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SHORT- AND LONG- TERM APPROACH COLLISION PROBABILITY OF THE OBJECTS IN SPACE DEBRIS CLOUD

Abstract. The main area of investigation is about self-production of the debris in orbit. This production is mainly done by mutual collisions between the orbital debris. The presented analysis on space debris cloud evolution and mutual collisions between the objects of the cloud have been based on the Gaussian distribution using position covariance ellipsoids and the available data. The collision probability was then compared with the breakup energy experimental results which can produce more and smaller objects. The probability density function (pdf), which describes the uncertainties of the two objects positions was used to determine the probability that they are within a specified volume. Then after the relative position of an object in space relative to another has been calculated their relative positions uncertainty pdf has been obtained. These procedures have lead to be able to integrate over volume part of the region where one object approaches another.

Key words: Space debris, collision probability, covariance ellipsoids, Gauss pdfs.

The idea behind this paper is in understanding of the consequences and aftermath of the collisions, explosions or breakups of artificial Earth satellites in space. In order to understand the behavior of the objects at the instant of the collision we must approach this problem gradually in stages.

This paper introduces the stage of the probability of collision between two objects orbiting in space. Generally this kind of collisions has three natures of outcome. First, the kinetic energy of particles (the debris of the satellite-satellite collision, satellite-meteorite collision, etc.) is still quite high, to transfer the energy within its reach region. To compute the probability of this event, it is necessary to formulate the problem within the framework of probability theory. The probability density function (pdf) describes the uncertainties of the positions of the two objects, this can give us the ability to determine the probability that they are within a specified region.

Suppose that the RV (random variables) associated with these pdf are independent (or uncorrelated), these pdf can be taken into product to obtain the joint pdf. We can then integrate over the product space of RVs that corresponds to the specified volume of interest. Even for simple pdfs, this approach is rather involved.

Another approach was to consider the relative position of one object with respect to the other and obtain the pdf describing the uncertainty of their relative positions. Then we integrate over the region of the space swept by the interested volume when an object moves towards another.

In this paper the presented analysis on space debris cloud mutual collisions probability are concerned with close encounters between two or more orbiting objects. They are based on Gaussian distribution because the primary object and secondary object can be tracked and their positions determined to within the errors associated with the corresponding position covariance ellipsoids. The collisions probability is then computed using this information.

1. Short-term encounters

The probability of collision for most short-term space objects encounters may be expressed $P = e^{\frac{-v}{2}}(1 - e^{\frac{-u}{2}})$. Where the dimensionless variables u and v defined in terms of the mean standard deviationΩ, the companion standard deviation Ω^*, miss distance x_e and the radius r_a of collision cross-section integration by

$$u \equiv \left(\frac{r_A}{\sigma}\right)^2 > 0 \quad v \equiv \left(\frac{x_e}{\sigma^*}\right)^2 > 0,$$

Where, $\sigma^2 \equiv \sigma_{x'}\sigma_{z'}$ and $\sigma^{*2} \equiv \sigma^2_{z'}\left\{1+\left[\left(\frac{\sigma_{z'}}{\sigma_{x'}}\right)^2 - 1\right]\left(\frac{x_p'^2}{x_p'^2 + z_p'^2}\right)\right\}^{-1}$.

When non-essential singularity at $x_p' = 0$ and $z_p' = 0$ is $v = \frac{x_p'^2}{\sigma_{x'}^2} + \frac{z_p'^2}{\sigma_{z'}^2}$.

2. Long-term encounters

Relative motion for in-plane motion and no drift $x_c = C + A\sin\tau$; $y_c = \frac{1}{2}A\cos\tau$ and $z_c = 0$, where $\tau \equiv \omega t + \phi$. The secondary object moves in an ellipse relative to the primary in the orbital plane of the primary with semi-major axis A and semi-minor axis A/2. The volume of integration for determining the collision probability is approximated by a circular torus using the Method of Equivalent Areas. R is the radius of the center of the cross-section from the axis of symmetry of the torus and let r be the radius of the cross-section.

It is seen that when $\phi < 60^\circ$, the largest side of the triangle is L; and when $\phi > 60^\circ$, the largest side of the triangle is S. When $\phi = 60^\circ$, all the three sides are equal. The extent M of the encounter region is defined to be the largest side accordingly.

For a LEO, when $\phi = 2^\circ$ and S = 85 km for 15 digit accuracy, we use equations (6), (12) and (10) to obtain L = 2,435.2 km and D = 403.3 km. Thus, the deflection angle $\alpha = 9.4^\circ$ and the time T to traverse L is 324.7 sec = 5.4 min = 5% of orbital period. This is an exceedingly large deflection angle and the transit time is too long. Consequently, the straight line approximation is invalid. Even if we relax the requirement to 2 digit accuracy by choosing the ingress separation S = 30 km, then L = 859.5 km, D = 51.5 km the deflection angle $\alpha = 3.4^\circ$ and T = 114.6 sec = 1.9 min = 2% of orbital period. This is probably on the verge for the straight line approximation to be valid.

For a GEO, when $\phi = 2^\circ$ and S = 255 km for 15 digit accuracy, we obtain L = 7,305.6 km and D = 628.2 km. Thus, the deflection angle $\alpha = 4.9^\circ$ and the time T to traverse L is 2,356.6 sec = 39 min = 3% of orbital period. Again, these figures are a little too much for the straight line approximation to be valid. Even if we relax the requirement to 2 digit accuracy by choosing the ingress separation S = 90 km, then L = 2,578.4 km, D = 78.8 km the deflection angle $\alpha = 1.8^\circ$ and T = 831.7 sec $\cong$ 13.9 min = 1% of orbital period. The straight line approximation is acceptable in this case.

$$\int_{-\infty}^{+\infty} f_3(x,y,z)\,dy = \frac{1}{2\pi\sigma_x\sigma_z\sqrt{1-\rho_{xz}^2}} e^{-\left(\frac{x^2}{\sigma_x^2} - \frac{2\rho_{xz}xz}{\sigma_x\sigma_z} + \frac{z^2}{\sigma_z^2}\right) \Big/ 2\left(1-\rho_{xz}^2\right)}$$

By integrating the general three-dimensional pdf with respect to the variable y from $-\infty$ to $+\infty$, we have obtained the general marginal two-dimensional pdf. This approach is along the lines of reasoning given by Papoulis [1] who considered the case of integrating a general two-dimensional bivariate Gaussian pdf over the same range to obtain a one-dimensional marginal pdf.

Table 1 - The calculation of preemptive maneuvers

INPUTS ARE IN BOLD NUMBERS	
Minimum Distance M between Primary and Secondary (km)	**0.106066017**
Distance H between Parallel Planes (km)	**0.1**
Distance L of Primary from Point Q when Secondary is at Q (km)	**0.05**
Semi-Major Axis Ap of Primary (km)	**7070.068481**
Primary Arrival Time Tp at Point Q (sec)	**0.02**
Secondary Arrival Time Ts at Point Q (sec)	**0.013333333**
Initial Time T0 (sec)	**0**
SigmaXPrime	**5.099**
SigmaZPrime	**0.707**
Angle Theta (deg)	**70.52877937**
Combined Radius Ra	**0.01**
Desired Probability of Collision PPrime	**1.00E-06**
Days before Maneuver (units of days)	**1.00E+00**
Gamma	**1**

OUTPUTS ARE IN REGULAR NUMBERS	
Delta Ap (m)	17.26519046
Delta Intrack Velocity (m/sec)	-0.027504014
Sigma (km)	1.898681911
SigmaStar (km)	0.748987321
New Minimum Distance M' between Primary and Secondary (km)	1.717679992
Hprime (km)	0.08273481
RhoPrime (dimensionless)	48.52693689
Mean Motion Np of Primary (Radians/sec)	0.001062022
Quantity T	0.007265434
Tau (km)	0.000363272
A (km^2)	-0.001250132
B (km^3)	-4.40893E-07
C (km^4)	3.87872E-07
DeltaAp1 (km)	-0.017970545
DeltaAp2 (km)	0.01726519

Here the results which compute the preemptive maneuvers under the assumption that DeltaAp is not necessarily negligible compared to distance H between the parallel planes.

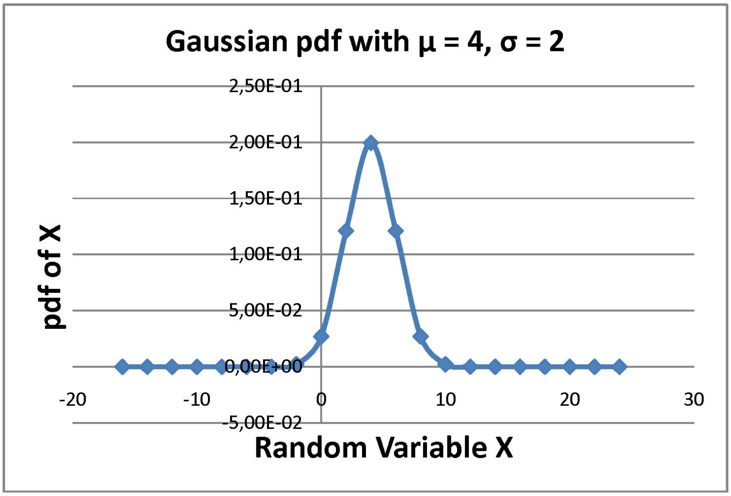
Gaussian Probability Density Function

The model computes the Gaussian probability density function.

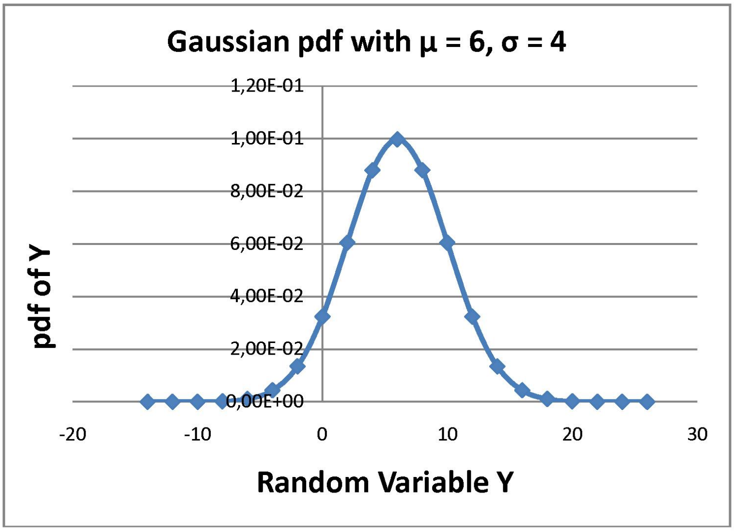
INPUTS	
X1	**-20**
X2	**20**
Sigma	**4.47213 595**

Table 2 - Output results

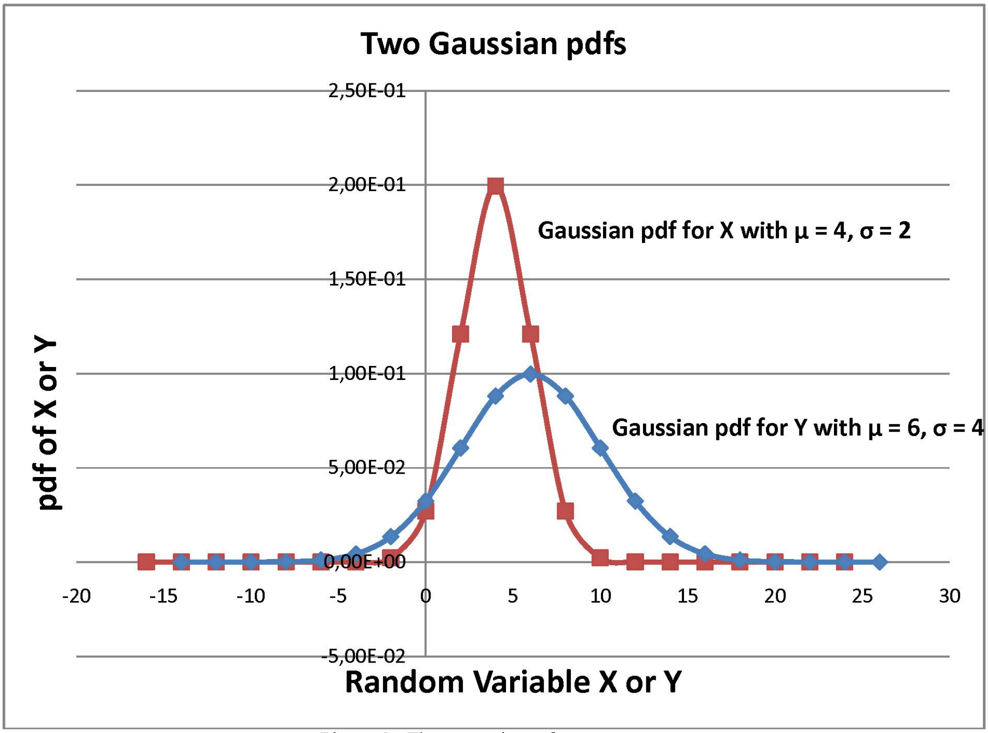
INPUTS (BOLD NUMBERS)										
Minimum Distance M between Primary and Secondary (km)	**2.121320344**	**2.121320344**	**1.060660172**	**1.060660172**	**0.501248441**	**0.501248441**	**0.367423461**	**0.367423461**	**0.106066017**	**0.106066017**
Distance H between Parallel Planes (km)	**0**	**0**	**1**	**1**	**0.5**	**0.5**	**0.1**	**0.1**	**0.1**	**0.1**
Distance L of Primary from Point Q when Secondary is at Q (km)	**3**	**3**	**0.5**	**0.5**	**0.05**	**0.05**	**0.5**	**0.5**	**0.05**	**0.05**
Semi-Major Axis Ap of Primary (km)	**7070.070733**	**7070.070733**	**7070.068795**	**7070.068795**	**7070.068481**	**7070.068481**	**7070.068795**	**7070.068795**	**7070.068481**	**7070.068481**
Primary Arrival Time Tp at Point Q (sec)	**0.533333333**	**0.533333333**	**0.2**	**0.2**	**0.02**	**0.02**	**0.2**	**0.2**	**0.02**	**0.02**
Secondary Arrival Time Ts at Point Q (sec)	**0.133333333**	**0.133333333**	**0.133333333**	**0.133333333**	**0.013333333**	**0.013333333**	**0.133333333**	**0.133333333**	**0.013333333**	**0.013333333**
Initial Time T0 (sec)	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
SigmaXPrime	**10.198**	**5.099**	**10.198**	**5.099**	**10.198**	**5.099**	**10.198**	**5.099**	**10.198**	**5.099**
SigmaZPrime	**1.414**	**0.707**	**1.414**	**0.707**	**1.414**	**0.707**	**1.414**	**0.707**	**1.414**	**0.707**
Angle Theta (deg)	**0**	**0**	**70.52877937**	**70.52877937**	**85.95530876**	**85.95530876**	**15.79316905**	**15.79316905**	**70.52877937**	**70.52877937**
Combined Radius Ra	**0.01**	**0.01**	**0.01**	**0.01**	**0.01**	**0.01**	**0.01**	**0.01**	**0.01**	**0.01**
Desired Probability of Collision PPrime	**1.00E-06**	**1.00E-06**	**1.00E-06**	**1.00E-06**	**1.00E-06**	**1.00E-06**	**1.00E-06**	**1.00E-06**	**1.00E-06**	**1.00E-06**
Days before Maneuver (units of days)	**1.00E+00**	**1.00E+00**	**1.00E+00**	**1.00E+00**	**1.00E+00**	**1.00E+00**	**1.00E+00**	**1.00E+00**	**1.00E+00**	**1.00E+00**
Gamma	**0.99999984**	**0.99999984**	**0.999999978**	**0.999999978**	**1**	**1**	**0.999999978**	**0.999999978**	**1**	**1**
OUTPUTS (REGULAR NUMBERS)										
Delta Ap (m)	143.4364451	98.35103706	18.44459554	10.79570873	22.07290597	15.57811666	71.95401623	51.32730297	23.89596725	17.26519046
Delta Intrack Velocity (m/sec)	-0.22849885	-0.156676351	-0.029382843	-0.017197917	-0.035162863	-0.024816451	-0.114625099	-0.081766071	-0.038067059	-0.027504014
Sigma (km)	3.797363822	1.898681911	3.797363822	1.898681911	3.797363822	1.898681911	3.797363822	1.898681911	3.797363822	1.898681911
SigmaStar (km)	10.198	5.099	1.497974643	0.748987321	1.417462467	0.708731233	4.66498762	2.33249381	1.497974643	0.748987321
New Minimum Distance M' between Primary and Secondary (km)	16.08187469	11.69372301	2.36225147	1.717679992	2.2352867	1.625359234	7.356515623	5.349193283	2.36225147	1.717679992
Hprime (km)	-0.143436422	-0.098351021	0.981555405	0.989204292	0.477927094	0.484421883	0.028045985	0.048672698	0.076104033	0.08273481
RhoPrime (dimensionless)	7.580766884	5.51227892	6.077355341	3.971800018	61.76142695	43.88282652	20.80721712	15.12917704	66.7798783	48.52693689
Mean Motion Np of Primary (Radians/sec)	0.001062021	0.001062021	0.001062022	0.001062022	0.001062022	0.001062022	0.001062022	0.001062022	0.001062022	0.001062022
Quantity T	0.007265437	0.007265437	0.007265434	0.007265434	0.007265434	0.007265434	0.007265434	0.007265434	0.007265434	0.007265434
Tau (km)	0.021796312	0.021796312	0.003632717	0.003632717	0.000363272	0.000363272	0.003632717	0.003632717	0.000363272	0.000363272
A (km^2)	-4.500475079	-4.500475079	-0.125013197	-0.125013197	-0.001250132	-0.001250132	-0.125013197	-0.125013197	-0.001250132	-0.001250132
B (km^3)	-0.098083402	-0.098083402	-0.000440893	-0.000440893	-3.88106E-07	-3.88106E-07	-0.00045277	-0.00045277	-4.40893E-07	-4.40893E-07
C (km^4)	0.120730305	0.062825973	5.87941E-05	2.40895E-05	6.26214E-07	3.15471E-07	0.000712398	0.000375825	7.34918E-07	3.87872E-07
DeltaAp1 (km)	-0.187024466	-0.141939058	-0.025498139	-0.017849252	-0.022693811	-0.016199021	-0.079197571	-0.058570858	-0.024601322	-0.017970545
DeltaAp2 (km)	0.143436445	0.098351037	0.018444596	0.010795709	0.022072906	0.015578117	0.071954016	0.051327303	0.023895967	0.01726519



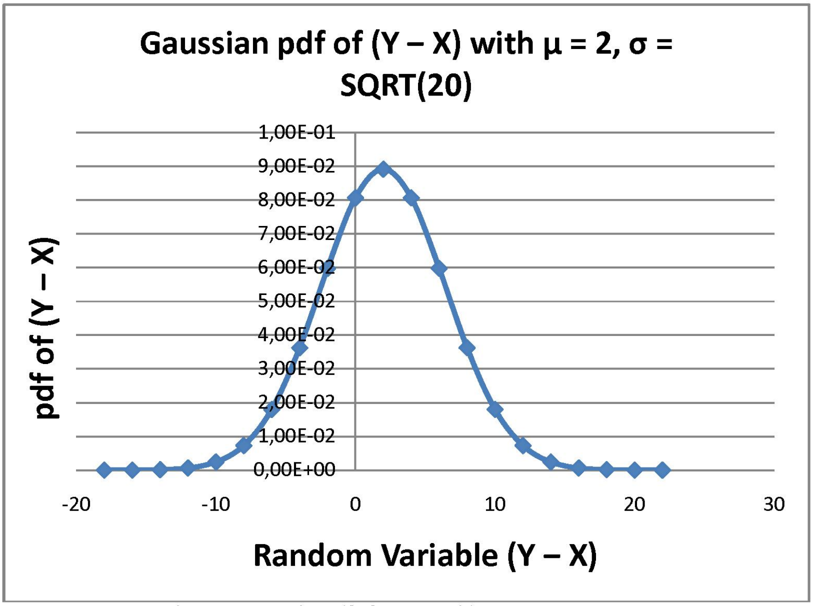
Picture 1 - The Gaussian Probability Density Function with μ = 4, σ = 2



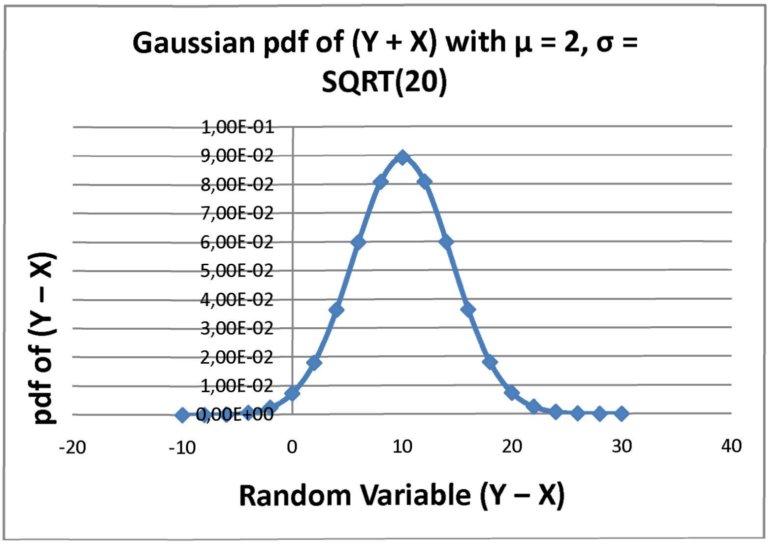
Picture 2 - The Gaussian Probability Density Function with μ = 6, σ = 4



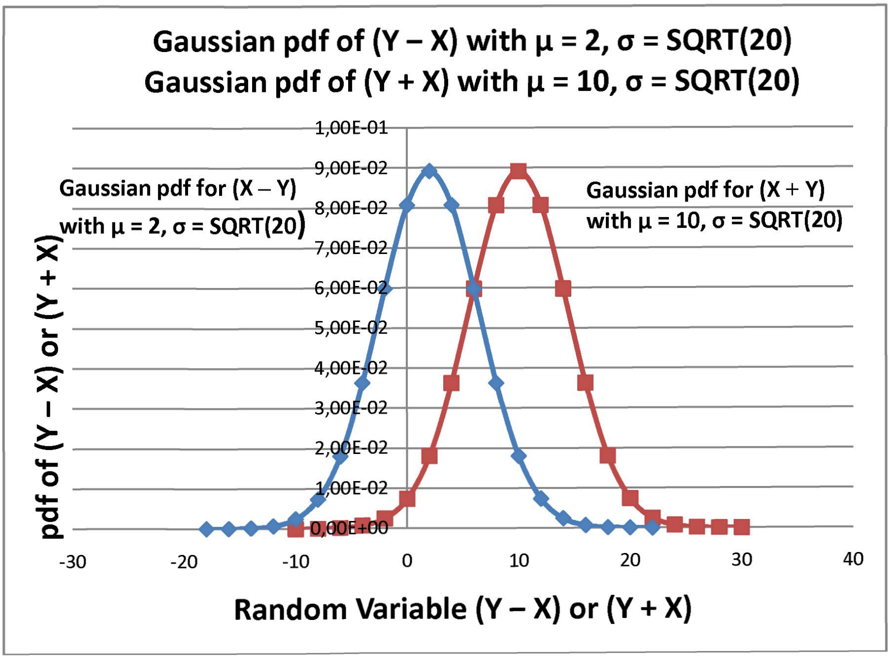
Picture 3 - The comparison of two outcomes



Picture 4 - Gaussian pdf of (Y – X) with $\mu = 2$, $\sigma = SQRT(20)$



Picture 5 - The Gaussian pdf of (Y + X) with μ = 2, σ = SQRT(20)



Picture 6 - Gaussian pdf of (Y – X) with μ = 2, σ = SQRT(20), Gaussian pdf of (Y + X) with μ = 10, σ = SQRT(20)

Conclusion

The study has shown that when assuming the probability of the collision between the objects and their differences in results obtained between long-term encounter modeling and short-term encounter modeling. It can be seen that the maximum probability of collision is when the amplitude A is about 1.4 times of the combined standard deviation.

The condition at which for LEO is the straight path of the relative motion over a distance of 85 km (or 30 km) where the deflection angle is less than 0.18 degrees (or 0.06 deg.). To satisfy the condition for the short-term encounters the relative velocities are of the order of several kilometers per second, the time spend in the encounter region is only a fraction of a second or at most a few seconds. On contrarily, the time spent for long-term encounters can take more than one orbital period and can take days. The parameters must be taken into consideration to obtain a maximum collision probability.

The project has been done in accordance to № 0003-1/ПЦФ-15 The study of optical and dynamic characteristics of space debris fragments in the geostationary orbit.

REFERENCES

[1] Popuolis A. Probability, Random Variables and Stochastic Processes, fourth edition. McGraw-Hill , U.S.A. ISBN: 978-0-070-48477-1

[2] Nazarenko A.I., et.al. Collision Of Spacecraft Of Various Shape With Debris Particles Assessment, e-mail correspondence, P. 11

[3] Назаренко А.И. Моделирование Космического Мусора. Федеральное Государственное Бюджетное Учреждение Науки Институт Космических Исследований Российской Академии Наук (ИКИ РАН), МОСКВА, 2013

[4] Kenneth Chan. Module 14: Miss Distance. Iridium33-Cosmos2251 Collision, Close Approach Statistics. e-mail correspondence.

[5] Kenneth Chan F. (2008). Spacecraft Collision Probability, Aerospace Press, U.S.A. ISBN: 978-1-884-98918-6

[6] Ken Chan, et.al. A Mathematical Formulation to Describe Density of Particles in an Inhomogeneous Distribution. Chan Aerospace Consultants. e-mail correspondence

[7] Chan F. K. (1997), Collision Probability Analyses for Earth-Orbiting Satellites. Proceedings of the 7th International Space Conference of Pacific Basin Societies. Nagasaki, Japan.

[8] Nazarenko A.I. (2007). Accuracy of Determination and Prediction Orbits in LEO. Estimation Errors Depending on Accuracy and Amount of Measurements. 7th US/ Russian Space Surveillance Workshop. Monterey, U.S.A.

[9] Назаренко А. И., Маркова Л. Г. Методы определения и прогнозирования орбит ИСЗ при наличии погрешностей в математическом описании движения. Прикладные задачи космической баллистики. Наука, 1973. С. 36–67.

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ҒАРЫШТЫҚ ҚОҚЫС БҰЛТЫНДАҒЫ ОБЪЕКТТЕРДІҢ СОҚТЫҒЫСУ ЫҚТИМАЛДЫЛЫҒЫН АНЫҚТАУДЫ БОЛЖАУҒА АРНАЛҒАН ҚЫСҚА ЖӘНЕ ҰЗАҚ МЕРЗІМДІ ӘДІС

Аннотация. Негізгі зерттеу аясы – орбитадағы сынықтардың өздігінен пайда болуы. Бұл жағдай негізінен орбиталды сынықтардың өзара соқтығысуынан болады. Ғарыштық қоқыс бұлттарының эволюциясына және өзара соқтығысқан объекттер бұлттарына берілген талдау ұстанымды ковариялық эллипсоидтарды және қолда бар мәліметтерді пайдалана отырып Гаусс таралуына негізделген. Сосын соқтығысу ықтималдылығын ұсақ объекттердің пайда болуына әкеліп соғатын энергияның ыдырауы бойынша экспериментальды нәтижелерімен салыстырылды. Екі объекттердің позициясының белгісіздігін сипаттайтын тығыздықтың ықтималдылық функциясы (pdf) олардың берілген көлемде екендігінің ықтималдылығын сипаттау үшін қолданылды. Кеңістіктегі объекттің салыстырмалық орны басқа объект орнымен салыстырмалы есептелген соң олардың салыстырмалық орын қателігі алынды. Бұл рәсімдер бір объекттің басқасына жақындау аймағының бір бөлігін көлем бойынша интегралдауға мүмкіндік берді.

Тірек сөздер: ғарыштық қоқыс, соқтығысу ықтималдылығы, ковариациялық эллипсоидтар, Гаусс таралулары.

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КОРОТКО- ВРЕМЕНОЙ И ДОЛГОВРЕМЕННОЙ ПОДХОД ДЛЯ ПРОГНОЗА ОПРЕДЕЛЕНИЯ ВЕРОЯТНОСТИ СТОЛКНОВЕНИЯ ОБЪЕКТОВ В ОБЛАКЕ КОСМИЧЕСКОГО МУСОРА

Аннотация. Основная область исследования - самопроизводство обломков на орбите. Это производство, в основном, осуществляется путем взаимных столкновений между орбитальными обломками. Представленный анализ эволюции облаков космического мусора и взаимных столкновений между объектами облака был основан на распределении Гаусса с использованием позиционных ковариационных эллипсоидов и имеющихся данных. Затем вероятность столкновения сравнивали с экспериментальными результатами по энергии разрыва, которые могут приводить к появлению все более и более мелких объектов. Функция плотности вероятности (pdf), которая описывает неопределенности позиций двух объектов, использовалась для определения вероятности того, что они находятся в заданном объеме. Затем, после того как относительное положение объекта в пространстве относительно другого было рассчитано, была получена их относительная погрешность положений. Эти процедуры позволили интегрировать по объему часть региона, где один объект приближается к другому.

Ключевые слова: космический мусор, вероятность столкновения, ковариационные эллипсоиды, распределения Гаусса.

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