

then these solutions are identically equal, moreover $x(\tau,t)=x(\tau+\theta,t+q\omega)$. Conversely, if the solution $x(\tau,t)$ is θ-periodic by τ, then the condition (5.3) is satisfy. Then by virtue of theorem 3, we have $x(\tau,t)=\varphi(\tau,t,u(h(0,\tau,t)))$, at that $x(0,t)=u(t)$ and $x(\theta,t)=\varphi(\theta,t,u(h(0,\theta,t)))\equiv\varphi(\theta,t,u(t+\Delta(t)))$, $\Delta(t)=h(0,\theta,t)-t$. Therefore, condition (5.3), depending on function $u(t)$, has the form (5.2). Q.E.D.

Theorem 5. *Linear system* (1.1) *with various operators* (1.1′)-(1.1″) *under the conditions (5.1) and*

$$\operatorname{Re}\lambda(A)<0, \tag{5.4}$$

has a unique (θ,ω)*-periodic solution* $x^*(\tau,t)$ *with integral representation*

$$x^*(\tau,t)=\int_{-\infty}^{\tau}X(\tau-s)Pf(s,h(s,\tau,t))ds. \tag{5.5}$$

Proof. By virtue of structure of the general solution (4.6), the system (5.2) has the form

$$u(t)=X(\theta)Pu(t+\Delta(t))+\psi(t), \tag{5.6}$$

arbitrary term defines by $\psi(t)=\int_0^{\theta}X(\theta-s)Pf(s,h(s,\theta,t))ds$, ω-periodic by t and $\Delta(t)=-a\theta$. If real parts of eigenvalues $\lambda(A)$ of A are negative (5.4), then after k of iterations, system (5.6) can be reduced

$$u(t)=X(k\theta)Pu(t+k\Delta(t))+\psi_k(t) \tag{5.7}$$

with matrix $X(k\theta)$ that is normally bounded by constant δ from interval $0<\delta<1$. Therefore, we have

$$\|X(k\theta)\|\le\delta<1. \tag{5.8}$$

Then by the method of successive approximations by virtue of conditions (5.4) and (5.8) it is easy to show that system (5.7) and, therefore, system (5.6) has unique smooth ω-periodic solution:

$$u(t)=\int_{-\infty}^{0}X(\theta-s)Pf(s,h(s,\theta,t))ds. \tag{5.9}$$

It's clear that corresponding homogeneous system (2.1) with various differentiation operators (1.1′) and (1.1″) under the conditions (5.4) has only a zero (θ,ω)-periodic solution.

Substituting (5.9) in the formula of general solution (4.6) we have integral representation of the unique (θ,ω)-periodic solution (5.5). Thus, theorem 5 is completely proved.

Conclusion. In article on the basis of the method of projection operator multiperiodic it is researched the solution of system with various operators of differentiation, it is built a matricant of linear system in the general case, the splitting Jordan block as a sum of two sub-blocks, it is established the conditions for existence of (θ,ω)-periodic solutions of the considered system and their integral representations. Note that the developed method can be generalized to the quasilinear case when the coefficients of linear part are multiperiodic. Meanwhile, it will be necessary to use generalizations of methods of works [4-10], [14-16] for the case under consideration.

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ӘРТҮРЛІ ДИФФЕРЕНЦИАЛДАУ ОПЕРАТОРЛЫ БЛОК-МАТРИЦАЛЫҚ ТҮРДЕГІ ЖҮЙЕНІҢ КӨППЕРИОДТЫ ШЕШІМІН ЗЕРТТЕУДІҢ БІР ӘДІСІ ТУРАЛЫ

Аннотация. Дербес туындылы теңдеулер жүйелерімен сипатталатын тербеліс процестерінің көппериодты және периодты дерлік шешімдерін зерттеу дифференциалдық теңдеулер теориясының қазіргі даму кезеңінде үлкен қызығушылық туғызуда. Бұл тұтас орта механика теориясы, сығылатын сұйықтық пен газдардың стационар емес ағысы, бөлшектер ағынының толқындық қозғалысының физикалық-техникалық есептерінің қолданыстарымен байланысты. Деформацияланатын, кристаллдық емес орталардағы процестерді зерттеу периодтылық немесе периодты дерлік қасиеттерді ескеру қажеттілігіне алып келетініне ескере кеткен жөн.

Векторлық өріс бағыты бойынша әртүрлі дифференциалдау операторлы сызықты тұрақты коэффициентті жүйенің барлық айнымалылары бойынша жалғыз көппериодты шешімінің бары және интеғралдық бейнесі туралы есеп зерттелген.

Интеғралдау сипаттауыштары проектор арқылы анықталатын тәуелсіз айнымалылар кеңістігінің екі түзуі бойында әртүрлі арнайы дифференциалдау операторлы сызықты тұрақты коэффициентті блок-матрицалық түрдегі жүйе үшін бастапқы есептің шешімдерін тұрғызу тәсілі Кошидің сипаттауыштар әдісі негізінде түзілді.

Үшбұрыш-блокты түрдегі біртекті жүйенің матрицантын құру тәсілдемесі келтірілген. Диағональды блоктардың интегралдық матрицалық теңдеулері табылды. Жордан блоғы екі блокшаның қосындысына ажырағанда жалпы жағдайдағы сызықты біртекті жүйенің матрицантын құрудың жаңа тәсілдемесі ұсынылған.

Құрылған әдістеме бойынша векторлар өрісі бағыты бойынша әртүрлі арнайы дифференциалдау операторлы біртекті және біртекті емес сызықты жүйелер үшін Коши есебінің жалғыз шешімінің бар болу туралы есебінің сұрағы шешілді және енетін деректер анықталған жатықтыққа ие болғандағы шарт орындалғанда интегралдық бейнелері келтірілді.

Осы тұста дифференциалдау және интеғралдау жүргізілетін сәйкес характеристикаларды анықтаушы енгізілген проекторлардың маңызы шешуші болды. Проекторлардың кейбір қасиеттері тағайындалды, оның ішінде проекторлардың екі характеристикалардың бірінде берілген вектор-функцияға және құрылған матрицантқа әсері көрсетілді.

Әртүрлі екі дифференциалдау операторлы сызықты біртекті және біртекті емес жүйелердің жалпы шешімдерін құрумен қатар, периодты жатық функциялар класындағы функционалдық-айырымдық жүйенің шешілімділігінің қажетті және жеткілікті шарттары орындалғанда қарастырылған арнайы дифференциалдау операторлы жүйенің көппериодты шешімінің бар болу шарттары дәлелденді. Алынған нәтиже теорема түрінде тұжырымдалды.

Жоғарыда қарастырылғандардың негізінде, енетін деректер көппериодтылық пен анықталған жатықтық қасиеттеріне қанағаттандырғанда, критикалық емес жағдайда біртекті емес сызықты блок-матрицалық түрдегі жүйенің көппериодты шешімінің бар және жалғыз болуы туралы теорема дәлелденді және оның проекциялау операторларынан тәуелді интеғралдық бейнесі келтірілді. Теореманы дәлелдеу барысында көппериодты функциялар класында функционалдық-айырымдық жүйенің шешілімділігі және енгізілген проекциялау операторы қолданылды. Теореманы дәлелдегенде біртіндеп жуықтау әдісі падаланылып, критикалық емес және анықталған жатықтық қасиеттеріне сүйене қарастырылған жүйенің жалғыз жатық көппериодты шешім табылды.

Векторлық өріс бағыты бойынша арнайы дифференциалдау операторлы блок-матрицалық түрдегі сызықты дифференциалдық теңдеулер жүйесінің көппериодты шешімін құру және проектор теориясын қолдану көппериодты функциялар кеңістігінде шешілімді есептер класының кеңеюіне келтіретінін ескереміз.

Бірінші ретті дербес туындылы сызықты теңдеулер жүйесін енетін деректердің матрицалық блоктары мен вектор-функциялары арқылы өрнектелетіндігі тәуелсіз айнымалылар кеңістігінің векторлық өріс бағыты бойынша әртүрлі арнайы сызықты дифференциалдау операторлы жүйенің

уақыт және кеңістік тәуелсіз айнымалылары бойынша көппериодты шешімінің бар болатындыгын зерттеудің жаца тәсілдемесін құруга әкелді.

Қолданылган әдістememен алынган нәтижелерді қарастырылган жүйенің жалпыланган квазисызықты жагдайында да, сондай-ақ, әртүрлі *n* дифференциалдау операторлы жүйе жагдайында және белгісіз вектор-функцияныц дербес туындыларыныц жанындагы коэффициенттері көппериодты матрицалар болган кезде де осы әдісті қолданып алуга болады.

Түйін сөздер: көппериодты шешім, характеристикалар әдісі, проекциялау операторлары, векторлық өрістер бойынша дифференциалдау операторлары, интегралдық бейне.

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ОБ ОДНОМ МЕТОДЕ ИССЛЕДОВАНИЯ МНОГОПЕРИОДИЧЕСКОГО РЕШЕНИЯ СИСТЕМЫ БЛОЧНО-МАТРИЧНОГО ВИДА С РАЗЛИЧНЫМИ ОПЕРАТОРАМИ ДИФФЕРЕНЦИРОВАНИЯ

Аннотация. На современном этапе развития теории дифференциальных уравнений наибольший интерес представляет исследование многопериодических и почти периодических решений колебательных процессов, описывающихся системами дифференциальных уравнений в частных производных. Это связано с приложениями физико-технических задач теории механики сплошной среды, нестационарных течений сжимаемых жидкостей и газов, волнового движения потока частиц. Заметим, что исследование процессов в деформируемых, некристаллических средах приводит к необходимости учета свойства периодичности или почти периодичности.

Исследована задача о существовании и интегральном представлении единственного многопериодического по всем независимым переменным решения линейной системы с постоянными коэффициентами и с различными операторами дифференцирования по направлению векторного поля пространства независимых переменных.

На основе метода характеристик Коши разработана методика построения решений начальной задачи для линейной системы с постоянными коэффициентами и с различными специальными операторами дифференцирования вдоль двух прямых пространства независимых переменных, где характеристики интегрирования определяются при помощи проектора.

Приведена методика построения матрицанта однородной системы блочно-треугольного вида. Найдены интегральные матричные уравнения диагональных блоков. Предложен новый подход построения матрицанта линейной однородной системы в общем случае, когда жордановый блок расщепляется на сумму двух подблоков.

По разработанной методике решен вопрос о существовании единственного решения задачи Коши для линейной однородной и неоднородной систем с различными специальными операторами дифференцирования по направлению векторного поля и найдены их интегральные представления, при условии, что входные данные обладают определенной гладкостью. При этом существенное значение имели введенные проекторы по определению соответствующих характеристик, по которым ведутся дифференцирование и интегрирование. Установлены некоторые свойства проекторов, в том числе действие проекторов на вектор-функцию заданных на одной из двух характеристик, а также воздействие на построенный матрицант.

Наряду с построением общих решений линейных систем с двумя операторами дифференцирования доказана теорема об условиях существования многопериодического решения рассматриваемой системы со специальным оператором дифференцирования при необходимом и достаточном условии разрешимости функционально-разностной системы в классе периодических гладких функций.

При предположении многопериодичности и определенной гладкости входных данных на основе вышеизложенного в некритическом случае доказана теорема о существовании и единственности многопериодического решения линейной неоднородной системы блочно-

матричного вида и дано его интегральное представление, зависящее от операторов проектирования. В процессе доказательства теоремы воспользовались разрешимостью функционально-разностной системы в классе многопериодических функций и введенным оператором проектирования. В ходе доказательства используя метод последовательных приближений, в силу условий некритичности и гладкости получено единственное гладкое многопериодическое решение рассматриваемой системы.

Отметим, что построение многопериодического решения системы дифференциальных уравнений блочно-матричного вида со специальными операторами дифференцирования по направлению векторного поля с применением теории проектора приводит к расширению класса разрешимых задач в пространстве многопериодических функций.

Представление линейной системы уравнений в частных производных первого порядка через блоки матричной и векторной функций входных данных привело к разработке нового подхода исследования вопроса существования многопериодического решения как по временным так и по пространственным переменным системы с различными операторами дифференцирования по направлению векторного поля пространства независимых переменных.

Разработанная методика имеет перспективу распространения полученных результатов на квазилинейный случай рассматриваемой системы, а также на случаи системы с n различными операторами дифференцирования и многопериодических матриц при частных производных искомой вектор-функции.

Ключевые слова: многопериодическое решение, метод характеристик, операторы проектирования, операторы дифференцирования по векторным полям, интегральное представление.

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