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CORRECTNESS OF THE MIXED PROBLEM FOR ONE CLASS OF DEGENERATE MULTIDIMENSIONAL HYPERBOLO-PARABOLIC EQUATIONS

Abstract. It is known that in mathematical modeling of electromagnetic fields in space, the nature of the electromagnetic process is determined by the properties of the medium. If the medium is non-conductive, we get degenerate multi-dimensional hyperbolic equations. If the medium has a high conductivity, then we go to degenerate multidimensional parabolic equations.

Consequently, the analysis of electromagnetic fields in complex media (for example, if the conductivity of the medium changes) reduces to degenerate multidimensional hyperbolic-parabolic equations.

Also, it is known that the oscillations of elastic membranes in space according to the Hamilton principle can be modeled by degenerating multidimensional hyperbolic equations.

Studying the process of heat propagation in a medium filled with mass leads to degenerate multidimensional parabolic equations.

Consequently, by studying the mathematical modeling of the process of heat propagation in oscillating elastic membranes, we also come to degenerate multidimensional hyperbolic-parabolic equations. When studying these applications, it is necessary to obtain an explicit representation of the solutions of the studied problems.

The mixed problem for degenerate multidimensional hyperbolic equations was previously considered.

As far as is known, these questions for degenerate multidimensional hyperbolic-parabolic equations have not been studied.

In this paper, unique solvability is shown and an explicit form of the classical solution of the mixed problem for one class of degenerate multidimensional hyperbolic-parabolic equations is obtained.

Keywords: mixed problem, classical solution, unique solvability, Bessel functions, spherical functions.

item 1. Introduction. The mixed problem for degenerate multidimensional hyperbolic equations in generalized spaces has been studied [1,2]. The correctness of this problem was proved in [3,4] and an explicit form of the classical solution was obtained.

As far as we know, these questions have not been studied for degenerate multidimensional hyperbolic-parabolic equations.

This article shows the unique solvability and obtains an explicit representation of the classical solution of the mixed problem for one class of degenerate multidimensional hyperbolic-parabolic equations.

item 2. Statement of the problem and results. Let $\mathbf{\Omega}_{\alpha\beta}$ – the cylindrical region of the Euclidean space E_{m+1} of points $(x_1,......x_m,t)$ bounded by the cylinder $\Gamma=\{(x,t):|x|=1\}$, the planes $t=\alpha>0$ and $t=\beta<0$, where $|x|$ – the length of the vector $x=(x_1,...,x_m)$.

We denote by $\mathbf{\Omega}_\alpha$ and $\mathbf{\Omega}_\beta$ the parts of the region $\mathbf{\Omega}_{\alpha\beta}$, and by Γ_α, Γ_β the parts of the surface Γ lying in the half-spaces $t>0$ and $t<0$; σ_α – the upper and σ_β – lower base of the area $\mathbf{\Omega}_{\alpha\beta}$.

Let further S be the common part of the boundaries of the regions $\mathbf{\Omega}_\alpha$ and $\mathbf{\Omega}_\beta$, representing the set $\{t=0,\ 0<|x|<1\}$ in E_m.

In the domain $\mathbf{\Omega}_{\alpha\beta}$, we consider degenerate multidimensional hyperbolic-parabolic equations.

$$0=\begin{cases} t^q\Delta_x u-u_t+\sum\limits_{i=1}^{m} d_i(x,t)u_{x_i}+e(x,t)u, & t>0,\\ |t|^p\,\Delta_x u-u_{tt}+\sum\limits_{i=1}^{m} a_i(x,t)u_{x_i}+b(x,t)+c(x,t)u, & t<0,\end{cases} \tag{1}$$

where $p,q=const$, $p>0$, $q\geq 0$, Δ_x - is the Laplace operator with respect to variables $x_1,...,x_m$, $m\geq 2$.

In the future, it is convenient for us to switch from Cartesian coordinates $x_1,...,x_m$, t to spherical $r,\theta_1,...,\theta_{m-1}$, t $r\geq 0, 0\leq\theta_1<2\pi$, $0\leq\theta_i\leq\pi$, $i=2,3,...,m-1$, $\theta=(\theta_1,...,\theta_{m-1})$.

Problem 1. Find the solution to equation (1) in the region $\mathbf{\Omega}_{\alpha\beta}$ when $t\neq 0$ the class $C(\overline{\mathbf{\Omega}}_{\alpha\beta})\cap C^1(\mathbf{\Omega}_{\alpha\beta})\cap C^1(\mathbf{\Omega}_\alpha)\cap C^2(\mathbf{\Omega}_\alpha\cup\mathbf{\Omega}_\beta)$, that satisfy the boundary conditions

$$u\big|_{\sigma_\alpha}=\varphi(r,\theta),\ u\big|_{\Gamma_\alpha}=\psi_1(t,\theta), \tag{2}$$

$$u\big|_{\Gamma_\beta}=\psi_2(t,\theta), \tag{3}$$

wherein $\varphi(1,\theta)=\psi_1(\alpha,\theta)$, $\psi_1(0,\theta)=\psi_2(0,\theta)$.

Let be $\{Y_{n,m}^k(\theta)\}$ - a system of linearly independent spherical functions of order n, $1\leq k\leq k_n$, $(m-2)!n!k_n=(n+m-3)!(2n+m-2)$, $W_2^l(S)$, $l=0,1,...$ - Sobolev space.

It takes place ([5]).

Lemma 1. Let $f(r,\theta)\in W_2^l(S)$. If $l\geq m-1$, then the series

$$f(r,\theta)=\sum_{n=0}^{\infty}\sum_{k=1}^{k_n} f_n^k(r)Y_{n,m}^k(\theta), \tag{4}$$

and also the series obtained from it by differentiating order $p\leq l-m+1$, converge absolutely and evenly.

Lemma 2. In order that $f(r,\theta)\in W_2^l(S)$, it is necessary and sufficient that the coefficients of the series (4) satisfy the inequalities

$$\left|f_0^1(r)\right|\leq c_1,\ \sum_{n=1}^{\infty}\sum_{k=1}^{k_n} n^{2l}\left|f_n^k(r)\right|^2\leq c_2,\qquad c_1,c_2=const.$$

By $\tilde{d}_{in}^k(r,t)$, $d_{in}^k(r,t)$, $\tilde{e}_n^k(r,t)$, $\tilde{d}_n^k(r,t)$, ρ_n^k, $\overline{\varphi}_n^k(r)$, $\psi_{1n}^k(t)$, $\psi_{2n}^k(t)$, we denote the expansion coefficients of the series (4) respectively, of the functions, $d_i(r,\theta,t)\rho$, $d_i\frac{x_i}{r}\rho$, $e(r,\theta,t)\rho$, $d(r,\theta,t)\rho$, $\rho(\theta)$, $i=1,...,m$, $\varphi(r,\theta)$, $\psi_1(t,\theta)$, $\psi_2(t,\theta)$, and $\rho(\theta)\in C^\infty(H)$, $H-$the unit sphere in E_m.

Let be $a_i(r,\theta,t), b(r,\theta,t), c(r,\theta,t)\in W_2^l(\mathbf{\Omega}_\beta)\subset C(\overline{\mathbf{\Omega}}_\beta)$, $d_i(r,\theta,t), e(r,\theta,t)\in W_2^l(\mathbf{\Omega}_\alpha)$, $i=1,...,m$, $l\geq m+1$, $e(r,\theta,t)\leq 0$, $\forall(r,\theta,t)\in\mathbf{\Omega}_\alpha$.

Then fair

Theorem. If $\varphi(r,\theta)\in W_2^p(S)$, $\psi_1(t,\theta)\in W_2^p(\Gamma_\alpha)$, $\psi_2(t,\theta)\in W_2^p(\Gamma_\beta)$, $p>\frac{3m}{2}$ and

$$\cos\mu_{s,n}\beta'\neq 0,\ s=1,2,..., \tag{5}$$

then Problem 1 is uniquely solvable, $\mu_{s,n}$ – where the positive zeros of the Bessel functions of the first kind $J_{n+\frac{(m-2)}{2}}(z)$, $\beta'=\frac{2}{2+p}|\beta|^{\frac{2+p}{2}}$, $n=0,1,...$.

item 3. Solvability of Problem 1. In the spherical coordinates of equation (1) in a region $\mathbf{\Omega}_\alpha$ it has the form

$$L_1u=t^q\left(u_{rr}+\frac{m-1}{r}u_r-\frac{1}{r^2}\delta u\right)-u_t+\sum_{i=1}^{m}d_i(r,\theta,t)u_{x_i}+e(r,\theta,t)u=0, \tag{6}$$

$$\delta\equiv-\sum_{j=1}^{m-1}\frac{1}{g_j\sin^{m-j-1}\theta_j}\frac{\partial}{\partial\theta_j}\left(\sin^{m-j-1}\theta_j\frac{\partial}{\partial\theta_j}\right),\quad g_1=1,\quad g_j=(\sin\theta_1...\sin\theta_{j-1})^2,\quad j>1.$$

It is known ([5]) that the spectrum of an operator δ consists of eigenvalues $\lambda_n=n(n+m-2)$, $n=0,1,....$, each of which corresponds k_n to orthonormal eigenfunctions $Y_{n,m}^k(\theta)$.

The desired solution to problem 1 in the domain $\mathbf{\Omega}_\alpha$ will be sought in the form

$$u(r,\theta,t)=\sum_{n=0}^{\infty}\sum_{k=1}^{k_n}\overline{u}_n^k(r,t)Y_{n,m}^k(\theta), \tag{7}$$

where $\overline{u}_n^k(r,t)$ are the functions to be determined.

Substituting (7) into (6), multiplying the resulting expression by $\rho(\theta)\neq 0$, and integrating over the unit sphere H , for $\overline{u}_n^k$ we obtain ([3,4])

$$t^q\rho_0^1\overline{u}_{0rr}^1-\rho_0^1\overline{u}_{0t}^1+\left(\frac{m-1}{r}t^q\rho_0^1+\sum_{i=1}^{m}d_{i0}^1\right)\overline{u}_{0r}^1+\widetilde{e}_{0r}^1\overline{u}_{0r}^1+$$

$$\sum_{n=1}^{\infty}\sum_{k=1}^{k_n}\left\{t^q\rho_n^k\overline{u}_{nrr}^k-\rho_n^k\overline{u}_{nt}^k+\left(\frac{m-1}{r}t^q\rho_n^k+\sum_{i=1}^{m}d_{in}^k\right)\overline{u}_{nr}^k+\left[\widetilde{e}_n^k-\lambda_n\frac{\rho_n^k}{r^2}t^q\sum_{i=1}^{m}(\widetilde{d}_{in}^k-nd_{in}^k\right]\overline{u}_n^k\right\}=0. \tag{8}$$

Now consider an infinite system of differential equations

$$t^q\rho_0^1\overline{u}_{0rr}^1-\rho_0^1\overline{u}_{0t}^1+\frac{(m-1)}{r}t^q\rho_0^1\overline{u}_{0r}^1=0, \tag{9}$$

$$t^q\rho_1^k\overline{u}_{1rr}^k-\rho_1^k\overline{u}_{1t}^k+\frac{(m-1)}{r}t^q\rho_1^k\overline{u}_{1r}^k-\frac{\lambda_1}{r^2}t^q\rho_1^k\overline{u}_1^k=-\frac{1}{k_1}\left(\sum_{i=1}^{m}(\widetilde{d}_{i0}^1\overline{u}_{0r}^1+\widetilde{e}_0^1\overline{u}_0^1\right),\ n=1,\ k=\overline{1,k_1},$$

$$t^q\rho_n^k\overline{u}_{nrr}^k-\rho_n^k\overline{u}_{nt}^k+\frac{(m-1)}{r}t^q\rho_n^k\overline{u}_{nr}^k-\frac{\lambda_n}{r^2}t^q\rho_n^k\overline{u}_n^k=-\frac{1}{k_n}\sum_{k=1}^{k_{n-1}}\left\{\sum_{i=1}^{m}(\widetilde{d}_{in-1}^k\overline{u}_{n-1r}^k+\right.$$

$$\left.+\left[\widetilde{e}_{n-1}^k+\sum_{i=1}^{m}(\widetilde{d}_{in-2}^k-(n-1)d_{in-1}^k\right]\overline{u}_{n-1}^k\right\},\ k=\overline{1,k_n},\ n=2,3.... \tag{10}$$

It is easy to verify that if $\{u_n^k\}$ $k=\overline{1,k_n}$, $n=0,1,...$ is a solution to system (9), (10), then it is a solution to equation (8).

It is easy to see that each equation of system (9), (10) can be represented as

$$t^q\left(\bar{u}^k_{nrr}+\frac{(m-1)}{r}\bar{u}^k_{nr}-\frac{\lambda_n}{r^2}\bar{u}^k_n\right)-\bar{u}^k_{nt}=\bar{f}^k_n(r,t)\,, \tag{11}$$

where $\bar{f}^k_n(r,t)$ are determined from the previous equations of this system, at that $\bar{f}^1_0(r,t)\equiv 0$.

Further, from the boundary condition (2), by virtue of (7), we have

$$\bar{u}^k_n(r,\alpha)=\bar{\varphi}^k_n(r),\ \ \bar{u}^k_n(1,t)=\bar{\psi}^k_{1n}(t),\ k=\overline{1,k_n},\quad n=0,1,\ldots. \tag{12}$$

In (11), (12), changing the variables $\bar{\upsilon}^k_n(r,t)=\bar{u}^k_n(r,t)-\bar{\psi}^k_{1n}(t)$ we obtain

$$t^q(\bar{\upsilon}^k_{nrr}+\frac{m-1}{r}\bar{\upsilon}^k_{nr}-\frac{\lambda_n}{r^2}\bar{\upsilon}^k_n)-\bar{\upsilon}^k_{nt}=f^k_n(r,t)\,, \tag{13}$$

$$\bar{\upsilon}^k_n(r,\alpha)=\varphi^k_n(r),\ \ \bar{\upsilon}^k_n(1,t)=0\ \ k=\overline{1,k_n},\quad n=0,1,\ldots, \tag{14}$$

$$f^k_n(r,t)=\bar{f}^k_n(r,t)+\psi^k_{1nt}+\frac{\lambda_n t^q}{r^2}\psi^k_{1n},\ \varphi^k_n(r)=\bar{\varphi}^k_n(r)-\psi^k_{1n}(\alpha).$$

Having replaced $\bar{\upsilon}^k_n(r,t)=r^{\frac{(1-m)}{2}}\upsilon^k_n(r,t)$ the problem (13), (14), we reduce to the following problem

$$L\upsilon^k_n\equiv t^q\left(\upsilon^k_{nrr}+\frac{\bar{\lambda}_n}{r^2}\upsilon^k_n\right)-\upsilon^k_{nt}=\tilde{f}^k_n(r,t), \tag{15}$$

$$\upsilon^k_n(r,\alpha)=\tilde{\varphi}^k_n(r),\ \ \upsilon^k_n(1,t)=0\,. \tag{16}$$

$$\bar{\lambda}_n=\frac{((m-1)(3-m)-4\lambda_n)}{4},\qquad \tilde{f}^k_n(r,t)=r^{\frac{(m-1)}{2}}f^k_n(r,t),\ \ \tilde{\varphi}^k_n(r)=r^{\frac{(m-1)}{2}}\varphi^k_n(r).$$

The solution to problem (15), (16) is sought in the form

$$\upsilon^k_n(r,t)=\upsilon^k_{1n}(r,t)+\upsilon^k_{2n}(r,t), \tag{17}$$

where $\upsilon^k_{1n}(r,t)-$ is the solution to the problem

$$L\upsilon^k_{1n}(r,t)=\tilde{f}^k_n(r,t), \tag{18}$$

$$\upsilon^k_{1n}(r,\alpha)=0,\ \ \upsilon^k_{1n}(1,t)=0, \tag{19}$$

and $\upsilon^k_{2n}(r,t)-$ the solution to the problem

$$L\upsilon^k_{2n}=0\,, \tag{20}$$

$$\upsilon^k_{2n}(r,\alpha)=\tilde{\varphi}^k_n(r),\ \ \upsilon^k_{2n}(1,t)=0, \tag{21}$$

The solution to the above tasks, consider in the form

$$\upsilon^k_n(r,t)=\sum_{s=1}^{\infty}R_s(r)T_s(t), \tag{22}$$

while let

$$\tilde{f}^k_n(r,t)=\sum_{s=1}^{\infty}a_{s,n}(t)R_s(r),\quad \tilde{\varphi}^k_{1n}(r)=\sum_{s=1}^{\infty}b_{s,n}R_s(r). \tag{23}$$

Substituting (22) into (18), (19), taking into account (23), we obtain

$$R_{srr}+\frac{\bar{\lambda}_n}{r^2}R_s+\mu R_s=0,\ \ 0<r<1. \tag{24}$$

$$R_s(1)=0, \ |R_s(0)|<\infty, \tag{25}$$

$$T_s+\mu t^q T_s(t)=-a_{s,n}(t), \ 0<t<\alpha, \tag{26}$$

$$T_s(\alpha)=0. \tag{27}$$

A limited solution to problem (24), (25) is ([6])

$$R_s(r)=\sqrt{r}J_v(\mu_{s,n}r), \tag{28}$$

where $v=\frac{n+(m-2)}{2}$, $\mu=\mu_{s,n}^2$.

The solution to problem (26), (27) is the function

$$T_{s,n}(t)=\left(\exp\left(-\frac{\mu_{s,n}^2}{q+1}t^{q+1}\right)\right)\int_t^a a_{s,n}(\xi)\left(\exp\frac{\mu_{s,n}^2}{q+1}\xi^{q+1}\right)d\xi. \tag{29}$$

Substituting (28) in (23) we obtain

$$r^{-\frac{1}{2}}\tilde{f}_n^k(r,t)=\sum_{s=1}^{\infty}a_{s,n}(t)J_v(\mu_{s,n}r), \ r^{-\frac{1}{2}}\tilde{\varphi}_n^k(r)=\sum_{s=1}^{\infty}b_{s,n}J_v(\mu_{s,n}r), \ 0<r<1. \tag{30}$$

Series (30) are expansions in Fourier-Bessel series ([7]), if

$$a_{s,n}(t)=2[J_{v+1}(\mu_{s,n})]^{-2}\int_0^1\sqrt{\xi}\tilde{f}_n^k(\xi,t)J_v(\mu_{s,n}\xi)d\xi, \tag{31}$$

$$b_{s,n}=2[J_{v+1}(\mu_{s,n})]^{-2}\int_0^1\sqrt{\xi}\tilde{\varphi}_n^k(\xi)J_v(\mu_{s,n}\xi)d\xi, \tag{32}$$

where $\mu_{s,n}$, $s=1,2,...$ – the positive zeros of the Bassel functions $J_v(z)$, are arranged in increasing order of magnitude.

From (22), (28), (29) we obtain the solution to problem (18), (19)

$$\upsilon_{1n}^k(r,t)=\sum_{s=1}^{\infty}\sqrt{r}T_{s,n}(t)J_v(\mu_{s,n}r), \tag{33}$$

where $a_{s,n}(t)$ is determined from (31).

Further, substituting (22) into (20), (21), taking into account (23), we will have the problem

$$T_{st}+\mu_{s,n}^2t^qT_s=0, \ 0<t<\alpha, \ T_s(\alpha)=b_{s,n},$$

which decision is

$$T_{s,n}(t)=b_{s,n}\exp\frac{\mu_{s,n}^2}{q+1}(\alpha^{q+1}-t^{q+1}). \tag{34}$$

From (28), (34) we obtain

$$\upsilon_{2n}^k(r,t)=\sum_{s=1}^{\infty}b_{s,n}\sqrt{r}\exp\left(\frac{\mu_{s,n}^2}{q+1}(\alpha^{q+1}-t^{q+1})\right)J_v(\mu_{s,n}r), \tag{35}$$

where $b_{s,n}$ are from (32).

Therefore, first solving problem (9), (12) $(n=0)$, and then (10), (12) $(n=1)$, etc. we find successively all $\upsilon_n^k(r,t)$ of (17), where $\upsilon_{1n}^k(r,t)$, $\upsilon_{2n}^k(r,t)$ they are determined from (33), (35).

So, in the field $\mathbf{\Omega}_\beta$ takes place

$$\int_H \rho(\theta) L_1 u dH = 0. \quad (36)$$

Let $f(r,\theta,t) = R(r)p(\theta)T(t)$, at that $R(r) \in V_0, V_0 -$ be dense in $L_2((0,1)), \rho(\theta) \in C^\infty(H)$, dense in $L_2(H)$, and $T(t) \in V_1, V_1 -$ dense in $L_2((0,\alpha))$. Then it is $f(r,\theta,t) \in V, V = V_0 \otimes H \otimes V_1 -$ dense in $L_2(\mathbf{\Omega}_\alpha)$ ([8]).

From this and (36) it follows that

$$\int_{\mathbf{\Omega}_\alpha} f(r,\theta,t) L_1 u d\mathbf{\Omega}_\alpha = 0$$

and

$$L_1 u = 0, \forall (r,\theta,t) \in \mathbf{\Omega}_\alpha .$$

Thus, the solution to problem (1), (2) in the domain $\mathbf{\Omega}_\alpha$ is the function

$$u(r,\theta,t) = \sum_{n=0}^{\infty}\sum_{k=1}^{k_n}\left\{\psi_{2n}^k(t) + r^{\frac{(1-m)}{2}}\left[\upsilon_{1n}^k(r,t) + \upsilon_{2n}^k(r,t)\right]\right\} Y_{n,m}^k(\theta), \quad (37)$$

where $\upsilon_{1n}^k(r,t)$, $\upsilon_{2n}^k(r,t)$ are determined from (33), (35).

Given the formula ([7]), $2J_\nu'(z) = J_{\nu-1}(z) - J_{\nu+1}(z)$ estimates ([9, 5])

$$J_\nu(z) = \sqrt{\frac{2}{\pi z}} \cos\left(z - \frac{\pi}{2}\nu - \frac{\pi}{4}\right) + 0\left(\frac{1}{z^{3/2}}\right), \ \nu \geq 0,$$

$$|k_n| \leq c_1 n^{m-2}, \ \left|\frac{\partial^q}{\partial \theta_j^q} Y_{n,m}^k(\theta)\right| \leq c_2 n^{\frac{m}{2}-1+q}, \ \ j = \overline{1, m-1}, \ q = 0,1,..., \quad (38)$$

as well as lemmas, restrictions on the coefficients of equation (1) and on given functions as $\psi_1(t,\theta)$, $\varphi(r,\theta)$, in [10], we can prove that the resulting solution (37) belongs to the class $C(\overline{\mathbf{\Omega}}_\alpha) \cap C^1(\mathbf{\Omega}_\alpha \cup S) \cap C^2(\mathbf{\Omega}_\alpha)$.

Further, from (33), (35), (37) for $t \to +0$ we have

$$u(r,\theta,0) = \tau(r,\theta) = \sum_{n=0}^{\infty}\sum_{k=1}^{k_n} \tau_n^k(r) Y_{n,m}^k(\theta),$$

$$\tau_n^k(r) = \psi_{1n}^k(0) + \sum_{s-1}^{\infty} r^{\frac{(2-m)}{2}}\left[\int_0^\alpha a_{s,n}(\xi)\left(\exp\frac{\mu_{s,n}^2}{q+1}\xi^{q+1}\right)d\xi + b_{s,n}\left(\exp\frac{\mu_{s,n}^2}{q+1}\alpha^{q+1}\right)\right] J_{n+\frac{(m-2)}{2}}(\mu_{s,n} r), \quad (39)$$

$$u_t(r,\theta,0) = \nu(r,\theta) = \sum_{n=0}^{\infty}\sum_{k=1}^{k_n} \nu_n^k(r) Y_{n,m}^k(\theta), \ (40)$$

$$\nu_n^k(r) = \psi_{1nt}^k(0) - \sum_{s=1}^{\infty} r^{\frac{(2-m)}{2}} a_{s,n}(0) \ J_{n+\frac{(m-2)}{2}}(\mu_{s,n} r).$$

From (31) - (33), (38), as well as the lemmas, it follows that $\tau(r,\theta)$, $\nu(r,\theta) \in W_2^l(S)$, $l > \frac{3m}{2}$.

Thus, taking into account the boundary conditions (3), (39), (40) in the domain $\mathbf{\Omega}_\beta$ we arrive at the mixed problem for degenerate hyperbolic equations

$$L_2 u = |t|^p \Delta_x u - u_{tt} + \sum_{i=1}^m a_i(r,\theta,t) u_{x_i} + b(r,\theta,t) u_t + c(r,\theta,t) u = 0 \quad (41)$$

with data

$$u\big|_S = \tau(r,\theta),\ u_t\big|_S = \nu(r,\theta),\ u\big|_{\Gamma_\beta} = \psi_2(t,\theta). \tag{42}$$

The following theorem was proved in [4]

Theorem 2. If $\tau(r,\theta), \nu(r,\theta) \in W_2^l(S),\ \psi_2(t,\theta) \in W_2^l(\Gamma_\beta),\ l > \frac{3m}{2}$, then problem (41), (42) has a unique solution if condition (5) is satisfied.

Further, using Theorem 2, we arrive at the solvability of Problem 1.

item 4. Uniqueness of the solution to Problem 1. First we consider the problem (1), (2) in the domain $\mathbf{\Omega}_\alpha$ and prove its uniqueness to the solution. For this, we construct a solution to the first boundary value problem for the equation

$$L_1^* \upsilon = t^q \Delta_x \upsilon - \upsilon_t - \sum_{i=1}^m d_i \upsilon_{x_i} + d\upsilon = 0, \ (6^*)$$

with data ξ

$$\upsilon\big|_S = \tau(r,\theta) = \sum_{n=0}^{\infty} \sum_{k=1}^{k_n} \overline{\tau}_n^k(r) Y_{n,m}^k(\theta),\ \upsilon\big|_{\Gamma_\alpha} = 0, \tag{43}$$

where $d(x,t) = e - \sum_{i=1}^m d_{ix_i}, \overline{\tau}_n^k(r) \in G,\ G-$ many functions $\tau(r)$ from the class $C([0,1]) \cap C^1((0,1))$. Scores G are dense everywhere in $L_2((0,1))([8])$. Solution of the problem (6 *). (43) we will search in the form (7), where the functions $\overline{\upsilon}_n^k(r,t)$ will be defined below. Then, similarly to item 2, the functions $\overline{\upsilon}_n^k(r,t)$ satisfy a system of equations of the form (9) - (10), where, respectively, $\widetilde{d}_{in}^k, d_{in}^k$ are replaced to $-\widetilde{d}_{in}^k, -d_{in}^k$, and $\widetilde{e}_n^k$ to $\widetilde{d}_n^k,\ i = 1,...,m,\ k = \overline{1,k_n},\ n = 0,1,...$.

Further, from the boundary condition (43), by virtue of (7), we arrive at the following problem

$$L_1 \upsilon_n^k = t^q \left(\upsilon_{nrr}^k + \frac{\overline{\lambda}_n}{r^2} \upsilon_n^k \right) + \upsilon_{nt}^k = \widetilde{f}_n^k(r,t) \tag{44}$$

$$\upsilon_n^k(r,0) = \tau_n^k(r),\ \upsilon_n^k(1,t) = 0, \tag{45}$$

$$\upsilon_n^k(r,t) = r^{\frac{(m-1)}{2}} \overline{\upsilon}_n^k(r,t),\ \widetilde{f}_n^k(r,t) = r^{\frac{(m-1)}{2}} \widetilde{\overline{f}}_n^k(r,t),\ \tau_n^k(r) = r^{\frac{(m-1)}{2}} \overline{\tau}_n^k(r).$$

The solution to problem (44), (45) will be sought in the form of (17), where $\upsilon_{1n}^k(r,t)$ is the solution to the problem for equation (18) with data

$$\upsilon_{1n}^k(r,0) = 0, \upsilon_{1n}^k(1,t) = 0, \tag{46}$$

a $\upsilon_{2n}^k(r,t)-$ solution of the problem for equation (20) with condition

$$\upsilon_{2n}^k(r,0) = \tau_n^k(r), \upsilon_{2n}^k(1,t) = 0, \tag{47}$$

The solutions to problem (18), (46) and (20), (47) respectively have the form

$$\upsilon_{1n}^k(r,t) = \sum_{s=1}^{\infty} \sqrt{r} \left[\left(\exp\left(\frac{\mu_{s,n}^2}{q+1} t^{q+1} \right) \right) \int_0^t a_{s,n}^k(\xi) \left(\exp\left(-\frac{\mu_{s,n}^2}{q+1} \xi^{q+1} \right) \right) d\xi \right] J_\nu(\mu_{s,n} r),$$

$$\upsilon_{2n}^k(r,t) = \sum_{s=1}^{\infty} \tau_{s,n}^k \sqrt{r} \left(\exp\left(\frac{\mu_{s,n}^2}{q+1} t^{q+1} \right) \right) J_\nu(\mu_{s,n} r),$$

where

$$\tau_{s,n}^{k} = 2\left[J_{\nu+1}(\mu_{s,n})\right]^{-2}\int_{0}^{1}\sqrt{\xi}\,\tau_{n}^{k}(\xi)J_{\nu}(\mu_{s,n}\xi)d\xi,\ \nu = n+\frac{(m-2)}{2}$$

Thus, the solution of problem (6*), (43) in the form of a series

$$\upsilon(r,\theta,t) = \sum_{n=0}^{\infty}\sum_{k=1}^{k_n} r^{-\frac{(m-1)}{2}}[\upsilon_{1n}^{k}(r,t)+\upsilon_{2n}^{k}(r,t)]J_{n,m}^{k}(\theta),$$

constructed, which, by virtue of estimates (38), belongs to the class $C(\overline{\mathbf{\Omega}}_{\alpha})\cap C^{1}(\overline{\mathbf{\Omega}}_{\alpha}\cap S)\cap C^{2}(\overline{\mathbf{\Omega}}_{\alpha})$.

$$\upsilon L_1 u - uL_1^{*}\upsilon = -\upsilon P(u) + uP(\upsilon) - u\upsilon Q,$$

where

$$P(u) = t^{q}\sum_{i=1}^{m}u_{x_i}\cos(N^{\perp},x_i),\ Q = \cos(N^{\perp},t) - \sum_{i=1}^{m}d_i\cos(N^{\perp},x_i),$$

And $N^{\perp}$ is the internal normal to the boundary $\partial\mathbf{\Omega}_{\alpha}$, according to Green's formula, we obtain

$$\int_{S}\tau(r,\theta)u(r,\theta,0)ds = 0. \tag{48}$$

Since the linear span of a system of functions $\left\{\overline{\tau}_{n}^{k}(r)J_{n,m}^{k}(\theta)\right\}$ is dense in $L_2(S)$ ([8]), we conclude from (48) that $u(r,\theta,0) = 0,\ \forall(r,\theta)\in S$. Therefore, by the extremum principle for parabolic equation (6) [12] $u \equiv 0$ in $\overline{\mathbf{\Omega}}_{\alpha}$.

It follows that $u_t(r,\theta,0) = \nu(r,\theta) = 0,\ \forall(r,\theta)\in S$.

Thus, we have arrived at the homogeneous mixed problem (41), (42), which, by virtue of Theorem 2, has a trivial solution.

Consequently, the uniqueness of the solution to Problem 1 is proved.

The theorem is completely proved.

Since an explicit form of the solution to problem (41), (42) was obtained in [4], it is possible to write an explicit representation for problem 1 as well.

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БІР КЛАСТАҒЫ АЗҒЫНДАЛҒАН КӨП ӨЛШЕМДІ ГИПЕРБОЛА-ПАРАБОЛАЛЫҚ ТЕҢДЕУЛЕР ҮШІН АРАЛАС ЕСЕПТЕРДІҢ КОРРЕКТІЛІГІ

Аңдатпа. Кеңістіктегі электромагниттік жазықтың математикалық моделдерін зерттегенде, электромагниттік процесстің негізі оның қасиеттерімен анықталады. Егер орта өткізбейтін болса, онда азғындалған көп өлшемді гиперболалық теңдеулерге келеміз. Егер де орта көп өткізгішті болса, онда азғындалған көп өлшемді параболалық теңдеулерге келтірілді.

Сондықтан, күрделі орталарда (мысалы, өткізетін ортада өзгермелі делік) электромагниттік жазықтықтарды зерттегенде біз азғындалған көп өлшемді гиперболалық-параболалық теңдеулерге келеміз.

Сонымен қатар, Гамильтон қағидасына сәйкес кеңістіктегі серпімді мембрана тербелістерінің азғындалған көп өлшемді гиперболалық теңдеулермен модельдеу мүмкін екендігі белгілі.

Массамен толтырылған ортада жылу тарату процесін зерттеу азғындалған көп өлшемді параболалық теңдеулерге алып келеді.

Сонымен серпімді мембранадағы жылу тарату процесінің математикалық модельдеуін зерттей отырып, азғындалған көп өлшемді гипербола-параболалық теңдеулерге келеміз. Осы қосымшаларды оқып үйрену кезінде зерттелген мәселелердің шешімдерін нақты түрде көрсету керектігі туындайды.

Азғындалған көп өлшемді гиперболалық теңдеулер үшін аралас есептер бұған дейін қарастырылды.

Азғындалған көп өлшемді гипербола-параболалық теңдеулер үшін мұндай есептердің нақты шешімі табылмағандығы белгілі.

Мақалада бірмәнді шешімділік көрсетілген және бір кластағы азғындалған көп өлшемді гипербола-параболалық теңдеулер үшін аралас есептің нақты классикалық шешімі келтірілген.

Түйін сөздер: аралас есеп, классикалық шешім, бірмәнді шешімділік, Бессель функциясы, сфералық функциялар.

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КОРРЕКТНОСТЬ СМЕШАННОЙ ЗАДАЧИ ДЛЯ ОДНОГО КЛАССА ВЫРОЖДАЮЩИХСЯ МНОГОМЕРНЫХ ГИПЕРБОЛО-ПАРАБОЛИЧЕСКИХ УРАВНЕНИЙ

Аннотация. Известно, что при математическом моделировании электромагнитных полей в пространстве, характер электромагнитного процесса определяется свойствами среды. Если среда непроводящая, то получаем вырождающихся многомерные гиперболические уравнения. Если же среда обладает большой проводимостью, то приходим к вырождающимся многомерным параболическим уравнениям.

Следовательно, анализ электромагнитных полей в сложных средах (например, если проводимость среды меняется) сводятся к вырождающимся многомерным гиперболо- параболическим уравнениям.

Известно, также что колебания упругих мембран в пространстве по принципу Гамильтона можно моделировать вырождающимися многомерными гиперболическими уравнениями.

Изучение процесса распространения тепла в среде, заполненной массой, приводят к вырождающимся многомерным параболическим уравнениям.

Следовательно, исследуя математическое моделирование процесса распространения тепла в колеблющихся упругих мембранах, также приходим к вырождающимся многомерным гиперболо- параболическим уравнениям. При изучении этих приложений, возникает необходимость получения явного представления решений исследуемых задач.

Смешанные задачи для вырождающихся многомерных гиперболических уравнений ранее рассмотрены.

Насколько известно, эти задачи для вырождающихся многомерных гиперболо- параболических уравнений не изучены.

В данной работе показана однозначная разрешимость и получен явный вид классического решения смешанной задачи для одного класса вырождающихся многомерных гиперболо- параболических уравнений.

Ключевые слова: смешанная задача, классическое решение, однозначная разрешимость, функций Бесселя, сферические функций.

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