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APPROXIMATION OF PROBLEM FOR FINDING THE BOUNDED SOLUTION TO SYSTEM OF NONLINEAR LOADED DIFFERENTIAL EQUATIONS

Abstract. On the whole axis the system of nonlinear loaded differential equations is considered. The questions of existence and approximation bounded solution to the system are studied. The definition of «limit as $t \to \pm\infty$» solution to the system of nonlinear loaded differential equations is introduced. Sufficient conditions for the existence of bounded solution to the system of nonlinear loaded differential equations and convergence of the function sequence composed by the bounded solutions to the linearized system of loaded differential equations are obtained. Regular nonlinear two-point boundary value problem for the system of nonlinear loaded differential equations on the finite interval is constructed, which approximate the problem of finding bounded solutions to the original system of loaded differential equations. It is given an estimate of the difference between the solution to initial singular problem and the solution to the approximating regular two-point boundary value problem.
Keywords: singular problem, nonlinear loaded differential equation, bounded solution, approximation.

Questions of existence and construction of approximate methods for finding of nonlinear ordinary differential equations, restricted on the whole axis, are considered by many authors [1-11]. Various problems for loaded differential equations and methods for their solutions are studied in [12-17].

In this article, nonlinear loaded differential equations is considered on $R = (-\infty, \infty)$

$$\frac{dx}{dt} = f(t,x) + f_0(t, x(\theta_{-m}), x(\theta_{-m+1}), \ldots, x(\theta_m)), \quad x \in R^n, \quad \|x\| = \max |x_i|, \tag{1}$$

where $f: R^{n+1} \to R^n$, $f_0 : R^{2n+2} \to R^n$ are continuous, $\theta_{-m} < \theta_{-m+1} < \ldots < \theta_0 = 0 < \theta_1 < \ldots < \theta_m$.

The aim of this research is to find conditions for the existence of system solutions of nonlinear loaded differential equations restricted on the whole axis (1) and the construction of the regular two-point boundary value problems on a finite interval, which allows determining the narrowing of the decision on the final interval with given accuracy.

In work [11] there was introduced the definition of "limit at $t \to \infty$" solution of nonlinear ordinary differential equations, and proved that, if the system is linearized along such solution is exponentially dichotomous on semi-axis, the "limit at $t \to \infty$" decision has the attractive property. This result allowed building approximate two-point boundary value problems on a finite interval for the singular boundary value problems for nonlinear ordinary differential equations on the whole axis. Methods and results [11] are used to find conditions for the existence of the equation solutions restricted on the whole axis (1) and for the construction of approximating regular boundary value problems on a finite interval.

The following symbols are used:

$\widetilde{C}(J, R^n)$ – space of continuous and restricted on $J \subseteq R$ functions $x: J \to R^n$ with norm $\|x\|_1 = \sup\limits_{t\in R}\|x(t)\|$;

$C(J,R^n)$ – set of continuous on J functions;

$S(x_0(t),J,r)=\{x(t)\in C(J,R^n):(x(t)-x_0(t))\in\widetilde{C}(J,R^n),\|x-x_0\|_1<r\}$, где $x_0(t)\in C(J,R^n)$;

$$G(x_0(t),J,r)=\{(t,x):t\in J,\ \|x-x_0(t)\|<r\};$$

$$G_0(x_0(t),J,r)=\{(t,v_{-m},\dots v_m):t\in J,\ \|v_k-x_0(\theta_k)\|<r,\ k=\overline{-m,m}\}.$$

We take a continuously differentiable on R function $x_0(t)$ so that

$$\left(\frac{d}{dt}x_0(t)-f(t,x_0(t))-f_0(t,x_0(\theta_{-m}),x_0(\theta_{-m+1}),\dots,x_0(\theta_m))\right)\in\widetilde{C}(R,R^n) \tag{2}$$

Restricted on R solution of the system of loaded differential equations (1) is defined as the limit of a sequence of functions compiled using linearized solutions of systems of loaded differential equations restricted on the whole axis. Therefore, we consider the linear loaded ordinary differential equation

$$\frac{dx}{dt}=A(t)x+\sum_{j=-m}^{m}A_j(t)x(\theta_j)+f(t),\quad x\in R^n,\quad t\in R, \tag{3}$$

where matrix $A(t)$, $A_j(t)$ $(j=\overline{-m,m})$ and vector-function $f(t)$ are continuous and restricted on R. Restricted solution of equation (3) is called solution of problem 1.

Definition 1. Task 1 is called correctly solvable if for any continuous and restricted on R function $f(t)\in C(R,R^n)$ equation (3) has only one restricted on R solution $x^*(t)$ and $\|x^*\|_1\le\gamma\|f\|_1$, inequality is carried out, where γ does not depend on $f(t)$.

Definition 2. Continuously differentiable on R function $x_0(t)$ is called limit at $t\to\mp\infty$ by equation solution (1), if

$$\lim_{t\to\mp\infty}\|\dot{x}_0(t)-f(t,x_0(t))-f_0(t,x_0(\theta_{-m}),x_0(\theta_{-m+1}),\dots,x_0(\theta_m))\|=0.$$

The following conditions should be carried out:

(A). Function $f(t,x)$ is continuous and has uniformly continuous derivatives $\frac{\partial}{\partial x}f(t,x)$ in $G(x_0(t),R,r)$, where $x_0(t)$ – limit at $t\to\mp\infty$ equation solution (1), and the following limit relations are correct

$$\lim_{t\to-\infty}f(t,x)=f_-(x),\quad \lim_{t\to+\infty}f(t,x)=f_+(x) \tag{4}$$

$$\lim_{t\to-\infty}x_0(t)=x_-,\quad \lim_{t\to\infty}x_0(t)=x_+, \tag{5}$$

where x_-, x_+ are the solutions of systems of nonlinear equations $f_-(x)=0$, $f_+(x)=0$, respectively.

(B). Function $f_0(t,v_{-m},\dots v_m)$ is continuous and has uniformly continuous derivatives $\frac{\partial}{\partial v_k}f_0(t,v_{-m},\dots v_m)$ $(k=\overline{-m,m})$ in $G_0(x_0(t),R,r)$ and for all $(t,v_{-m},\dots,v_m)\in G_0(x_0(t),R,r)$ has relation point

$$\sup_{t\in(-\infty,-T]}\|f_0(t,v_{-m},\dots,v_m)\|\le\delta_0(-T),\quad \sup_{t\in[T,\infty)}\|f_0(t,v_{-m},\dots,v_m)\|\le\delta_0(T),$$

$$\lim_{T\to\mp\infty}\delta_0(T)=0,\quad \lim_{t\to\mp\infty}\left\|\frac{\partial}{\partial v_k}f_0(t,v_{-m},\dots,v_m)\right\|=0,\quad k=\overline{-m,m}.$$

(C). Task 1 for the linearized loaded differential equation

$$\frac{dy}{dt}=\frac{\partial}{\partial x}f(t,x_0(t))y+\sum_{k=-m}^{m}\left(\frac{\partial}{\partial v_k}f_0(t,v_{-m},\dots,v_m)\Bigg|_{\substack{v_{-m}=x_0(\theta_{-m})\\ \dots \\ v_m=x_0(\theta_m)}}\cdot J_{\theta_k}\right)y,\quad y\in R^n,\qquad (6)$$

can be correctly solved, where $J_{\theta_k}y(t)=y(\theta_k),\ k=\overline{-m,m}$.

(B). Functions $f_-(x)$, $f_+(x)$ in $S(x_-,r)$, $S(x_+,r)$ respectively, have derivatives $f'_-(x)$, $f'_+(x)$ and uniformly relative x is correct limit relations

$$\lim_{t\to-\infty}\frac{\partial}{\partial x}f(t,x)=f'_-(x),\qquad x\in S(x_-,r),$$

$$\lim_{t\to+\infty}\frac{\partial}{\partial x}f(t,x)=f'_+(x),\qquad x\in S(x_+,r),$$

and $f'_\mp(x_\mp)=A_{(\mp)}$, $Re\xi_j^\mp\neq 0$, where $\xi_j^\mp$ - the eigenvalues of $A_{(\mp)}$, $j=\overline{1,n}$.

Theorem 1. Functions $f(t,x)$ and $f_0(t,v_{-m},\dots v_m)$ are continuous and have uniformly continuous derivatives $\frac{\partial}{\partial x}f(t,x)$ and $\frac{\partial}{\partial v_k}f_0(t,v_{-m},\dots v_m)$ respectively in $G(x_0(t),R,r)$ and $G_0(x_0(t),R,r)$. At any $\hat{x}(t)\in S(x_0(t),R,r)$ task 1 for linearized loaded differential equation

$$\frac{dy}{dt}=\frac{\partial}{\partial x}f(t,\hat{x}(t))y+\sum_{k=-m}^{m}\left(\frac{\partial}{\partial v_k}f_0(t,v_{-m},\dots,v_m)\Bigg|_{\substack{v_{-m}=\hat{x}(\theta_{-m})\\ \dots \\ v_m=\hat{x}(\theta_m)}}\cdot J_{\theta_k}\right)y+\hat{f}(t),\quad y\in R^n,\quad t\in R,\qquad (7)$$

can be correctly solved with constant γ.

Then, at inequality solutions

$$\gamma\,\|\,\dot{x}_0(t)-f(t,x_0(t))-f_0(t,x_0(\theta_{-m}),x_0(\theta_{-m+1}),\dots,x_0(\theta_m))\,\|_1<r$$

there is a such number $\alpha\geq 1$, the sequence of continuously differentiable on R functions

$$x_{n+1}(t)=x_n(t)+\Delta x_n(t),\qquad n=0,1,\dots,\qquad (8)$$

where $\Delta x_n(t)$ – restricted on R solution of linear equation

$$\frac{dy}{dt}=\frac{\partial}{\partial x}f(t,x_n(t))y+\sum_{k=-m}^{m}\left(\frac{\partial}{\partial v_k}f_0(t,v_{-m},\dots,v_m)\Bigg|_{\substack{v_{-m}=x_n(\theta_{-m})\\ \dots \\ v_m=x_n(\theta_m)}}\cdot J_{\theta_k}\right)y-$$

$$-\frac{1}{\alpha}\left(\frac{d}{dt}x_n(t)-f(t,x_n(t))-f_0(t,x_n(\theta_{-m}),\dots,x_n(\theta_m))\right),\quad y\in R^n,\quad t\in R,\qquad (9)$$

by norm $\widetilde{C}(R,R^n)$ is consistent to $x^*(t)$ of the equation solution (1) in $S(x_0(t),R,r)$.

Corroboration. In equation (1) we substitute $u=x-x_0(t)$, then have

$$\frac{du}{dt}=f(t,u+x_0(t))+f_0(t,u(\theta_{-m})+x_0(\theta_{-m}),\dots,u(\theta_m)+x_0(\theta_m))-\frac{d}{dt}x_0(t),\quad u\in R^n,\quad t\in R,\qquad (10)$$

The problem of finding the equation solution (10), belonging to the ball $S(0,R,r)\subset\widetilde{C}(R,R^n)$ may be written as the operational equation

$$A(u) \equiv Hu + F(u) = 0, \quad u \in S(0,R,r),$$

where $H = \frac{d}{dt}$, $F(u) = -f(t,u(t)+x_0(t)) - f_0(t,u(\theta_{-m})+x_0(\theta_{-m}),\dots,u(\theta_m)+x_0(\theta_m)) + \frac{d}{dt}x_0(t)$.

Taking into account, that the correct solvability of a constant γ of the task 1 for equation (7) provide the assessment $\| (H+F'(u))^{-1} \|_{L(Y,X)} \le \gamma$ at all $u \in S(0,R,r)$, and relations (8), (9) are equivalent to iterative process (1.18) [11, p. 18], on the basis of the theorem 5 [11, c. 18] we obtain theorem conclusion.

Further, the questions of approximation of the nonlinear loaded equation solutions restricted on the whole axis (1) by the solutions of the regular boundary value problems on a finite interval. For this purpose, a nonlinear two-point boundary value problem for the loaded differential equation is considered

$$\frac{dx}{dt} = f(t,x) + f_0(t, x(\theta_{-m}), x(\theta_{-m+1}),\dots,x(\theta_m)), \quad t \in [-T,T], \quad x \in R^n, \tag{11}$$

$$P_1 S_- f_-(x(-T)) + P_2 S_+ f_+(x(T)) = 0. \tag{12}$$

where S_-, S_+ – real-valued nonspecial $(n \times n)$-matrices, resulting matrix $f'_-(x_-)$, $f'_+(x_+)$ to a generalized Jordan form.

$$\tilde{A}_- = S_- f'_-(x_-) S_-^{-1} = \begin{pmatrix} \tilde{A}_{-,11} & 0 \\ 0 & \tilde{A}_{-,22} \end{pmatrix}, \quad \tilde{A}_+ = S_+ f'_+(x_+) S_+^{-1} = \begin{pmatrix} \tilde{A}_{+,11} & 0 \\ 0 & \tilde{A}_{+,22} \end{pmatrix},$$

where $\tilde{A}_{\mp,11}$ and $\tilde{A}_{\mp,22}$ consist of generalized Jordan cells corresponding to the eigenvalues of the matrices $f'_{\mp}(x_{\mp})$ with negative and positive real parts, which number is $n_1^{\mp}$ and $n_2^{\mp}$ respectively. We introduce $(n \times n)$- matrices $P_1 = \begin{pmatrix} I_{n_1^-} & 0 \\ 0 & 0 \end{pmatrix}$, $P_2 = \begin{pmatrix} 0 & 0 \\ 0 & I_{n_2^+} \end{pmatrix}$, where $I_{n_1^-}$, $I_{n_2^+}$ – identity matrices of dimension n_1^-, n_2^+ respectively.

Restriction of solution $x^*(t)$ of the equation restricted on R (1) on interval $[-T,T]$ is denoted through $x_T^*(t)$ and the $S(x_T^*(t),[-T,T],\rho^*) = \{x(t) \in C([-T,T],\mathrm{R}^n) : \| x - x_T^* \|_{0,T} < \rho^*\}$ functional ball is introduced.

Theorem 2. The conditions (A)-(D) and $x^*(t) \in S(x^{(0)}(t),r)$ should be carried out – solution of the nonlinear differential equations loaded restricted on R (1). There are such numbers $T_0 > 0$, $\rho^* > 0$ so that for all $T \ge T_0$ regular two-point boundary value problem (11), (12) in $S(x_T^*(t),[-T,T],\rho^*)$ has only one solution $x_T(t)$, and the following assessment is correct

$$\|x_T - x^*\|_{0,T} \le 2\gamma\{\|\mathrm{S}_-\| \cdot (\|f_-(x^*(-T))\| + \delta_0(-T)) + \|\mathrm{S}_+\| \cdot (\|f_+(x^*(T))\| + \delta_0(T))\}.$$

Corroboration. Nonlinear two-point boundary value problem (11), (12) may be written as the operational equation

$$Ax \equiv Hx + F(x) = 0, \tag{13}$$

where

$$H = \begin{pmatrix} \frac{d}{dt} \\ 0 \end{pmatrix}, \quad F(x) = \begin{pmatrix} -f(t,x(t)) - f_0(t,x(\theta_{-m}),x(\theta_{-m+1}),\dots,x(\theta_0),\dots,x(\theta_m)) \\ P_1 S_- f_-(x(-T)) + P_2 S_+ f_+(x(T)) \end{pmatrix}.$$

A operator represents a Banach space $X = C([-T,T],R^n)$ with norm $\| x \|_{0,T} = \max_{t \in [-T,T]} \| x(t) \|$ into the Banach space $Y = \tilde{C}([-T,T],R^n)\dot{+}R^n$ with norm $\| y\|_Y = \max\{\| f\|_{0,T}, \| d\|\}$.

The condition of the theorem implies the existence of $\rho_0 > 0$ such, that $S(x^*(t),\rho_0)\subset S(x_0(t),r)$, and functions $f(t,x)$, $f_0(t,v_{-m},\ldots,v_m)$, $f_-(x)$, $f_+(x)$ have uniformly continuous derivatives: $f'_x(t,x)$, $\frac{\partial}{\partial v_k}f_0(t,v_{-m},\ldots,v_m)$ $(k=\overline{-m,m})$, $f'_-(x)$, $f'_+(x)$ in the respective sets. This implies the existence and uniform continuous Frechet derivatives $F'(x)$ in $S(x_T^*(t),[-T,T],\rho^*)$.

According to the condition (C), the task 1 for the linearized equation (7) is correctly solved. Then theorem 1 implies the existence of $T_1 > 0$ so that regular linear two-point boundary value problem is

$$\frac{dz}{dt} = \frac{\partial}{\partial x}f(t,x^*(t))z + \sum_{k=-m}^{m}\left(\frac{\partial}{\partial v_k}f_0(t,v_{-m},\ldots,v_m)\Big|_{\substack{v_{-m}=x^*(\theta_{-m})\\ \ldots \\ v_m=x^*(\theta_m)}}\cdot J_{\theta_k}\right)z+\varphi(t),\quad t\in[-T,T],\quad z\in \mathrm{R}^n,$$

$$P_1S_-f'_-(x^*(-T))\cdot z(-T)+P_2S_+f'_+(x^*(T))\cdot z(T)=\tilde{d},\quad \tilde{d}\in \mathrm{R}^n,$$

for all $T > T_1$ is correctly solved with constant K_1 that does not depend on T. This is equivalent to the invertibility of the linear operator $H+F'(x_T^*):X\to Y$ and the solution of the inequality $\|(H+F'(x_T^*))^{-1}\|_{L(Y,X)}\le K_1$.

As x_0 we consider function $x_T^*(t)$ and use theorm 6 to the operational equation (13) [11, p. 18].

The first condition of the theorem is carried out with $\gamma_0 = K_1$.

We consider the number $\varepsilon = \frac{1}{2K_1}$ and, by reason of the uniform continuity of the Frechet derivative, choose $\rho_* \in (0,\rho_0]$ so that the following inequality is solved

$$\| F'(x)-F'(x_0)\|_{L(X,Y)}\le \varepsilon = \frac{1}{2K_1}.$$

Then

$$\varepsilon\gamma_0 = \frac{1}{2K_1}\cdot K_1 = \frac{1}{2} < 1$$

and

$$\| Hx_0 + F(x_0)\|_Y = \| P_1S_-f_-(x^*(-T)) + P_2S_+f_+(x^*(T))\|$$

(the function $x^*(t)$ satisfies the differential equation (1) at all $t\in \mathrm{R}$ is taken into account), $\lim_{t\to\mp\infty} x^*(t) = \hat{x}_\mp$, $f_-(\hat{x}_-) = 0$, $f_+(\hat{x}_+) = 0$, we choose $T_0 > T_1$ so that

$$2K_1\| Hx_0 + F(x_0)\|_Y < \rho_*.$$

All the conditions of the theorem 6 [11, p. 18] are implemented and conclusions are made.

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СЫЗЫҚСЫЗ ЖҮКТЕЛГЕН ДИФФЕРЕНЦИАЛДЫҚ ТЕҢДЕУЛЕР ЖҮЙЕСІНІҢ БҮКІЛ ӨСТЕ ШЕКТЕЛГЕН ШЕШІМІН ТАБУ ЕСЕБІНІҢ АППРОКСИМАЦИЯСЫ

Аннотация. Сызықсыз жүктелген дифференциалдық теңдеулер жүйесі бүкіл өсте қарастырылады. Қарастырылып отырған теңдеулер жүйесінің шектелген шешімінің бар болуы мен оны аппроксимациялау мәселелері зерттеледі. Сызықсыз жүктелген дифференциалдық теңдеулер жүйесінің «$t \to \pm\infty$ болғандағы шекті» шешімінің анықтамасы енгізіледі. Сызықсыз жүктелген дифференциалдық теңдеулер жүйесінің шектелген шешімінің бар болуының және сызықтандырылған жүктелген дифференциалдық теңдеулер жүйесінің шектелген шешімдері көмегімен құрылған функциялар тізбегінің осы шешімге жинақтылығының жеткілікті шарттары алынған. Жүктелген дифференциалдық теңдеулердің бастапқы жүйесінің шектелген шешімін табу есебін аппроксимациялайтын ақырлы аралықтағы сызықсыз жүктелген дифференциалдық теңдеулер жүйесі үшін регулярлы сызықсыз екінүктелі шеттік есеп тұрғызылған. Бастапқы сингулярлы есептің шешімі мен аппроксимациялаушы регулярлы екінүктелі шеттік есептің шешімінің арасындағы айырманың бағалауы тағайындалған.

Түйін сөздер: сингулярлы есеп, сызықсыз жүктелген дифференциалдық теңдеу, шектелген шешім, аппроксимациялау.
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АППРОКСИМАЦИЯ ЗАДАЧИ НАХОЖДЕНИЯ ОГРАНИЧЕННОГО РЕШЕНИЯ СИСТЕМЫ НЕЛИНЕЙНЫХ НАГРУЖЕННЫХ ДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ

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Аннотация. На всей оси рассматривается система нелинейных нагруженных дифференциальных уравнений. Исследуются вопросы существования и аппроксимации ограниченного решения рассматриваемой системы уравнений. Вводится определение «предельного при $t \to \pm\infty$» решения системы нелинейных нагруженных дифференциальных уравнений. Получены достаточные условия существования ограниченного решения системы нелинейных нагруженных дифференциальных уравнений и сходимости к нему последовательности функций, составленной с помощью ограниченных решений линеаризованной системы нагруженных дифференциальных уравнений. Построена регулярная нелинейная двухточечная краевая задача для системы нелинейных нагруженных дифференциальных уравнений на конечном интервале, аппроксимирующая задачу нахождения ограниченного решения исходной системы нагруженных дифференциальных уравнений. Установлена оценка разности между решением исходной сингулярной задачи и решением аппроксимирующей регулярной двухточечной краевой задачей.

Ключевые слова: сингулярная задача, нелинейное нагруженное дифференциальное уравнение, ограниченное решение, аппроксимация.