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ON THE NONLOCAL PROBLEM FOR A SYSTEM OF THE PARTIAL INTEGRO-DIFFERENTIAL EQUATIONS OF HYPERBOLIC TYPE

Abstract. The nonlocal problem with data on the characteristics for the system of integro-differential equations of hyperbolic type second order is considered. The questions of the existence and uniqueness of a classical solution to the nonlocal problem are studied. The considered problem is reduced to an equivalent nonlocal problem with integral condition by introducing a new unknown function instead of a integral term in the system of equations. The problem with parameter consists of a nonlocal problem for a system of hyperbolic equations with parameter and the integral relation. Algorithms for finding an approximate solution of the equivalent problem with parameter are constructed and the conditions for their convergence are proved. Sufficient conditions for the existence of unique solution to the problem with parameter are established. Conditions of existence of unique classical solution to the nonlocal problem for the system of integro-differential equations of hyperbolic type are obtained in the terms of initial data. Earlier, the method of reduced to an equivalent family of problems for partial differential equations is applied to study of this problem. Sufficient conditions for the existence of unique classical solution of this problem are found in the terms of some matrix compiled by the initial data.

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Key words: nonlocal problem, system of partial integro-differential equations, parameter, algorithm, approximate solution, unique solvability.

1. Introduction. Note that the interest of nonlocal problems for partial integro-differential equations of hyperbolic type has grown. Nonlocal problems are called boundary value problems, in which instead of the classical boundary conditions for the partial integro-differential equations it is given specified combination of values of the unknown function on the boundary of the domain and within it. Boundary conditions are set on the characteristics of the system of hyperbolic equations. The existence and uniqueness of the classical solutions to nonlocal problems for system of hyperbolic integro-differential equations are set. In the present work we consider the system of hyperbolic integro-differential equations of second order in a rectangular domain. Boundary conditions are specified as a combination of values from the required solution and their partial derivatives on first order. We investigate the questions of existence and uniqueness of the classical solution to nonlocal problem for system of hyperbolic integro-differential equations and its applications. For solve to considered problem we use a method of introduction additional functional parameters. The original problem is reduced to an equivalent problem consisting from Goursat problem for system of hyperbolic equations with functional parameters and integral relations. Sufficient conditions of the unique solvability to investigated problem are established in the terms of initial data. Algorithms of finding solution to the nonlocal problem are constructed. The applicability of the obtained results in an optimal control problems are showed.

2. Statement of problem. On the domain $\Omega = [0,T]\times[0,\omega]$ we consider the nonlocal problem with data on the characteristics for system of partial integro-differential equations of hyperbolic type of second order

$$\frac{\partial^2 u}{\partial t \partial x} = A(t,x)\frac{\partial u}{\partial x} + B(t,x)\frac{\partial u}{\partial t} + C(t,x)u +$$

$$+\int_0^\theta \left[K_1(t,\xi)\frac{\partial u(t,\xi)}{\partial \xi} + K_2(t,\xi)\frac{\partial u(t,\xi)}{\partial t} + K_3(t,\xi)u(t,\xi)\right] d\xi + f(t,x),\ (t,x)\in\Omega, \qquad (1)$$

$$u(t,0) = \psi(t),\ t\in[0,T], \qquad (2)$$

$$P_2(x)\frac{\partial u(0,x)}{\partial x} + P_1(x)\frac{\partial u(t,x)}{\partial t}\Big|_{t=0} + P_0(x)u(0,x) + L_2(x)\frac{\partial u(\theta,x)}{\partial x} + L_1(x)\frac{\partial u(t,x)}{\partial t}\Big|_{t=\theta} +$$

$$+ L_0(x)u(\theta,x) + S_2(x)\frac{\partial u(T,x)}{\partial x} + S_1(x)\frac{\partial u(t,x)}{\partial t}\Big|_{t=T} + S_0(x)u(T,x) = \varphi(x),\ x\in[0,\omega], \qquad (3)$$

where $u(t,x) = col(u_1(t,x), u_2(t,x),\ldots,u_n(t,x))$, the $n\times n$-matrices $A(t,x)$, $B(t,x)$, $C(t,x)$, $K_1(t,x)$, $K_2(t,x)$, $K_3(t,x)$ and n-vector-function $f(t,x)$ are continuous on Ω, the n-vector-function $\psi(t)$ is continuously differentiable on $[0,T]$, the $n\times n$-matrices $P_i(x)$, $L_i(x)$, $S_i(x)$, and n-vector-function $\varphi(x)$ are continuous on $[0,\omega]$, $i=0,1,2$, $0\le\theta\le\min(T,\omega)$. The initial data satisfy the condition of approval.

A function $u(t,x)\in C(\Omega,R^n)$ having partial derivatives $\frac{\partial u(t,x)}{\partial x}\in C(\Omega,R^n)$, $\frac{\partial u(t,x)}{\partial t}\in C(\Omega,R^n)$, $\frac{\partial^2 u(t,x)}{\partial t\partial x}\in C(\Omega,R^n)$ is called a classical solution to problem (1)-(3) if for all $(t,x)\in\Omega$ it satisfies the system (1), boundary conditions (2) and (3).

In the present paper we investigate the questions of existence and uniqueness of the classical solutions to the nonlocal problem for system of hyperbolic integro-differential equations (1)-(3) and the approaches of constructing its approximate solutions. For this goals, we applied the method of introduction additional functional parameters proposed in [1-18] for the solve of nonlocal boundary value problems for systems of hyperbolic equations with mixed derivative. Considered problem is provided to nonlocal problem with integral condition for system of hyperbolic equations including additional function. Hence, this problem is reduced to an equivalent problem, consisting of Goursat problem for the system of hyperbolic equations with functional parameters and Cauchy problem for system of ordinary differential equations with respect to the entered parameters by introducing new unknown functions. The algorithm of finding the approximate solution of the investigated problem is proposed and its convergence proved. Sufficient conditions of the existence of unique classical solution to problem (1)-(3) are obtained in the terms of initial data. The applicability of the obtained results in the optimal control problems is showed.

3. Reduction to nonlocal problem with integral condition for system of hyperbolic equations including special function. We introduce an additional special function

$$\mu(t) = \int_0^\theta \left[K_1(t,\xi)\frac{\partial u(t,\xi)}{\partial \xi} + K_2(t,\xi)\frac{\partial u(t,\xi)}{\partial \xi} + K_3(t,\xi)u(t,\xi)\right] d\xi$$

and write the problem (1)-(3) in the following form

$$\frac{\partial^2 u}{\partial t\partial x} = A(t,x)\frac{\partial u}{\partial x} + B(t,x)\frac{\partial u}{\partial t} + C(t,x)u + \mu(t) + f(t,x),\ (t,x)\in\Omega, \qquad (4)$$

$$u(t,0) = \psi(t),\ t\in[0,T], \qquad (5)$$

$$P_2(x)\frac{\partial u(0,x)}{\partial x} + P_1(x)\frac{\partial u(t,x)}{\partial t}\Big|_{t=0} + P_0(x)u(0,x) + L_2(x)\frac{\partial u(\theta,x)}{\partial x} + L_1(x)\frac{\partial u(t,x)}{\partial t}\Big|_{t=\theta} +$$

$$+ L_0(x)u(\theta,x) + S_2(x)\frac{\partial u(T,x)}{\partial x} + S_1(x)\frac{\partial u(t,x)}{\partial t}\Big|_{t=T} + S_0(x)u(T,x) = \varphi(x),\ x\in[0,\omega], \qquad (6)$$

$$\mu(t)=\int_0^{\theta}\left[K_1(t,\xi)\frac{\partial u(t,\xi)}{\partial \xi}+K_2(t,\xi)\frac{\partial u(t,\xi)}{\partial t}+K_3(t,\xi)u(t,\xi)\right]d\xi,\ t\in[0,T]. \quad (7)$$

4. Scheme of the method and algorithm. Let $\lambda(x)=u(0,x)$. In the problem (4)-(7) we change the function $u(t,x)$ by $u(t,x)=\tilde{u}(t,x)+\lambda(x)$ and proceed to the following equivalent problem

$$\frac{\partial^2\tilde{u}}{\partial t\partial x}=A(t,x)\frac{\partial\tilde{u}}{\partial x}+B(t,x)\frac{\partial\tilde{u}}{\partial t}+C(t,x)\tilde{u}+A(t,x)\dot{\lambda}(x)+C(t,x)\lambda(x)+\mu(t)+f(t,x), \quad (8)$$

$$\tilde{u}(t,0)+\lambda(0)=\psi(t),\ t\in[0,T],\ (9)$$

$$\tilde{u}(0,x)=0,\ x\in[0,\omega],\ (10)$$

$$[P_2(x)+L_2(x)+S_2(x)]\dot{\lambda}(x)+[P_0(x)+L_0(x)+S_0(x)]\lambda(x)+$$
$$+P_1(x)\frac{\partial\tilde{u}(t,x)}{\partial t}\Big|_{t=0}+L_2(x)\frac{\partial\tilde{u}(\theta,x)}{\partial x}+L_1(x)\frac{\partial\tilde{u}(t,x)}{\partial t}\Big|_{t=\theta}+L_0(x)\tilde{u}(\theta,x)+S_2(x)\frac{\partial\tilde{u}(T,x)}{\partial x}+$$
$$+S_1(x)\frac{\partial\tilde{u}(t,x)}{\partial t}\Big|_{t=T}+S_0(x)\tilde{u}(T,x)=\varphi(x),\qquad x\in[0,\omega]. \quad (11)$$

$$\mu(t)=\int_0^{\theta}\left[K_1(t,\xi)\frac{\partial\tilde{u}(t,\xi)}{\partial\xi}+K_2(t,\xi)\frac{\partial\tilde{u}(t,\xi)}{\partial t}+K_3(t,\xi)\tilde{u}(t,\xi)+K_1(t,\xi)\dot{\lambda}(\xi)+K_3(t,\xi)\lambda(\xi)\right]d\xi. \quad (12)$$

A triple functions $(\tilde{u}(t,x),\lambda(x),\mu(t))$, where the function $\tilde{u}(t,x)\in C(\Omega,R^n)$ has partial derivatives $\frac{\partial\tilde{u}(t,x)}{\partial x}\in C(\Omega,R^n)$, $\frac{\partial\tilde{u}(t,x)}{\partial t}\in C(\Omega,R^n)$, $\frac{\partial^2\tilde{u}(t,x)}{\partial t\partial x}\in C(\Omega,R^n)$, the function $\lambda(x)\in C([0,\omega],R^n)$ has derivative $\dot{\lambda}(x)\in C([0,\omega],R^n)$, the function $\mu(t)\in C([0,T],R^n)$ determine from relations (12) for all $t\in[0,T]$, is called a solution to problem (8)-(12) if for all $(t,x)\in\Omega$ it satisfies of the system of hyperbolic equations with parameters (8), the boundary conditions (9), (10), the functional relation (11) and the integral condition (12).

From the compatibility condition at the point (0,0) of initial data is yield: $\lambda(0)=\psi(0)$. Then the condition (9) may be rewrite in the following form

$$\tilde{u}(t,0)=\psi(t)-\psi(0),\ t\in[0,T]. \quad (13)$$

The problem (8), (10), (13) at fixed $\lambda(x)$, $\mu(t)$ is the Goursat problem with respect to $\tilde{u}(t,x)$ on Ω. The relation (11) allows us to determine the unknown functional parameter $\lambda(x)$. The integral condition (12) allows us to determine the unknown function $\mu(t)$ for all $t\in[0,T]$.

We introduce new unknown functions $\tilde{v}(t,x)=\frac{\partial\tilde{u}(t,x)}{\partial x}$, $\tilde{w}(t,x)=\frac{\partial\tilde{u}(t,x)}{\partial t}$. Goursat problem (8), (10), (13) is equivalent to a three systems integral equations

$$\tilde{v}(t,x)=\int_0^t\{A(\tau,x)\tilde{v}(\tau,x)+B(\tau,x)\tilde{w}(\tau,x)+C(\tau,x)\tilde{u}(\tau,x)+$$
$$+A(\tau,x)\dot{\lambda}(x)+C(\tau,x)\lambda(x)+\mu(\tau)+f(\tau,x)\}d\tau,\ (14)$$

$$\tilde{w}(t,x)=\dot{\psi}(t)+\int_0^x\{A(t,\xi)\tilde{v}(t,\xi)+B(t,\xi)\tilde{w}(t,\xi)+C(t,\xi)\tilde{u}(t,\xi)+$$
$$+A(t,\xi)\dot{\lambda}(\xi)+C(t,\xi)\lambda(\xi)+\mu(t)+f(t,\xi)\}d\xi,\ (15)$$

$$\tilde{u}(t,x)=\psi(t)-\psi(0)+\int_0^t\int_0^x\{A(\tau,\xi)\tilde{v}(\tau,\xi)+B(\tau,\xi)\tilde{w}(\tau,\xi)+C(\tau,\xi)\tilde{u}(\tau,\xi)+$$
$$+A(\tau,\xi)\dot{\lambda}(\xi)+C(\tau,\xi)\lambda(\xi)+\mu(\tau)+f(\tau,\xi)\}d\xi d\tau. \quad (16)$$

In the relation (11) instead of the functions $\tilde{v}(\theta,x)$, $\tilde{v}(T,x)$, we substitute the appropriate expressions of the integral relation under $t=\theta$, $t=T$, respectively. Then we obtain

$$\left[P_2(x)+L_2(x)+L_2(x)\int_0^\theta A(\tau,x)d\tau+S_2(x)+S_2(x)\int_0^T A(\tau,x)d\tau\right]\dot{\lambda}(x)=$$

$$=-\left[P_0(x)+L_0(x)+S_0(x)+L_2(x)\int_0^\theta C(\tau,x)d\tau+S_2(x)\int_0^T C(\tau,x)d\tau\right]\lambda(x)-$$

$$-L_2(x)\int_0^\theta \mu(\tau)d\tau-S_2(x)\int_0^T \mu(\tau)d\tau-P_1(x)\tilde{w}(0,x)-L_1(x)\tilde{w}(\theta,x)-S_1(x)\tilde{w}(T,x)-$$

$$-L_2(x)\int_0^\theta \{A(\tau,x)\tilde{v}(\tau,x)+B(\tau,x)\tilde{w}(\tau,x)+C(\tau,x)\tilde{u}(\tau,x)\}d\tau-L_0(x)\tilde{u}(\theta,x)-$$

$$-S_2(x)\int_0^T \{A(\tau,x)\tilde{v}(\tau,x)+B(\tau,x)\tilde{w}(\tau,x)+C(\tau,x)\tilde{u}(\tau,x)\}d\tau-S_0(x)\tilde{u}(T,x)-$$

$$-L_2(x)\int_0^\theta f(\tau,x)d\tau-S_2(x)\int_0^T f(\tau,x)d\tau+\varphi(x),\ x\in[0,\omega]. \qquad (17)$$

From the compatibility condition follows the initial condition
$\lambda(0)=\psi(0)$. (18)

The unknown functional parameter $\lambda(x)$ will be determined from Cauchy problem for system of ordinary differential equations (17), (18). The unknown special function $\mu(t)$ will be determined from integral relation (12).

If we know the functional parameter $\lambda(x)$, the special function $\mu(t)$, then from integral systems (14)-(16) find the functions $\tilde{u}(t,x)$, $\tilde{v}(t,x)$, $\tilde{w}(t,x)$. Conversely, if we know functions $\tilde{u}(t,x)$, $\tilde{v}(t,x)$, $\tilde{w}(t,x)$, then from Cauchy problem (17), (18) and integral condition (12) we find the functional parameter $\lambda(x)$ and special function $\mu(t)$. Since the functions $\tilde{u}(t,x)$, $\tilde{v}(t,x)$, $\tilde{w}(t,x)$ and $\lambda(x)$, $\mu(t)$ are unknown together for finding of the solution to problem (8)--(12) we use an iterative method. The solution to problem (8)--(12) is the triple functions $(\tilde{u}^*(t,x),\lambda^*(x),\mu^*(t))$ we defined as a limit of sequence of triples $(\tilde{u}^{(m)}(t,x),\lambda^{(m)}(x),\mu^{(m)}(t,x))$, $m=0,1,2,...$, according to the following algorithm:

Step 0. 1) Let the matrix $D_1(x)=P_2(x)+L_2(x)+L_2(x)\int_0^\theta A(\tau,x)d\tau+S_2(x)+S_2(x)\int_0^T A(\tau,x)d\tau$ is invertible for all $x\in[0,\omega]$. Suppose in the right-hand part of the system (17) $\mu(t)=0$, $\tilde{u}(t,x)=\psi(t)-\psi(0)$, $\tilde{v}(t,x)=0$, $\tilde{w}(t,x)=\dot{\psi}(t)$, from Cauchy problem (17), (18) we find the initial approximation $\lambda^{(0)}(x)$ for all $x\in[0,\omega]$:

$$\lambda^{(0)}(x)=\psi(0)-\int_0^x D_1^{-1}(\xi)D_2(\xi)\lambda^{(0)}(\xi)d\xi-$$

$$-\int_0^x D_1^{-1}(\xi)[P_1(\xi)\dot{\psi}(0)+L_1(\xi)\dot{\psi}(\theta)+S_1(\xi)\dot{\psi}(T)+L_0(\xi)\{\psi(\theta)-\psi(0)\}+S_0(\xi)\{\psi(T)-\psi(0)\}]d\xi-$$

$$-\int_0^x D_1^{-1}(\xi)L_2(\xi)\int_0^\theta \{B(\tau,\xi)\dot{\psi}(\tau)+C(\tau,\xi)[\psi(\tau)-\psi(0)]\}d\tau d\xi-$$

$$-\int_0^x D_1^{-1}(\xi)S_2(\xi)\int_0^T \{B(\tau,\xi)\dot{\psi}(\tau)+C(\tau,\xi)[\psi(\tau)-\psi(0)]\}d\tau d\xi -$$

$$-\int_0^x D_1^{-1}(\xi)\left[L_2(\xi)\int_0^\theta f(\tau,\xi)d\tau + S_2(\xi)\int_0^T f(\tau,\xi)d\tau - \varphi(\xi)\right]d\xi ,$$

where $$D_2(x) = P_0(x) + L_0(x) + S_0(x) + L_2(x)\int_0^\theta C(\tau,x)d\tau + S_2(x)\int_0^T C(\tau,x)d\tau .$$

2) From the system of integral equations (14)--(16) under $\mu(t)=0$, $\lambda(x)=\lambda^{(0)}(x)$, $\dot{\lambda}(x)=\dot{\lambda}^{(0)}(x)$ we find the functions $\tilde{u}^{(0)}(t,x)$, $\tilde{v}^{(0)}(t,x)$, $\tilde{w}^{(0)}(t,x)$ for all $(t,x)\in\Omega$:

$$\tilde{v}^{(0)}(t,x) = \int_0^t \{A(\tau,x)\tilde{v}^{(0)}(\tau,x) + B(\tau,x)\tilde{w}^{(0)}(\tau,x) + C(\tau,x)\tilde{u}^{(0)}(\tau,x) +$$
$$+ A(\tau,x)\dot{\lambda}^{(0)}(x) + C(\tau,x)\lambda^{(0)}(x) + f(\tau,x)\}d\tau ,$$

$$\tilde{w}^{(0)}(t,x) = \dot{\psi}(t) + \int_0^x \{A(t,\xi)\tilde{v}^{(0)}(t,\xi) + B(t,\xi)\tilde{w}^{(0)}(t,\xi) + C(t,\xi)\tilde{u}^{(0)}(t,\xi) +$$
$$+ A(t,\xi)\dot{\lambda}^{(0)}(\xi) + C(t,\xi)\lambda^{(0)}(\xi) + f(t,\xi)\}d\xi ,$$

$$\tilde{u}^{(0)}(t,x) = \psi(t) - \psi(0) + \int_0^t \int_0^x \{A(\tau,\xi)\tilde{v}^{(0)}(\tau,\xi) + B(\tau,\xi)\tilde{w}^{(0)}(\tau,\xi) + C(\tau,\xi)\tilde{u}^{(0)}(\tau,\xi) +$$
$$+ A(\tau,\xi)\dot{\lambda}^{(0)}(\xi) + C(\tau,\xi)\lambda^{(0)}(\xi) + f(\tau,\xi)\}d\xi d\tau .$$

From integral relation (12) under $\tilde{u}(t,x)=\tilde{u}^{(0)}(t,x)$, $\tilde{v}(t,x)=\tilde{v}^{(0)}(t,x)$, $\tilde{w}(t,x)=\tilde{w}^{(0)}(t,x)$, $\lambda(x)=\lambda^{(0)}(x)$, $\dot{\lambda}(x)=\dot{\lambda}^{(0)}(x)$ we find the initial approximation $\mu^{(0)}(t)$ for all $t\in[0,T]$:

$$\mu^{(0)}(t) = \int_0^\theta \left[K_1(t,\xi)\tilde{v}^{(0)}(t,\xi) + K_2(t,\xi)\tilde{w}^{(0)}(t,\xi) + K_3(t,\xi)\tilde{u}^{(0)}(t,\xi) + K_1(t,\xi)\dot{\lambda}^{(0)}(\xi) + K_3(t,\xi)\lambda^{(0)}(\xi)\right]d\xi .$$

Step 1. 1) Suppose in the right-hand part of the system (17) $\mu(t)=\mu^{(0)}(t)$, $\tilde{u}(t,x)=\tilde{u}^{(0)}(t,x)$, $\tilde{v}(t,x)=\tilde{v}^{(0)}(t,x)$, $\tilde{w}(t,x)=\tilde{w}^{(0)}(t,x)$, from Cauchy problem (17), (18) we find the first approximation $\lambda^{(1)}(x)$ for all $x\in[0,\omega]$:

$$\lambda^{(1)}(x) = \psi(0) - \int_0^x D_1^{-1}(\xi)D_2(\xi)\lambda^{(1)}(\xi)d\xi -$$

$$-\int_0^x D_1^{-1}(\xi)L_2(\xi)d\xi\int_0^\theta \mu^{(0)}(\tau)d\tau - \int_0^x D_1^{-1}(\xi)S_2(\xi)d\xi\int_0^T \mu^{(0)}(\tau)d\tau -$$

$$-\int_0^x D_1^{-1}(\xi)[P_1(\xi)\tilde{w}^{(0)}(0,\xi) + L_1(\xi)\tilde{w}^{(0)}(\theta,\xi) + S_1(\xi)\tilde{w}^{(0)}(T,\xi) + L_0(\xi)\tilde{u}^{(0)}(\theta,\xi) + S_0(\xi)\tilde{u}^{(0)}(T,\xi)]d\xi -$$

$$-\int_0^x D_1^{-1}(\xi)L_2(\xi)\int_0^\theta \{A(\tau,\xi)\tilde{v}^{(0)}(\tau,\xi)B(\tau,\xi)\tilde{w}^{(0)}(\tau,\xi) + C(\tau,\xi)\tilde{u}^{(0)}(\tau,\xi)\}d\tau d\xi -$$

$$-\int_0^x D_1^{-1}(\xi)S_2(\xi)\int_0^T \{A(\tau,\xi)\tilde{v}^{(0)}(\tau,\xi)B(\tau,\xi)\tilde{w}^{(0)}(\tau,\xi) + C(\tau,\xi)\tilde{u}^{(0)}(\tau,\xi)\}d\tau d\xi -$$

$$-\int_0^x D_1^{-1}(\xi)\left[L_2(\xi)\int_0^\theta f(\tau,\xi)d\tau + S_2(\xi)\int_0^T f(\tau,\xi)d\tau - \varphi(\xi)\right]d\xi .$$

2) From the system of integral equations (14)--(16) under $\mu(t)=\mu^{(0)}(t)$, $\lambda(x)=\lambda^{(1)}(x)$, $\dot{\lambda}(x)=\dot{\lambda}^{(1)}(x)$ we find the functions $\tilde{u}^{(1)}(t,x)$, $\tilde{v}^{(1)}(t,x)$, $\tilde{w}^{(1)}(t,x)$ for all $(t,x)\in\Omega$:

$$\tilde{v}^{(1)}(t,x)=\int_0^t\{A(\tau,x)\tilde{v}^{(1)}(\tau,x)+B(\tau,x)\tilde{w}^{(1)}(\tau,x)+C(\tau,x)\tilde{u}^{(1)}(\tau,x)+$$
$$+A(\tau,x)\dot{\lambda}^{(1)}(x)+C(\tau,x)\lambda^{(1)}(x)+\mu^{(0)}(\tau)+f(\tau,x)\}d\tau,$$

$$\tilde{w}^{(1)}(t,x)=\psi(t)+\int_0^x\{A(t,\xi)\tilde{v}^{(1)}(t,\xi)+B(t,\xi)\tilde{w}^{(1)}(t,\xi)+C(t,\xi)\tilde{u}^{(1)}(t,\xi)+$$
$$+A(t,\xi)\dot{\lambda}^{(1)}(\xi)+C(t,\xi)\lambda^{(1)}(\xi)+\mu^{(0)}(t)+f(t,\xi)\}d\xi,$$

$$\tilde{u}^{(1)}(t,x)=\psi(t)-\psi(0)+\int_0^t\int_0^x\{A(\tau,\xi)\tilde{v}^{(1)}(\tau,\xi)+B(\tau,\xi)\tilde{w}^{(1)}(\tau,\xi)+C(\tau,\xi)\tilde{u}^{(1)}(\tau,\xi)+$$
$$+A(\tau,\xi)\dot{\lambda}^{(1)}(\xi)+C(\tau,\xi)\lambda^{(1)}(\xi)+\mu^{(0)}(\tau)+f(\tau,\xi)\}d\xi d\tau.$$

From integral relation (12) under $\tilde{u}(t,x)=\tilde{u}^{(1)}(t,x)$, $\tilde{v}(t,x)=\tilde{v}^{(1)}(t,x)$, $\tilde{w}(t,x)=\tilde{w}^{(1)}(t,x)$, $\lambda(x)=\lambda^{(1)}(x)$, $\dot{\lambda}(x)=\dot{\lambda}^{(1)}(x)$ we find the first approximation $\mu^{(1)}(t)$ for all $t\in[0,T]$:

$$\mu^{(1)}(t)=\int_0^\theta\left[K_1(t,\xi)\tilde{v}^{(1)}(t,\xi)+K_2(t,\xi)\tilde{w}^{(1)}(t,\xi)+K_3(t,\xi)\tilde{u}^{(1)}(t,\xi)+K_1(t,\xi)\dot{\lambda}^{(1)}(\xi)+K_3(t,\xi)\lambda^{(1)}(\xi)\right]d\xi.$$

And so on.

Step m. 1) Suppose in the right-hand part of the system (17) $\mu(t)=\mu^{(m-1)}(t)$, $\tilde{u}(t,x)=\tilde{u}^{(m-1)}(t,x)$, $\tilde{v}(t,x)=\tilde{v}^{(m-1)}(t,x)$, $\tilde{w}(t,x)=\tilde{w}^{(m-1)}(t,x)$, from Cauchy problem (17), (18) we find the m-th approximation $\lambda^{(m)}(x)$ for all $x\in[0,\omega]$:

$$\lambda^{(m)}(x)=\psi(0)-\int_0^x D_1^{-1}(\xi)D_2(\xi)\lambda^{(m)}(\xi)d\xi-\int_0^x D_1^{-1}(\xi)\left[L_2(\xi)\int_0^\theta\mu^{(m-1)}(\tau)d\tau+S_2(\xi)\int_0^T\mu^{(m-1)}(\tau)d\tau\right]d\xi-$$

$$-\int_0^x D_1^{-1}(\xi)[P_1(\xi)\tilde{w}^{(m-1)}(0,\xi)+L_1(\xi)\tilde{w}^{(m-1)}(\theta,\xi)+S_1(\xi)\tilde{w}^{(m-1)}(T,\xi)+L_0(\xi)\tilde{u}^{(m-1)}(\theta,\xi)+S_0(\xi)\tilde{u}^{(m-1)}(T,\xi)]d\xi-$$
$$-\int_0^x D_1^{-1}(\xi)L_2(\xi)\int_0^\theta\{A(\tau,\xi)\tilde{v}^{(m-1)}(\tau,\xi)+B(\tau,\xi)\tilde{w}^{(m-1)}(\tau,\xi)+C(\tau,\xi)\tilde{u}^{(m-1)}(\tau,\xi)\}d\tau d\xi-$$
$$-\int_0^x D_1^{-1}(\xi)S_2(\xi)\int_0^T\{A(\tau,\xi)\tilde{v}^{(m-1)}(\tau,\xi)+B(\tau,\xi)\tilde{w}^{(m-1)}(\tau,\xi)+C(\tau,\xi)\tilde{u}^{(m-1)}(\tau,\xi)\}d\tau d\xi-$$
$$-\int_0^x D_1^{-1}(\xi)\left[L_2(\xi)\int_0^\theta f(\tau,\xi)d\tau+S_2(\xi)\int_0^T f(\tau,\xi)d\tau-\varphi(\xi)\right]d\xi,\ x\in[0,\omega].$$

2) From the system of integral equations (14)--(16) under $\mu(t)=\mu^{(m-1)}(t)$, $\lambda(x)=\lambda^{(m)}(x)$, $\dot{\lambda}(x)=\dot{\lambda}^{(m)}(x)$ we find the functions $\tilde{u}^{(m)}(t,x)$, $\tilde{v}^{(m)}(t,x)$, $\tilde{w}^{(m)}(t,x)$ for all $(t,x)\in\Omega$:

$$\tilde{v}^{(m)}(t,x)=\int_0^t\{A(\tau,x)\tilde{v}^{(m)}(\tau,x)+B(\tau,x)\tilde{w}^{(m)}(\tau,x)+C(\tau,x)\tilde{u}^{(m)}(\tau,x)+$$
$$+A(\tau,x)\dot{\lambda}^{(m)}(x)+C(\tau,x)\lambda^{(m)}(x)+\mu^{(m-1)}(\tau)+f(\tau,x)\}d\tau,$$

$$\tilde{w}^{(m)}(t,x)=\psi(t)+\int_0^x\{A(t,\xi)\tilde{v}^{(m)}(t,\xi)+B(t,\xi)\tilde{w}^{(m)}(t,\xi)+C(t,\xi)\tilde{u}^{(m)}(t,\xi)+$$

$$+ A(t,\xi)\dot{\lambda}^{(m)}(\xi) + C(t,\xi)\lambda^{(m)}(\xi) + \mu^{(m-1)}(t) + f(t,\xi)\}d\xi,$$

$$\widetilde{u}^{(m)}(t,x) = \psi(t) - \psi(0) + \int_0^t \int_0^x \{A(\tau,\xi)\widetilde{v}^{(m)}(\tau,\xi) + B(\tau,\xi)\widetilde{w}^{(m)}(\tau,\xi) + C(\tau,\xi)\widetilde{u}^{(m)}(\tau,\xi) +$$

$$+ A(\tau,\xi)\dot{\lambda}^{(m)}(\xi) + C(\tau,\xi)\lambda^{(m)}(\xi) + \mu^{(m-1)}(\tau) + f(\tau,\xi)\}d\xi d\tau.$$

From integral relation (12) under $\widetilde{u}(t,x) = \widetilde{u}^{(m)}(t,x)$, $\widetilde{v}(t,x) = \widetilde{v}^{(m)}(t,x)$, $\widetilde{w}(t,x) = \widetilde{w}^{(m)}(t,x)$, $\lambda(x) = \lambda^{(m)}(x)$, $\dot{\lambda}(x) = \dot{\lambda}^{(m)}(x)$ we find the m-th approximation $\mu^{(m)}(t)$ for all $t \in [0,T]$:

$$\mu^{(m)}(t) = \int_0^\theta \left[K_1(t,\xi)\widetilde{v}^{(m)}(t,\xi) + K_2(t,\xi)\widetilde{w}^{(m)}(t,\xi) + K_3(t,\xi)\widetilde{u}^{(m)}(t,\xi)\right]d\xi +$$

$$+ \int_0^\theta \left[K_1(t,\xi)\dot{\lambda}^{(m)}(\xi) + K_3(t,\xi)\lambda^{(m)}(\xi)\right]d\xi, \quad m = 1,2,3,\dots .$$

5. The main result.

Let $a = \max_{(t,x)\in\Omega} \| A(t,x) \|$, $b = \max_{(t,x)\in\Omega} \| B(t,x) \|$, $c = \max_{(t,x)\in\Omega} \| C(t,x) \|$, $H = a + b + c$,

$\alpha_0 = \max_{x\in[0,\omega]} \| [D_1(x)]^{-1} \|$, $\alpha = \max_{x\in[0,\omega]} \| [D_1(x)]^{-1} D_2(x) \|$,

$$a_1 = \max_{x\in[0,\omega]} \| P_1(x) \| + \max_{x\in[0,\omega]} \| L_1(x) \| + \max_{x\in[0,\omega]} \| S_1(x) \| + \max_{x\in[0,\omega]} \| L_0(x) \| + \max_{x\in[0,\omega]} \| S_0(x) \|,$$

$$b_1 = \max_{x\in[0,\omega]} \| L_2(x) \| \max(T,\omega)\left[e^{H(\theta+\omega)} - e^{H\omega}\right] + \max_{x\in[0,\omega]} \| S_2(x) \| \max(T,\omega)\left[e^{H(T+\omega)} - e^{H\omega}\right],$$

$$a_2 = \theta \max_{x\in[0,\omega]} \| L_2(x) \| + T \max_{x\in[0,\omega]} \| S_2(x) \|, \quad b_2 = \max(T,\omega)e^{H(T+\omega)}(a + c + 1),$$

$$c_1 = \theta\left[(b_2 + 1)\left(\max_{(t,x)\in\Omega} \| K_1(t,x) \| + \max_{(t,x)\in\Omega} \| K_3(t,x) \|\right) + b_2 \max_{(t,x)\in\Omega} \| K_2(t,x) \|\right].$$

The following theorem gives conditions of realizability and convergence of the constructed algorithm and the conditions of the existence of unique solution to problem (8)--(12).

Theorem 1. *Suppose that*

i) the matrix $D_1(x)$ *is invertible for all* $x \in [0,\omega]$;

ii) the inequality fulfilled

$$q(T,\omega) = \max([\alpha e^{\alpha\omega}\omega + 1]\alpha_0(a_2c_1 + (a_1 + b_1)b_2), e^{\alpha\omega}\omega\alpha_0(a_2c_1 + (a_1 + b_1)b_2), c_1) < 1.$$

Then the problem for system of hyperbolic equations with parameters (8)--(12) has a unique solution.

Theorem 2. *Suppose that the conditions i) - ii) of Theorem 1 are fulfilled.*

Then the nonlocal problem for system of partial integro-differential equations (1)-(3) has a unique classical solution.

The proof of the theorem is similar to the scheme of the proof of theorem [12, p. 26].

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ГИПЕРБОЛАЛЫҚ ТЕКТЕС ДЕРБЕС ТУЫНДЫЛЫ ИНТЕГРАЛДЫҚ-ДИФФЕРЕНЦИАЛДЫҚ ТЕҢДЕУЛЕР ЖҮЙЕСІ ҮШІН БЕЙЛОКАЛ ЕСЕП ТУРАЛЫ

Аннотация. Екінші ретті гиперболалық тектес интегралдық-дифференциалдық теңдеулер жүйесі үшін бейлокал есеп қарастырылады. Бейлокал есептің классикалық шешімінің бар болуы мен жалғыздығы мәселелері зерттелген. Интегралдық қосылғыштың орнына жаңа белгісіз функция енгізу жолымен зерттеліп отырған есеп пара-пар интегралдық шарты бар бейлокал есепке келтірілген. Параметрі бар есеп гиперболалық теңдеулер жүйесі үшін параметрі бар бейлокал есептен және интегралдық қатынастан тұрады. Параметрі бар пара-пар есептің жуық шешімін табу алгоритмдері тұрғызылған және олардың жинақтылығы дәлелденген. Параметрі бар есептің жалғыз шешімінің бар болуының жеткілікті шарттары тағайындалған. Гиперболалық тектес интегралдық-дифференциалдық теңдеулер жүйесі үшін бейлокал есептің жалғыз классикалық шешімінің бар болуының шарттары бастапқы берілімдер терминінде алынған. Қарастырылып отырған есепті зерттеу үшін бұрын дербес туындылы дифференциалдық теңдеулер үшін есептер әулетіне келтіру әдісі пайдаланылған болатын. Зерттеліп отырған есептің жалғыз классикалық шешімінің бар болуының шарттары бастапқы берілімдер арқылы тұрғызылатын матрица терминінде табылған.

Түйін сөздер: бейлокал есеп, дербес туындылы интегралдық-дифференциалдық теңдеулер жүйесі, параметр, алгоритм, жуық шешім, бірмәнді шешілімділік.

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О НЕЛОКАЛЬНОЙ ЗАДАЧЕ ДЛЯ СИСТЕМЫ ИНТЕГРО-ДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ В ЧАСТНЫХ ПРОИЗВОДНЫХ ГИПЕРБОЛИЧЕСКОГО ТИПА

Аннотация. Рассматривается нелокальная задача с данными на характеристиках для системы интегро-дифференциальных уравнений гиперболического типа второго порядка. Исследуются вопросы существования и единственности классического решения нелокальной задачи. Путем введения новой неизвестной функции вместо интегральной слагаемой исследуемая задача сведена к эквивалентной нелокальной задаче с интегральным условием. Задача с параметром состоит из нелокальной задачи для системы гиперболических уравнений с параметром и интегрального соотношения. Построены алгоритмы нахождения приближенного решения эквивалентной задачи с параметром и доказана их сходимость. Установлены достаточные условия существования единственного решения задачи с параметром. Получены условия существования единственного классического решения нелокальной задачи для системы интегро-дифференциальных уравнений гиперболического типа в терминах исходных данных. Ранее к исследованию рассматриваемой задачи был применен метод сведения к эквивалентному семейству задач для дифференциальных уравнений в частных производных. Были найдены достаточные условия существования единственного классического решения исследуемой задачи в терминах некоторой матрицы, составляемой по исходным данным.

Ключевые слова: нелокальная задача, система интегро-дифференциальных уравнений в частных производных, параметр, алгоритм, приближенное решение, однозначная разрешимость.