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# MULTIDIMENSIONAL PROBLEMS OF SOILS' CONSOLIDATION WITH MODULUS OF DEFORMATION, VARIABLE IN ITS DEPTH

**Abstract.** In unconsolidated condition, the tension applied to the soil is perceived both by a skeleton of soil and pore fluid. Wherein speaking about the one-dimensional case the pressure on the elementary area consists of the tension on the soil's skeleton and the pressure in the water. Sometimes the tension in the soil's skeleton is called effective, the pressure in the water is called neutral, and the unit pressure applied on the whole soil's surface is called total.

All problems in this article are directed to the definition of neutral pressure. Subsequent values are calculated easily. Consequently, it is necessary to find the pore fluid pressure. This pressure must be figured out in order to determine it.

The filtration theory of consolidation states that compression curve is taken as the main rheological equation of soil's condition, and phases' interaction is described by the equation of equilibrium, according to which the unit pressure, applied toward the soil is made up of an efficient and neutral pressure. And here the air movement obeys the law of Boyle-Mariotte, which explains the relationship between pressure and volume, and the gas, dissolved in water, obeys the law of Henry. This model allows us to simplify the mathematical formulation of the problem and makes it easier to determine the solution of soil consolidation problems.

The compaction of soil is mainly determined by its compressibility. Compressibility of the base depends on the soil type and the nature of the tension. The phenomenon of the soil's compressibility is very important in the design of engineering structures on a consolidated basis. Wherein this soil compression deformation occurs mainly due to the convergence of solid particles together and is evaluated by the change in porosity coefficient when the compressive pressures in the soil skeleton are changed. The determining of the relationship between porosity coefficient and compressive tensions in the soil skeleton is usually performed in the laboratory in compression devices.

Below we consider the process of compaction of heterogeneous saturated soil's layer which has capacity h, and which lies beneath a sandy pillow. At the initial time ($t=\tau_1$) the soil's layer immediately gets a distributed tension with the intensity q (z, t). Then the mathematical formulation of this problem is as follows: it is required to determine the pressure in pore fluid P (z, t), the tension in the soil skeleton $\sigma(z,t)$ and vertical movement of the upper surface S (t) (the sediment) of the compacted soil base.

The paper also studies the flat compaction of soil, which is mainly determined by its compressibility. The spatial problems of the deformed solid's mechanics are also considered by the authors.

**Key words:** Evaluation, equation in integral form, the process, compaction, soil, rectangle, pressure, basis, foundation, boundary conditions.

During estimation of different kinds of soil consolidation, it is necessary to identify the nature of the change in time of constructions and bases' sediments. Preliminary it is needed to determine the dissipation of pore pressure arising upon application of tension to the soil and the amount of the main tensions exerting on the soil skeleton; i.e. it is necessary to evaluate the bearing capacity of soil foundations, which are in an unstabilized condition.

We should mention that in unconsolidated condition the tension applied to the soil is perceived both by the soil's skeleton and pore fluid. Wherein speaking about the one-dimensional case the pressure on the elementary area consists of the tension on the soil's skeleton and the pressure in the water. Sometimes the tension in the soil's skeleton is called effective, the pressure in the water is called neutral, and the unit pressure applied on the whole soil's surface is called total.

Thus, the soil consolidation is determined not only by physic-mechanical properties, but also by the size of the compaction area as the dissipation speed of pore pressure depends on the length of filtration path.

In the calculations of consolidation it is necessary to identify the nature of the change in time of bases and constructions' sediments, which are determined by the gradual soil's compaction with decreasing pore pressure. Then, the resulting value of sediment is compared with the limit value, i.e., [11].

$$S_p \le S_{np}$$

After that, it will be possible to talk about the strength of the entire construction's system - soil.

Currently, there are three main estimating models of consolidation theory, i.e. filtration consolidation theory, the theory of body forces and osmotic theory, which are used in the mechanics of elastic and elastic creeping soils' compaction, depending on the used species of the equation of condition, describing the relationship between the tensions in the soil skeleton and its deformation, and description of the nature of the phases' interaction.

The compression curve is taken as the main rheological equation of soil condition in the filtration consolidation theory, and the phase equilibrium interaction is described by the equation, according to which the unit pressure applied to the soil is made up of an efficient and neutral pressure. And here the air movement obeys the law of Boyle-Mariotte, which explains the relationship between pressure and volume, and the gas, dissolved in water, obeys the law of Henry. This model allows us to simplify the mathematical formulation of the problem and makes it easier to determine the solution of soil consolidation problems. The filtration consolidation theory was formulated by K.Terzaghi and further developed in the works of Soviet scientists. This theory is sometimes fairly called the theory of Terzaghi-Gersevanov-Florin, because N.M.Gersevanov [6] and V.A. Florin [4] not only received partial solution of problems of this theory, but also could formulate a general statement for the two- and three-dimensional cases.

The compaction of soil is mainly determined by its compressibility. Compressibility of the base depends on the soil type and the nature of the tension. The phenomenon of the soil's compressibility is very important in the design of engineering structures on a consolidated basis. Wherein this soil compression deformation occurs mainly due to the convergence of solid particles together and is evaluated by the change in porosity coefficient when the compressive pressures in the soil skeleton σ are changed. The determining of the relationship between porosity coefficient and compressive tensions in the soil skeleton is usually performed in the laboratory in compression devices. In addition, the deformation properties of soils, in general, vary with the point's coordinates, and the assumption on their homogeneity is the idealization of the real conditions of earth massives' compaction. In this respect, theoretical and experimental researches of B.N. Barshevskyi [2], L.A. Galin [6], G.K. Klein [7] and other researchers have shown that the soil, on which constructions are built, is heterogeneous for its mechanical properties, and this soil's heterogeneity varies in depth according to the law:

$$E(z) = E_m z^m, \qquad (1)$$

where $E_m$ is a module of soil deformation at depth $z = 1$; index $m$ in most cases ranges in limits $0 < m < 2$ and is connected with a Poisson coefficient $\mu_0$, i.e.

$$\mu_0 \ (2 + m) = 1 .$$

G.K. Klein [7] has worked out the method of calculation of beams lying on the soil base, the deformation modulus of which varies according to the law (1). He has derived the following formula for the determination of the half-space's surface's sediment:

$$W_m = \frac{P}{\pi_m D_m r^{m+1}},$$

where $P$ – the concentrated force applied to the surface of the half-space ; $D_m = \frac{E_m}{\alpha^*}$ – characteristics of heterogeneous half-space's rigidity; $r$ – distance from the place of application of force $P$ to the point of the half-point's surface, where the sediment is determined by:

$$\alpha = \frac{3+m}{2}\left(\frac{1}{1+m} - \mu_0\right).$$

On the basis of these studies, in contrast to (1) herein in order to study the compaction process the soil deformation's module is the following

$$E = E_m(1+\beta z)^m \quad (\alpha > 0,\ E_m > 0,\ \alpha + \beta z > 0), \tag{2}$$

where $E_m, \beta, m$ are the parameters of an experience.

The parameters $E_m, \beta, m$, in (2), can be determined, if we know three values $E_1, E_2, E_3$ of deformation modulus for three different values $z_1, z_2, z_3$.

**One-dimensional problem**. Below we consider the process of compaction of the heterogeneous saturated soil's layer, which has capacity h, and which lies beneath a sandy pillow. At the initial time ($t=\tau_1$) the soil's layer immediately gets a distributed tension with the intensity q (z, t). Then the mathematical formulation of this problem is as follows: it is required to determine the pressure in pore fluid P (z, t), the tension in the soil skeleton $\sigma(z,t)$ and vertical movement of the upper surface S (t) (the sediment) of the compacted soil base. Wherein we assume: the possibility of the part of the tension for highly compressible saturated clay soils at the initial time, tension q, instantly applied to the soil, which is equal in magnitude of the structural strength of the compression $p_{стр}$, immediately perceived by soil skeleton. The soil is structurally heterogeneous, i.e. soil heterogeneity may be conditioned by continuous increase in its density, and therefore in rigidity by the depth under the influence of its own weight. This means that the properties of the soil are not constant, but vary depending on the position of coordinates. And the soil, deformation modulus of which increases continuously with depth, is called continually heterogeneous; the soil bases deform under the tension in a vertical direction; the earthen surroundings are water saturated, i.e. it consists of solid soil particles and fills its pores with water; the viscous nature of the clay soil deformation is not expressed clearly enough, so in some cases the skeleton creep phenomenon, simply can be ignored. Water filtering, extracted from the strongly compacted water saturated clay soil, passes according to the generalized Darcy's law.

Then the value of the pore pressure $p(z,t)$ at $t=\tau_1$ is equal to [1]

$$p\big|_{t=\tau_1} = q(z,t) - p_{cmp} = q_0(z,t), \tag{3}$$

i.e. the part of the tension, which is equal to the value of the structural strength of the compression $p_{mp}$ is immediately accepted by the soil skeleton. The rate of change of porosity coefficient $\varepsilon(z,t)$ has the form

$$\frac{\partial \varepsilon}{\partial t} = \frac{\kappa}{\gamma_в}(1+\varepsilon_{cp})\frac{\partial^2 p}{\partial z^2}, \tag{4}$$

where $\varepsilon_{cp}$ - the average coefficient of porosity; $k$ - filtration coefficient, $\gamma_в$ - volumetric weight of water;

If the soil is deformed only in the vertical direction, then according to the filtration consolidation theory, the amount of excess pore pressure $p(z,t)$ and the effective tension in the soil $\sigma(z,t)$ at any given time is equal to the external tension, i.e.

$$p + \sigma = q \tag{5}$$

In the linear soils' consolidation theory the compression dependence for heterogeneous soil has the following form

$$\varepsilon(z,t) = \varepsilon_0 - a(z)\sigma(z,t) \tag{6}$$

Here, the compressibility coefficient for heterogeneous compacted soil $a(z)$ depends on the coordinate z, i.e. the depth of the studied point's location of the compacted soil massif; $\varepsilon_0$ - initial rate of porosity.

Using (3) - (6), the equation (4) leads to the following form:

$$\frac{\partial p}{\partial t} = C_{1v}(1+z)^m \frac{\partial^2 p}{\partial z^2}, \tag{7}$$

where

$$C_{1v} = \frac{k(1+\varepsilon_{cp})}{\gamma_в a_0}.$$

Boundary conditions in laminar Darcy law will take the following form

$$p\big|_{z=0} = 0; \qquad \frac{\partial p}{\partial z}\bigg|_{z=h} = 0. \tag{8}$$

The second boundary condition applies to the depth h, the filtration does not occur below it. Thus, the solution of the studied problem reduces to the solution of differential equation (7) at the edge (3) and (8) conditions.

The solution (7) with the boundary conditions (8) is the following

$$p(z,t) = \sqrt{1+\beta z}\sum_{i=0}^{\infty} C_i V_{\frac{1}{2-m}}\left[ v_i (1+\beta z)^{\frac{2-m}{2}} \right] e^{-C_{1v}\lambda_i^2 t}, \tag{9}$$

where $m \neq 2$;

$$C_i = \frac{\int_1^{1+\beta h} (q_0 + bz) z^{\frac{1}{2}-m} V_{\frac{1}{2-m}}\left[ v_i (1+\beta z)^{\frac{2-m}{2}} \right] dz}{\int_1^{1+\beta h} z^{1-m} V^2_{\frac{1}{2-m}}\left[ v_i (1+\beta z)^{\frac{2-m}{2}} \right] dz} \tag{10}$$

Wherein the function $V_{\frac{1}{2-m}}(x)$ depends on the value $\frac{1}{2-m}$. If it is even, then

$$V_{\frac{1}{2-m}}\left[ v(1+\beta z)^{\frac{2-m}{2}} \right] = J_{\frac{1}{2-m}}\left[ v(1+\beta z)^{\frac{2-m}{2}} \right] Y_{\frac{1}{2-m}}(v) - J_{\frac{1}{2-m}}(v) Y_{\frac{1}{2-m}}\left[ v(1+\beta z)^{\frac{2-m}{2}} \right]. \tag{11}$$

When it is fractional $\frac{1}{2-m}$, then

$$V_{\frac{1}{2-m}}\left[ v(1+\beta z)^{\frac{2-m}{2}} \right] = J_{\frac{1}{2-m}}\left[ v(1+\beta z)^{\frac{2-m}{2}} \right] Y_{-\frac{1}{2-m}}(v) - J_{-\frac{1}{2-m}}\left[ v(1+\beta z)^{\frac{2-m}{2}} \right] Y_{\frac{1}{2-m}}(v). \tag{12}$$

where $J_{\frac{1}{2-m}}$, $Y_{\frac{1}{2-m}}$ - Bessel functions of the first and second kinds correspondingly. And the parameter $\nu$ in (9) - (12) is of the following transcendental equation:

for the even index $\frac{1}{2-m}$

$$J_{\frac{1}{2-m}}(\nu)Y_{\frac{m-1}{2-m}}\left[\nu(1+\beta h)^{\frac{2-m}{2}}\right]-Y_{\frac{1}{2-m}}(\nu)J_{\frac{m-1}{2-m}}\left[\nu(1+\beta h)^{\frac{2-m}{2}}\right]=0, \quad (13)$$

for fractional index

$$J_{\frac{1}{2-m}}(\nu)Y_{\frac{m-1}{2-m}}\left[\nu(1+\beta h)^{\frac{2-m}{2}}\right]-Y_{\frac{1}{2-m}}(\nu)J_{\frac{m-1}{2-m}}\left[\nu(1+\beta h)^{\frac{2-m}{2}}\right]=0. \quad (14)$$

Equation (13), (14) using concrete numbers $m$ have countless varieties of material radicals $\nu$.

Due to the expression (9) and (5) the tension in soil $\sigma(z,t)$ at any given time is equal to

$$\sigma(z,t)=q_0-\sqrt{1+\beta z}\sum_{i=0}^{\infty}C_iV_{\frac{1}{2-m}}\left[\nu_i(1+\beta z)^{\frac{2-m}{2}}\right]e^{-C_{1\nu}\lambda_i^2 t}, \quad (15)$$

From (9) and (15) we can obtain the solution of the problem for soil, the deformation modulus of which will vary depending on coordinate, i.e. on the depth. To do this, you must assume that $\beta=1$ and $m=0$. Then the index of the Bessel functions $\frac{1}{2-m}$ is equal to $\frac{1}{2}$.

Due to the fact that the exponential function $e^{-x}$ rapidly decreases at higher values of the index, then (9) is limited to only the first member of the series. Wherein the solution of the problem relatively to pore pressure according to (9) can be written as follows:

$$p(z,t)=C_0\sqrt{1+\beta z}V_{\frac{1}{2-m}}\left[\frac{2\lambda_0}{2-m}(1+\beta z)^{\frac{2-m}{2}}\right]e^{-C_{1\nu}\lambda_0 t}, \quad (16)$$

Equation (16) describes a dispersion of the pore pressure in time and depth. This expression is a generalized result of M.Yu. Abelev [7] and K.Terzaghi [8].

The tension in the soil skeleton comes from the coefficient (15), i.e.

$$\sigma(z,t)=q-C_0\sqrt{\alpha+\beta z}V_{\frac{1}{2-m}}\left[\nu_0(\alpha+\beta z)^{\frac{2-m}{2}}\right]e^{-C_{1\nu}\lambda_0 t}, \quad (17)$$

The obtained expressions (16) and (17) respectively allow to determine the pressure changes in the pore fluid and tensions in the soil skeleton for any point of the considered heterogeneous two-phase soil's final area of compaction, having an elastic property. After the specified tension in the skeleton of compaction heterogeneous soil massif, we can calculate vertical displacements of the points of the soil compacted layer's upper surface (sediment).

Indeed, if a certain vertical tension is applied to the surface of soil layer, then the corresponding sediments $S(t)$ can be determined by formula [3], i.e.

$$S(t)=\int_0^h\frac{\varepsilon_0-\varepsilon(z,t)}{1+\varepsilon_0}dz, \quad (18)$$

where $h$ - capacity of heterogeneous compacted soil massif. Since $\varepsilon(\tau_1)-\varepsilon(z,t)=a(z)\sigma(z,t)$, then (18) takes the form

$$S^{(H)}(t)=\frac{1}{1+\varepsilon_0}\int_0^h a(z)\sigma(z,t)dz \qquad (19)$$

In (19) instead of $\sigma(z,t)$ by substituting (17), we find

$$S^{(H)}(t)=\frac{a_0}{1+\varepsilon_0}\int_0^h (1+\beta z)^{-m}\{q-\sqrt{1+\beta z}V_{\frac{1}{2-m}}[\nu_0(1+\beta z)^{\frac{2-m}{2}}]e^{-C_{1V}\lambda_0 t}\}dz$$

from whence

$$S^{(H)}(t)=\frac{a_0}{1+\varepsilon_0}\{\frac{q}{\beta(1-m)}[(1+\beta h)^{1-m}-1]-$$
$$-\frac{\gamma_в}{\beta(1-m)}[1-\frac{1}{\beta(2-m)}][(1+\beta h)^{2-m}-1]- \qquad (20)$$
$$-\int_0^h (1+\beta)^{1-m}V_{\frac{1}{2-m}}[\nu_0(1+\beta z)^{\frac{2-m}{2}}]e^{-C_{1V}\lambda_0 t}\}dz$$

With $t\to\infty$ from (20) we have

$$S^{(H)}(\infty)=\frac{a_0}{\beta(1+\varepsilon_0)(1-m)}\{q[(1+\beta h)^{1-m}-1]-$$
$$-\gamma_в\left[1-\frac{1}{\beta(2-m)}\right][(1+\beta h)^{2-m}-1]\} \qquad (21)$$

From (21) for a homogeneous soil we obtain

$$S^{(0)}(\infty)=\frac{a_0 h}{1+\varepsilon_{cp}}q, \qquad (22)$$

Equation (22) depends only on the thickness of the compacted layer, tension compressibility coefficient and does not depend on heterogeneous soil parameters.

Thus, the expression (16), (17) and (20) make it possible to determine the numerical values of pressure in pore fluid, tensions in the soil skeleton and sediments of the compacted heterogeneous soil.

**Two-dimensional problem.** With the construction of facilities on the sandy soils its sediment, mainly, stops in the end of the construction season. Absolutely diverse thing happens when the construction takes place on clay soils, which often lead to difficult situations, causing deformations and sometimes even crashes of erected structures on them.

This is due to underestimation of their more complex nature and uniqueness of the interactions of these soils' solid particulates. Therefore, in this respect we need to provide the models with calculations, considering the compaction of clay soils, which in advance would allow setting the strength and stability of the structures being built on these grounds.

In this regard, we consider the process of two-dimensional compaction of heterogeneous water saturated soils at their heterogeneous boundary conditions. The compacted soil heterogeneity is expressed through the module of its deformation, which varies according to the depth of the soil massif exponentially, i.e.,

$$E=E_0e^{\alpha z}, \quad 0<\alpha<1, \quad z\in[0.h], \qquad (23)$$

where $E_0$, $\alpha$ - experimental data.

G.Ya.Popov [8] has formulated and solved the concrete problem of elasticity theory.

Thus, if we choose the modulus of compacted soil massif in the form (23), then the equation of compaction (7) takes the form

$$\frac{\partial p}{\partial t} = C_v^{(2)} \cdot e^{\alpha \cdot y}\left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2}\right), \tag{24}$$

where

$$C_v^{(2)} = \frac{k(1+\varepsilon_{cp})\cdot(1+\zeta)}{2a\gamma_{\text{в}}}.$$

Let's consider the process of water permeable soil massif's compaction in the form of a rectangle, thickness h, having a waterproof bounding walls by the sides and a waterproof layer at a depth h. Suppose also that evenly-distributed tension of intensity q is applied at a certain part of the rectangle's top surface at the time $t=\tau_1$.

At the boundaries of the studied soil rectangle we have the following boundary conditions:

$$\left.\begin{aligned} &\lim_{x\to\pm\ell}\frac{\partial p_0}{\partial x} = 0, \quad \lim_{y\to 0}\frac{\partial p_0}{\partial y} = 0; \\ &\lim_{\substack{y\to h\\ |x|<a}} p_0 = p, \qquad \lim_{\substack{y\to h\\ -\ell<x<-a\\ a<x<\ell}} p_0 = 0. \end{aligned}\right\} \tag{25}$$

Besides (25) due to the symmetry $p_0$ must be even with respect to x, i.e. $p_0(x,y) = p_0(-x,y)$. In order to determine the pressure in the pore fluid, corresponding to the initial moment of time, it is needed to solve the differential equation of the following form

$$\frac{\partial^2 p_0}{\partial x^2} + \frac{\partial^2 p_0}{\partial y^2} = 0. \tag{26}$$

This solution will in the form of

$$p_0(x,y) = \frac{qa}{\ell} + 2q\sum_{j=1}^{\infty}\frac{\sin\frac{j\pi a}{\ell}}{j\pi}\cdot\frac{ch\left(\cdot\frac{j\pi}{\ell}\right)y}{ch\left(\frac{j\pi}{\ell}\right)h}\cos\frac{j\pi}{\ell}x. \tag{27}$$

The expression (27) fully satisfies the boundary conditions (25) of the problem. The abovementioned soil massif's strained condition for the time $t=\tau_1$ is studied in [9].

According to the condition of the studied problem the boundary conditions with respect to the pore pressure $p(x, y, t)$ are heterogeneous:

$$\left.\begin{aligned} &p = 0 \quad \text{при} \quad y = h \quad a < x < \ell \quad \text{и} \quad -\ell < x < -a, \\ &p = x(t)\frac{q}{\ell} \quad \text{при} \quad y = h \quad -a < x < a \quad \text{и} \quad 0 > x(t) \ge 0, \\ &\frac{\partial p}{\partial x} = 0 \quad \text{при} \quad x = \pm\ell, \\ &\frac{\partial p}{\partial y} = 0 \quad \text{при} \quad y = h, \quad -\ell < x < \ell. \end{aligned}\right\} \tag{28}$$

Solving the equation (24) at (28), we obtain the following calculation formula for estimating the pore pressure in the compaction of heterogeneous soil massif, which has elastic properties.

$$p(x,y,t)=\frac{aq\cdot æ(t)}{\ell}+2qæ(t)\cdot\sum\frac{\sin\frac{n\pi a}{\ell}}{n\pi}\cos\frac{n\pi x}{n\pi}+$$

$$+\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\left[A_{ij}\cdot e^{-C_V^{(1)}\lambda_{ij}^2\cdot(t-\tau_1)}+\int_{\tau_1}^{t}Q_{ij}(\tau)\cdot e^{-C_V^{(1)}\lambda_{ij}^2\cdot(t-\tau)}d\tau\right]\cdot W_\nu\left(\mu_j e^{-\frac{\alpha}{2}y}\right)\cdot\cos\frac{i\pi}{\ell}x, \quad (29)$$

where

$$A_{ij}=\frac{1}{\int_0^h W_\nu^2\left(\mu_j e^{-\frac{\alpha}{2}y}\right)dy}\left\{\frac{\sin\frac{i\pi\, a}{\ell}}{i\pi}\left[\frac{\int_0^h ch\frac{i\pi}{\ell}y\cdot W_\nu\left(\mu_j e^{-\frac{\alpha}{2}y}\right)dy}{ch\frac{i\pi}{\ell}h}-æ\right]\right\}. \quad (30)$$

It should be noted that the expression (30) at $t\to\infty$ will be equal 0, as æ(t) → 0.

The sum of the main tensions in the soil skeleton is defined by the formula

$$\theta(x,y,t)=\frac{aq}{\ell}[1-æ(t)]+2q\cdot\sum_{n=1}^{\infty}\frac{\sin\frac{n\pi\, a}{\ell}}{n\pi}\cos\frac{n\pi x}{\ell}\times$$

$$\times\left[\frac{ch\frac{h\pi y}{\ell}}{n\pi}-æ(t)\right]-q\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\left[A_{ij}\cdot e^{-C_V^{(1)}\lambda_{ij}^2\cdot(t-\tau_1)}+\int_{\tau_1}^{t}Q_{ij}(\tau)\cdot e^{-C_V^{(1)}\lambda_{ij}^2\cdot(t-\tau)}d\tau\right]\times$$

$$\times W_\nu\left(\mu_j e^{-\frac{\alpha}{2}y}\right)\cdot\cos\frac{i\pi}{\ell}x. \quad (31)$$

The vertical movement of the top surface's points, i.e. sediment of the compacted soil layer will be defined by the formula (18).

Substituting (31) into (18) we find

$$S(x,t)=\frac{a_0qh}{1+\varepsilon(\tau_1)}\left\{\frac{1}{\ell}[1-æ(t)]-2æ(t)\sum_{n=1}^{\infty}\frac{\sin\frac{n\pi\, a}{\ell}}{n\pi}\cos\frac{n\pi x}{\ell}-\right.$$

$$-\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\left[A_{ij}\cdot e^{-C_V^{(1)}\lambda_{ij}^2\cdot(t-\tau_1)}+\int_{\tau_1}^{t}Q_{ij}(\tau)\cdot e^{-C_V^{(1)}\lambda_{ij}^2\cdot(t-\tau)}d\tau\right]W_\nu\left(\mu_j e^{-\frac{\alpha}{2}y}\right)\cdot\cos\frac{i\pi}{\ell}x. \quad (32)$$

The obtained expressions (29), (31) and (32) at (33) give us respectively an opportunity to calculate pressure in pore fluid, the amount of main tensions and sediment of compacted heterogeneous soil massif, possessing elastic property.

**Three-dimensional problem.** Let's consider the compaction of the soil massif in the form of a parallelepiped with a confining layer at a depth of $h$ and waterproof walls $2\ell_1$ and $2\ell_2$. Evenly distributed tension with intensity q is applied instantaneously on top of the surface of the parallelepiped with sides 2a and 2b.

For this problem, the range of variation of the independent variables is the parallelepiped formed by planes

$$x = \pm\ell_1; \quad y = \pm\ell_2; \quad z = 0; \quad z = h.$$

According to the problem, the boundary conditions at $t = \tau_1$ will be:

$$\left.\begin{aligned} &\lim_{z\to h} p_0(x,y,z) = \begin{cases} q \text{ при } & |x| < a, \ |y| < b \\ 0 \text{ при } & |x| > a, \ |y| > b \text{ или} \end{cases} \\ &\qquad\qquad |x| > a, \ |y| < b \quad \text{или} \quad |x| < a, \ |y| > b \\ &\left.\frac{\partial p_0}{\partial x}\right|_{x=\pm\ell_1} = 0; \quad \left.\frac{\partial p_0}{\partial y}\right|_{y=\pm\ell_2} = 0; \quad \left.\frac{\partial p_0}{\partial z}\right|_{z=0} = 0. \end{aligned}\right\} \tag{33}$$

Also because of the symmetry function $p_0(x, y, z)$ must be an even with respect to *x* and *y* separately, i.e.

$$p(x,y,z) = \begin{Bmatrix} p(-x,y,z) \\ p(x,-y,z) \end{Bmatrix} \tag{34}$$

In order to determine the distribution of instantaneous pressures in the pore liquid in the indicated layer of soil, it is necessary to solve a differential equation of the following kind

$$\frac{\partial^2 p_0}{\partial x^2} + \frac{\partial^2 p_0}{\partial y^2} + \frac{\partial^2 p_0}{\partial z^2} = 0. \tag{35}$$

Here, the function $p_0(x, y, z)$ is dependent on spatial coordinates. The solution of equation (35) at (33), (34) was studied in [9]. Solving (35) under the conditions of (33), (34), we obtain

$$p_0(x,y,z) = q \cdot \left[ \frac{ab}{\ell_1 \ell_2} + \frac{2b}{\ell_2} \sum_{m=1}^{\infty} a_m ch\left(\frac{m\pi}{\ell_1} z\right) \cdot \cos\frac{m\pi}{\ell_1} x + \frac{2a}{\ell_1} \sum_{n=1}^{\infty} b_n ch\left(\frac{n\pi}{\ell_2} z\right) \times \right.$$
$$\left. \times \cos\frac{n\pi}{\ell_2} y + 4 \cdot \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} a_m b_m c_{mn} ch\left(\frac{m\pi}{\ell_1} h\right) \cdot ch\left(\frac{n\pi}{\ell_2} h\right) \cdot \cos\frac{m\pi}{\ell_1} x \cdot \cos\frac{n\pi}{\ell_2} y \cdot ch\alpha_{mn} z \right], \tag{36}$$

where

$$\left.\begin{aligned} a_m &= \frac{\sin\dfrac{m\pi a}{\ell_1}}{m\pi \cdot ch\left(\dfrac{m\pi}{\ell_1} h\right)}; \\ b_n &= \frac{\sin\dfrac{n\pi b}{\ell_2}}{n\pi \cdot ch\left(\dfrac{n\pi}{\ell_2} h\right)}; \\ c_{mn}^{-1} &= ch\alpha_{mn} h; \\ \alpha_{mn} &= \left[\left(\frac{m\pi}{\ell_1}\right)^2 + \left(\frac{n\pi}{\ell_2}\right)^2\right]^{1/2}. \end{aligned}\right\}$$

Then the solution of equation

$$\frac{\partial p}{\partial t}=C_V^{(3)}e^{\alpha\cdot z}\cdot\left(\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial y^2}+\frac{\partial^2 p}{\partial z^2}\right),\qquad(37)$$

where

$$C_V^{(3)}=\frac{k(1+\varepsilon_{cp})\cdot(1+2\xi)}{3a_0\gamma_{\text{в}}}$$

It will look like:

$$=p(x,y,z,t)=æ(t)q+\left[\frac{ab}{\ell_1\ell_2}+\frac{ab}{\ell_2}\sum_{m=1}^{\infty}\frac{\sin\frac{m\pi a}{\ell_1}}{m\pi}\cos\frac{m\pi}{\ell_1}x+\frac{2a}{\ell_1}\cdot\sum_{n=1}^{\infty}\frac{\sin\frac{n\pi b}{\ell_2}}{n\pi}\cos\frac{n\pi}{\ell_2}y+\right.$$

$$\left.+4\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{\sin\frac{m\pi a}{\ell_1}}{m\pi}\cdot\frac{\sin\frac{n\pi b}{\ell_2}}{n\pi}\cos\frac{m\pi}{\ell_1}x\cos\frac{n\pi}{\ell_2}y\right]+\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}T_{mnk}(t)\cdot W_v\left(\mu_k e^{-\frac{\alpha}{2}z}\right)\times$$

$$\times\cos\frac{m\pi}{\ell_1}x\cos\frac{n\pi}{\ell_2}y.\qquad(38)$$

where

$$T_{mnk}(t)=C_{mnk}e^{-C_V^{(3)}\lambda_{mnk}^2(t-\tau_1)}+\int_{\tau_1}^{t}Q_{mnk}(\tau)\cdot e^{-C_V^{(3)}\lambda_{mnk}^2(t-\tau)}d\tau.\qquad(39)$$

$$C_{mnk}=\frac{4\int_0^{\ell_1}\int_0^{\ell_2}\int_0^{h}\left[\left(\frac{\theta^*}{3}+p^*\right)-æ(t)p_{mn}(x.y.h,\tau_1)\right]\cdot W_v\left(\mu_k e^{-\frac{\alpha}{2}z}\right)\cos\frac{m\pi}{\ell_1}x\cos\frac{n\pi}{\ell_2}ydxdy}{\ell_1\ell_2\int_0^{h}W_v\left(\mu_k e^{-\frac{\alpha}{2}z}\right)dz}.$$

The sum of the main tensions will be calculated by the formula [9]. Knowing the amount of the main tensions, we determine sediment of the compacted soil massif, presented in the form of a parallelepiped having an elastic property. Wherein the modulus of its deformation is considered variable in its depth.

Using these data, numerical values of pore pressure, the amount of the main tensions, and also the value of the compacted massif's sediment can be calculated. Analysis of the curves shows that the values of pore pressure for biphasic soil ground in the initial period is almost two times smaller than the values for the three-phase ground. The values of the sediment for the three-phase soil are much more than for the two-phase soil base.

It should also be noted that recently the impact of the variability of the filtration coefficient in the compression process is considered in the following works [3], [12].

## REFERENCES

[1] M.Yu. Abelev. Contruction of industrial and civil constructions on weak water-saturated ground. M.: Contruction publishing, 1983. p.247.

[2] B.N.Barshevsky. Determination of the characteristics of the deformable axis of ground, considered as a continuous inhomogeneous environment on depth// Basics, Foundations and mechanism of Soil. **1969**. №1. p.46-58.

[3] A. D. Dasibekov, A. Asibekov, G. Makasheva, G. A. Takibaeva. The variability impact of coefficient of filtration of module deformation on consolidation process // Works of Int. scientific – method. Conf. at University of Friendship of Nation. Shymkent, **2006**. Part 1.

[4] V.A.Fljrin. Fundamentals of soil mechanism.-M.: State Construction Publishing. Part 1.2. 1959. P.357.: 1961-P.543.

[5] N.M. Gersevanov. Collected Works.-M.: Construction and Naval Publishing. **1948**. Part1. 2. P.644.

[6] L.A.Galin. Contact problems of the theory of elasticity for bodies with variable elastic modulus// Procrrdings of the All-Union Meeting on implementation of methods of theoretical physics to the problems of mathematical physics. Tbilisi, **1961**. p.123-138.

[7] G.K.Kelin. Analysis of beam on a solid basis, which is continuously inhomogeneous on depth// Contruction,Mechanics and design: Collected works MIITS Moskoww City Executive Committee. **1954**. No.3. p.131-137.

[8] Ya. A. Macheret. Distribution of instant push and pressures in soil mass caused by the instantaneous thrust // Works VIOS. **1934**, № 4. p. 65-121. p. 228-232.

[9] G. Ya. Popov. By the theory of bending of plates on elastic inhomogeneous halfspace. // Constrution and architure. **1959**, № 12. p. 11 – 19.

[10] G. A. Takibaeva. Three- dimensional task of mechanics of consolidated sil mass at a pressure which is depending on the filtration coefficient // Pub. NAS of RK. Part "Physics and Mathematics" . **2006**, № 1. p. 54-57.

[11] N.A.Tsytovich. Soil Mechanics.-M.; Literature on construction, architecture and construction materials.-1963.-P.633.

[12] K.Tertsaghi. Structural Mecanism of Soil. M. : State construction publishing., p. **1933**. 510.

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## ТЕРЕҢДІК БОЙЫНША АЙНЫМАЛЫ ДЕФОРМАЦИЯ МОДУЛДІ ГРУНТТЕР КОНСОЛИДАЦИЯСЫНЫҢ КӨПӨЛШЕМДІ ЕСЕПТЕРІ

**Аннотация:** Грунтқа жүктелген нагрузка консолидациялық емес жағдайда грунт скелетіменде жәнеқуыстық сұйықтықпенде қабылданады. Бұнда, бір өлшемді жағдай үшін элементар аландағы қысым грунт скелетіне напряжениядан және судағы қысымнан құралады. Лейде грунт скелетіне түсетін напряженисні эффектті, судағы қысымды нетраль , ал грунт бетіне қыйылған меншікті қысымды-тоталдық деп атайды.

Барлықесептер нейтераль қысымды анық тауға келтіріледі.Келесі шамалар жай табылады. Демек, қуыстық сұйықтағы қысымды табу қажет. Оны анықтау үшін бұл қысымды табу қажет.

Консолидацияның фильтрациялық теориясында грунт жағдайының негізгі реалогиялық теңдеу ретінде компрессиялық қисық қабылданды, ал фазалардың өзара әрекеттесуі тепе-теңдіктің теңдеуімен жазылады, осыған сәйкес грунтқа қойылған меншікті жүк эффектті және нейтраль қысымдардан құралады. Мұнда ауа көшуі қысым мен көлем арасындағы байланысты тағайындайтын Бойл-Мориотт заңына бағынады, ал судағы еріген газ Генри заңына бағынады. Бұл модель есептің математикалық қойылуын ықшамдауға мүмкіндік береді және грунт консолидациясы есебінің шешімін анықтауды жеңілдетеді.

Грунт тығыздалуы негізінен өзінің сығылуымен анықталады. Негіздің сығылуы грунт типіне, сондай-ақ жүк характеріне байланысты. Грунтардың сығылу құбылысы консолидацияланған негіздегі инженерлік құрлыстарды жобалауда айтарлықтай үлкен маңызға ие. Мұнда грунттардың сығылу деформациясы негізіне қатты бөлшектердің өзара жақындасуы салдарынан болады және грунт склетінде сығушының өзгеруінен қуыстық коэффиценттінің өзгеруімен бағаланады. Грунт склетіндегі сығушы және қуыстық коэффицент арасындағы байланысты анықтау әдетте лабораториялық жолмен компрессор құрылғыларында жүргізіледі.Төменде құмды жастық астында жататын h қуатты суға тойған біртекті емес грунт қабатының тығыздалу процесін қарастырамыз. Бастапқы уақыт ($t=\Omega$ ) моментінде грунт қабатында бір сәтте интенсивтілігі $q(z,t)$ таралған жүк қойылады. Бұл жағдайда осы есептің математикалық қойылымы келесіге келтіріледі: қуыстық сұйықтағы $p(z,t)$ қысымды, грунт склетіндегі $\Omega(z,t)$ және тығыздалып жатқан грунттық негіздің жоғары бетінің вертикал көшуі анықтау талап етіледі.

Негіздің сығылуы грунт типіне, сондай-ақ жүк характеріне (сипатына )байланысты. Жұмыста және де деформацияланған қатты денелер механикасының жазық және кеңістік есептерін зерттелген.

**Тірек сөз:** интегральдай формуласы,топырақ, тіктортбұрыш, негізі, шекаралық шарт.

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## МНОГОМЕРНЫЕ ЗАДАЧИ КОНСОЛИДАЦИИ ГРУНТОВ С ПЕРЕМЕННЫМ ПО ГЛУБИНЕ МОДУЛЕМ ДЕФОРМАЦИИ

**Аннотация:** В неконсолидированном состоянии нагрузка, приложенная к грунту, воспринимается как скелетом грунта, так и поровой жидкостью. При этом, для одномерного случая давление на элементарной площадке складывается из напряжения в скелет грунта и давления в воде. Иногда напряжение, приходящее в скелете грунта называют эффективным, давление в воде нейтральным, а удельное давление, приложенное в целом на поверхность грунта - тотальным.

Все задачи здесь сводятся к определению нейтрального давления. Последующие величины находятся просто. Следовательно, необходимо находить давление в поровой жидкости. Чтобы его определить следует находить это давление.

В фильтрационной теории консолидации в качестве основного реологического уравнения состояния грунта принимается компрессионная кривая, а взаимодействие фаз описывается уравнением равновесия, в соответствии с которым приложенная к грунту удельная нагрузка складывается из эффективного и нейтрального давлений. Причем здесь перемещение воздуха подчиняется закону Бойля-Мариотта, устанавливающему связь между давлением и объемом, а растворенный газ в воде подчиняется закону Генри. Эта модель позволяет упростить математическую постановку задачи и облегчает определения решений задач консолидации грунтов.

Уплотнение грунта в основном определяется своей сжимаемостью. Сжимаемость основания зависит как от типа грунта, так и от характера нагрузки. Явление сжимаемости грунтов имеет весьма большое значение при проектировании инженерных сооружений на консолидируемом основании. При этом деформация сжатия грунтов в основном происходит вследствие сближения твердых частиц между собой и оценивается изменением коэффициента пористости при изменении сжимающих напряжений в скелете грунта. Определение зависимости между коэффициентом пористости и сжимающими напряжениями в скелете грунта обычно производится лабораторным путем в компрессионных приборах.

Ниже рассмотрим процесс уплотнения слоя неоднородного водонасыщенного грунта мощностью $h$, залегающего под песчаной подушкой. В начальный момент времени ($t=\tau_1$) к слою грунта мгновенно прикладывается распределенная нагрузка с интенсивностью $q(z,t)$. Тогда математическая постановка данной задачи сводится к следующему: требуется определить давление в поровой жидкости $p(z,t)$, напряжение в скелете грунта $\sigma(z,t)$ и вертикальные перемещения верхней поверхности $S(t)$ (осадок) уплотняемого грунтового основания.

В работе также исследованы плоское и пространственное задачи механики деформированного твердого тела.

**Ключевые слова:** уравнение, интегральное форма, процесс, уплотнение, почва, прямоугольник, основа, граничное условие.