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ON THE UNIQUE SOLVABILITY OF A MULTI-POINT PROBLEM FOR SYSTEM OF THE LOADED DIFFERENTIAL EQUATIONS HYPERBOLIC TYPE

Annotation. The nonlocal multi-point problem for the system of loaded differential equations of hyperbolic type second order is considered. The load lines in the system of equations and lines that given of the boundary conditions, can be arranged as you like. The considered problem is reduced to an equivalent multi-point problem with parameter by introducing a new unknown function instead of a loaded term in the system of equations. The problem with parameter consists of a nonlocal problem for a system of hyperbolic equations with parameter and the functional relation. Algorithms for finding an approximate solution of the equivalent problem with parameter are constructed and the conditions for their convergence are set. Sufficient conditions for the existence of unique solution to the problem with parameter are established. Conditions of existence of unique classical solution to the multi-point problem for the system of loaded differential equations of hyperbolic type are obtained in the terms of initial data. Earlier, the method of reduced to an equivalent family of problems for partial differential equations is applied to study of this problem. Sufficient conditions for the existence unique classical solution of this problem are find in the terms of some matrix compiled by the initial data.

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Key words: nonlocal multi-point problem, system of loaded differential equations, system of hyperbolic equations, parameter, algorithm, approximate solution, unique solvability.

Problem statement. Consider the nonlocal multi-point problem for the system of loaded differential equations of hyperbolic type second order on the rectangular domain $\Omega = [0,T]\times[0,\omega]$

$$\frac{\partial^2 u}{\partial t\partial x} = A(t,x)\frac{\partial u}{\partial x} + B(t,x)\frac{\partial u}{\partial t} + C(t,x)u + f(t,x) +$$

$$+\sum_{i=1}^{k}\left\{P_i(x)\frac{\partial u(t,x)}{\partial x}\Big|_{t=\theta_i} + Q_i(x)\frac{\partial u(t,x)}{\partial t}\Big|_{t=\theta_i} + S_i(x)u(\theta_i,x)\right\}, \tag{1}$$

$$u(t,0)=\psi(t),\ t\in[0,T], \tag{2}$$

$$\sum_{j=1}^{m}\left\{K_j(x)\frac{\partial u(t,x)}{\partial x}\Big|_{t=t_j} + L_j(x)\frac{\partial u(t,x)}{\partial t}\Big|_{t=t_j} + M_j(x)u(t_j,x)\right\} = \varphi(x),\ x\in[0,\omega], \tag{3}$$

where $u = col(u_1,u_2,....,u_n)$ is unknown function, the $(n\times n)$ matrices $A(t,x)$, $B(t,x)$, $C(t,x)$, $P_i(x)$, $Q_i(x)$, $S_i(x)$, $i=\overline{1,k}$, $K_j(x)$, $L_j(x)$, $M_j(x)$, $j=\overline{1,m}$, the n vector functions $f(t,x)$, $\varphi(x)$ are continuous on Ω, $[0,\omega]$, respectively, the load lines $0<\theta_1<\theta_2<\ldots<\theta_{k-1}<\theta_k<T$, the

lines in the boundary condition $0 = t_1 < t_2 < ... < t_{m-1} < t_m = T$, n vector function $\psi(t)$ is continuously differentiable on $[0,T]$.

Let $C(\Omega, R^n)$ $\left(C([0,\omega], R^n)\right)$ be a space of continuous on Ω $([0,\omega])$ vector functions $u(t,x)$ $(\varphi(x))$ with norm $\|u\|_0 = \max\limits_{(t,x)\in\Omega} \|u(t,x)\|$ $\left(\|\varphi\|_1 = \max\limits_{x\in[0,\omega]} \|\varphi(x)\|\right)$.

A function $u(t,x) \in C(\Omega, R^n)$, having the partial derivatives $\dfrac{\partial u(t,x)}{\partial x} \in C(\Omega, R^n)$, $\dfrac{\partial u(t,x)}{\partial t} \in C(\Omega, R^n)$, $\dfrac{\partial^2 u(t,x)}{\partial t \partial x} \in C(\Omega, R^n)$ is called classical solution to problem (1)--(3), if its is satisfied of system (1) for all $(t,x) \in \Omega$, and boundary condition (2) and multi-point condition (3).

The questions for the existence of unique solution and the construction of algorithms for finding approximate solutions to problem (1)--(3) are investigated. Loaded partial differential equations arise in the study of various processes of physics, chemistry, biology, ecology and others [1-9]. A special role is played of a loaded hyperbolic equations and boundary value problems for them. Some classes nonlocal and boundary value problems for the loaded differential equations of hyperbolic type studied in the papers [10-14]. We can also find a bibliography and analysis results with respect to the loaded differential equations in the works [1-5]. Multi-point problems for the differential equations frequently appear in the study of natural science and engineering problems [15, 16]. Multi-point problems for the ordinary differential equations and equations of hyperbolic type are considered in the papers [17-22]. Questions of existence, uniqueness and continuous dependence of solution from data for the new classes of nonlocal problems for the system of loaded hyperbolic equations second-order are important and urgent problems of the theory of nonlocal problems for the loaded differential equations.

In present paper the results for nonlocal problems with integral conditions for the system of hyperbolic equations with mixed derivatives are developed to a class of multi-point problems for the system of loaded hyperbolic equations. Algorithms of finding approximate solutions of multi-point problem for the system of loaded hyperbolic equations are constructed and their convergence is proved.

Reduction to an equivalent problem with parameter. We introduce a special loading function $L(x) = \sum\limits_{i=1}^{k} \left\{ P_i(x) \dfrac{\partial u(t,x)}{\partial x}\Big|_{t=\theta_i} + Q_i(x) \dfrac{\partial u(t,x)}{\partial t}\Big|_{t=\theta_i} + S_i(x) u(\theta_i, x) \right\}$ and problem (1)--(3) is reduced to the following equivalent problem

$$\frac{\partial^2 u}{\partial t \partial x} = A(t,x)\frac{\partial u}{\partial x} + B(t,x)\frac{\partial u}{\partial t} + C(t,x)u + L(x) + f(t,x), \tag{4}$$

$$u(t,0) = \psi(t),\ t \in [0,T], \tag{5}$$

$$\sum_{j=1}^{m} \left\{ K_j(x) \frac{\partial u(t,x)}{\partial x}\Big|_{t=t_j} + L_j(x) \frac{\partial u(t,x)}{\partial t}\Big|_{t=t_j} + M_j(x) u(t_j, x) \right\} = \varphi(x),\ x \in [0,\omega], \tag{6}$$

$$L(x) = \sum_{i=1}^{k} \left\{ P_i(x) \frac{\partial u(t,x)}{\partial x}\Big|_{t=\theta_i} + Q_i(x) \frac{\partial u(t,x)}{\partial t}\Big|_{t=\theta_i} + S_i(x) u(\theta_i, x) \right\},\ x \in [0,\omega]. \tag{7}$$

A pair functions $\left(u^*(t,x), L^*(x)\right)$, where $u^*(t,x) \in C(\Omega, R^n)$, $L^*(x) \in C([0,\omega], R^n)$, is called a solution to problem (4)—(7), if the function $u^*(t,x)$ has a partial derivatives $\dfrac{\partial u^*(t,x)}{\partial x} \in C(\Omega, R^n)$, $\dfrac{\partial u^*(t,x)}{\partial t} \in C(\Omega, R^n)$, $\dfrac{\partial^2 u^*(t,x)}{\partial t \partial x} \in C(\Omega, R^n)$, satisfies to system (4) for all $(t,x) \in \Omega$ and

$L(x) = L^*(x)$, the boundary condition (5), the multi-point condition (6) and relation for loading function (7) are valid.

At fixed $L(x)$ the problem (4)—(6) is nonlocal problem for the system of hyperbolic equations with mixed derivatives [22]. In this work were studied of the questions for existence of unique classical solution to the investigated problem. Conditions of unique solvability to considered problem were established and the algorithms for finding its solutions are proposed. Further, for investigating to the problem (4)—(7) we applied the method of introduction functional parameters [23-38]. Problem (1)—(3) is studied in [39] by reducing the equivalent family of problems for partial differential equations with parameters.

Scheme of the method of introduction functional parameter. Introduce a new unknown parameter $\lambda(x)$ as value of function $u(t,x)$ for $t=0$: $\lambda(x) = u(0,x)$. We replace $u(t,x) = \widetilde{u}(t,x) + \lambda(x)$ in the problem (4)—(7) and pass to the following equivalent problem

$$\frac{\partial^2 \widetilde{u}}{\partial t \partial x} = A(t,x)\frac{\partial \widetilde{u}}{\partial x} + B(t,x)\frac{\partial \widetilde{u}}{\partial t} + C(t,x)\widetilde{u} + A(t,x)\dot{\lambda}(x) + C(t,x)\lambda(x) + L(x) + f(t,x), \quad (8)$$

$$\widetilde{u}(0,x) = 0, \; x \in [0,\omega], \quad (9)$$

$$\widetilde{u}(t,0) = \psi(t) - \psi(0), \; t \in [0,T], \quad (10)$$

$$\sum_{j=1}^{m} K_j(x)\dot{\lambda}(x) + \sum_{j=1}^{m} M_j(x)\lambda(x) + \sum_{j=2}^{m}\{K_j(x)\widetilde{v}(t_j,x) + M_j(x)\widetilde{u}(t_j,x)\} + \sum_{j=1}^{m} L_j(x)\widetilde{w}(t_j,x) = \varphi(x),$$

$$x \in [0,\omega], \quad (11)$$

$$L(x) = \sum_{i=1}^{k}\{P_i(x)\dot{\lambda}(x) + P_i(x)\widetilde{v}(\theta_i,x) + Q_i(x)\widetilde{w}(\theta_i,x) + S_i(x)\lambda(x) + S_i(x)\widetilde{u}(\theta_i,x)\}, \; x \in [0,\omega], \quad (12)$$

where $\widetilde{v}(t,x) = \dfrac{\partial \widetilde{u}(t,x)}{\partial x}$, $\widetilde{w}(t,x) = \dfrac{\partial \widetilde{u}(t,x)}{\partial t}$.

A triple functions $(\widetilde{u}^*(t,x), \lambda^*(x), L^*(x))$, where $\widetilde{u}^*(t,x) \in C(\Omega, R^n)$, $\lambda^*(x) \in C([0,\omega], R^n)$, $L^*(x) \in C([0,\omega], R^n)$, is called a solution to problem (8)—(12), if the function $\widetilde{u}^*(t,x)$ has a partial derivatives $\dfrac{\partial \widetilde{u}^*(t,x)}{\partial x} \in C(\Omega, R^n)$, $\dfrac{\partial \widetilde{u}^*(t,x)}{\partial t} \in C(\Omega, R^n)$, $\dfrac{\partial^2 \widetilde{u}^*(t,x)}{\partial t \partial x} \in C(\Omega, R^n)$, satisfies of system (8) for all $(t,x) \in \Omega$ and $\lambda(x) = \lambda^*(x)$, $L(x) = L^*(x)$, the conditions on the characteristics (9), (10), relation for functional parameter (11) and relation for loading function (12) are true.

The initial data of problem satisfies of the compatibility condition in the point $(0,0)$:

$$\lambda(0) = \psi(0). \quad (13)$$

At fixed $\lambda(x)$, $L(x)$ problem (8)—(10) is Goursat problem for the system of hyperbolic equations with mixed derivatives. Relations (11) and (12) allow us to determine of the unknown parameters $\lambda(x)$, $L(x)$.

Algorithms for finding approximate solution to problem (8)—(12). The solution to problem (8)—(12) is triple functions $(\widetilde{u}^*(t,x), \lambda^*(x), L^*(x))$, which we determine as a limit of the sequence of triples functions $(\widetilde{u}^{(s)}(t,x), \lambda^{(s)}(x), L^{(s)}(x))$, $s = 0,1,2,...$, based on the following algorithm:

Step 0. 1) Suppose that $\widetilde{u}(t,x) = \psi(t) - \psi(0)$, $\widetilde{v}(t,x) = 0$, $\widetilde{w}(t,x) = \dot{\psi}(t)$, $L(x) = 0$ in the relation (11), and solving Cauchy problem for system of ordinary differential equations (11), (13) we find initial approximation of parameter $\lambda^{(0)}(x)$ for all $x \in [0,\omega]$. 2) Suppose that $\lambda(x) = \lambda^{(0)}(x)$,

$\dot{\lambda}(x)=\dot{\lambda}^{(0)}(x)$, $L(x)=0$ in the system (8) and solving Goursat problem for system of hyperbolic equations (8)—(10), we find $\widetilde{u}^{(0)}(t,x)$ and its derivatives $\widetilde{v}^{(0)}(t,x)$, $\widetilde{w}^{(0)}(t,x)$ для всех $(t,x)\in\Omega$. 3) From relation (12) for $\lambda(x)=\lambda^{(0)}(x)$, $\dot{\lambda}(x)=\dot{\lambda}^{(0)}(x)$, $\widetilde{u}(t,x)=\widetilde{u}^{(0)}(t,x)$, $\widetilde{v}(t,x)=\widetilde{v}^{(0)}(t,x)$, $\widetilde{w}(t,x)=\widetilde{w}^{(0)}(t,x)$, we determine initial approximation of loading function $L^{(0)}(x)$ for all $x\in[0,\omega]$. And so on.

Step s. 1)Suppose that $\widetilde{u}(t,x)=\widetilde{u}^{(s-1)}(t,x)$, $\widetilde{v}(t,x)=\widetilde{v}^{(s-1)}(t,x)$, $\widetilde{w}(t,x)=\widetilde{w}^{(s-1)}(t,x)$, $L(x)=L^{(s-1)}(x)$ in the relation (11), and solving Cauchy problem for system of ordinary differential equations (11), (13) we find $\lambda^{(s)}(x)$ for all $x\in[0,\omega]$. 2) Suppose that $\lambda(x)=\lambda^{(s)}(x)$, $\dot{\lambda}(x)=\dot{\lambda}^{(s)}(x)$, $L(x)=L^{(s-1)}(x)$ in the system (8) and solving Goursat problem for system of hyperbolic equations (8)—(10) we find $\widetilde{u}^{(s)}(t,x)$ and its derivatives $\widetilde{v}^{(s)}(t,x)$, $\widetilde{w}^{(s)}(t,x)$ for all $(t,x)\in\Omega$. 3) From relation (12) for $\lambda(x)=\lambda^{(s)}(x)$, $\dot{\lambda}(x)=\dot{\lambda}^{(s)}(x)$, $\widetilde{u}(t,x)=\widetilde{u}^{(s)}(t,x)$, $\widetilde{v}(t,x)=\widetilde{v}^{(s)}(t,x)$, $\widetilde{w}(t,x)=\widetilde{w}^{(s)}(t,x)$, we determine s th approximation of loading function $L^{(s)}(x)$ for all $x\in[0,\omega]$, $s=1,2,3,...$.

Conditions of the feasibility and convergence of the proposed algorithm are established. These conditions are also conditions of the existence unique solution to problem (8)—(12) and unique solvability to the original problem (1)--(3).

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ОБ ОДНОЗНАЧНОЙ РАЗРЕШИМОСТИ МНОГОТОЧЕЧНОЙ ЗАДАЧИ ДЛЯ СИСТЕМЫ НАГРУЖЕННЫХ ДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ

Аннотация. Рассматривается нелокальная многоточечная задача для системы нагруженных дифференциальных уравнений гиперболического типа второго порядка. Линии нагрузки в системе уравнений и линии, в которых задаются краевые условия, могут располагаться как угодно. Путем введения новой неизвестной функции вместо нагруженной слагаемой исследуемая задача сведена к эквивалентной многоточечной задаче с параметром. Задача с параметром состоит из нелокальной задачи для системы гиперболических уравнений с параметром и функционального соотношения. Построены алгоритмы нахождения приближенного решения эквивалентной задачи с параметром и найдены условия их сходимости. Установлены достаточные условия существования единственного решения задачи с параметром. Получены условия существования единственного классического решения многоточечной задачи для системы нагруженных дифференциальных уравнений гиперболического типа в терминах исходных данных. Ранее к исследованию рассматриваемой задачи был применен метод сведения к эквивалентному семейству задач для дифференциальных уравнений в частных производных. Были найдены достаточные условия существования единственного классического решения исследуемой задачи в терминах некоторой матрицы, составляемой по исходным данным.

Ключевые слова: нелокальная многоточечная задача, система нагруженных дифференциальных уравнений, система гиперболических уравнений, параметр, алгоритм, приближенное решение, однозначная разрешимость.

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ЖҮКТЕЛГЕН ДИФФЕРЕНЦИАЛДЫҚ ТЕҢДЕУЛЕР ЖҮЙЕСІ ҮШІН КӨПНҮКТЕЛІ ЕСЕПТІҢ БІРМӘНДІ ШЕШІЛІМДІЛІГІ ТУРАЛЫ

Аннотация. Екінші ретті гиперболалық тектес жүктелген дифференциалдық теңдеулер жүйесі үшін бейлокал көпнүктелі есеп қарастырылады. Жүктеу сызықтары мен шеттік шарттар берілген сызықтар кез келген түрде орналасуы мүмкін. Жүктелген қосылғыштың орнына жаңа белгісіз функция енгізу жолымен зерттеліп отырған есеп пара-пар көпнүктелі параметрі бар есепке келтірілген. Параметрі бар есеп гиперболалық теңдеулер жүйесі үшін параметрі бар бейлокал есептен және функционалдық қатынастан тұрады. Параметрі бар пара-пар есептің жуық шешімін табу алгоритмдері тұрғызылған және олардың жинақтылық шарттары табылған. Параметрі бар есепің жалғыз шешімінің бар болуының жеткілікті шарттары тағайындалған. Гиперболалық тектес жүктелген дифференциалдық теңдеулер жүйесі үшін көпнүктелі есептің жалғыз классикалық шешімінің бар болуының шарттары бастапқы берілімдер терминінде алынған. Қарастырылып отырған есепті зерттеу үшін бұрын дербес туындылы дифференциалдық теңдеулер үшін есептер әулетіне келтіру әдісі пайдаланылған болатын. Зерттеліп отырған есептің жалғыз классикалық шешімінің бар болуының шарттары бастапқы берілімдер арқылы тұрғызылатын матрица терминінде табылған.

Кілттік сөздер: бейлокал көпнүктелі есеп, жүктелген дифференциалдық теңдеулер жүйесі, гиперболалық теңдеулер жүйесі, параметр, алгоритм, жуық шешім, бірмәнді шешілімділік.