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REDUCTION OF LINEAR HOMOGENEOUS *D_e*-SYSTEMS TO THE JORDAN CANONICAL FORM

Abstract. In this note we prove a theorem about reducibility to the canonical form of a linear homogeneous system with differentiation operator on diagonal and multiperiodic matrix constant on the diagonal. On the basis of the results obtained, it is possible to find out the structure of the solutions and investigate the conditions of the existence and uniqueness of the (θ, ω, ω) - periodic solution of the linear D_e-system of equations. When investigating periodic solutions of linear systems of first order partial differential equations, it becomes necessary to reduce matrices with variable elements to convenient form. In this connection, we note the results of [1-2] and commentaries on them in monographs [3-5]. It is known that the study of the problems of multiperiodic solutions of systems of first order partial D_e -equations with the same principal part originates in works [6-7]. On their basis, further qualitative studies have been continued in [8-11].

Key words: linear homogeneous system, differentiation operator, Jordan canonical form, multiperiodic matrix, main diagonal, vector-period.

The article is devoted investigation of reduction of a linear D_e-system of the form

$$D_e x = A(\sigma)x \tag{1}$$

with the differential operator $D_e = \dfrac{\partial}{\partial \tau} + \left\langle e, \dfrac{\partial}{\partial t} \right\rangle$ to the canonical form

$$D_e x = J(\sigma)x, \quad (1_*)$$

where $\tau \in (-\infty, +\infty) = R$, $t = (t_1, ..., t_m) \in R \times ... \times R = R^m$, $\dfrac{\partial}{\partial t} = \left(\dfrac{\partial}{\partial t_1}, ..., \dfrac{\partial}{\partial t_m} \right)$ is a vector operator, $e = (1, ..., 1)$ – m-vector, $\langle , \rangle$ denotes the scalar product, $\sigma = t - e\tau$, $A(\sigma)$ an $n \times n$-matrix, which satisfies condition

$$A(\sigma + k\omega) = A(\sigma) \in C_\sigma^{(e)}(R^m), \ \forall k \in Z^m \tag{2}$$

with multiple vector-periods $k\omega = (k_1\omega_1, ..., k_m\omega_m)$, $\omega = (\omega_1, ..., \omega_m)$, $k = (k_1, ..., k_m)$ from the set of integer vectors Z^m. $J(\sigma)$ an $n \times n$-matrix of the Jordan form possessing the properties of multiperiodicity with the same ω period and smoothness e in $\sigma \in R^m$:

$$J(\sigma + k\omega) = J(\sigma) \in C_\sigma^{(e)}(R^m), \ \forall k \in Z^m. \quad (2_*)$$

Variable matrices $A(\sigma)$ and $J(\sigma)$ are called constants on the diagonal $t = e\tau$.

Let $\lambda_j(\sigma)$ be eigenvalues of the matrix $A(\sigma)$ of multiplicity k_j, $j = \overline{1, s}$, possessing the following properties.

1^0. Continuous differentiability: $\lambda_j(\sigma) \in C_\sigma^{(e)}(R^m)$, $j=\overline{1,n}$.

2^0. Periodicity with period $\omega=(\omega_1,...,\omega_m)$: $\lambda_j(\sigma+k\omega)=\lambda_j(\sigma)$, $j=\overline{1,n}$, $\sigma \in R^m$, $k \in Z^m$.

3^0. Property of having fixed sign $\lambda_j(\sigma)$ for each $j=\overline{1,n}$:

a) $\lambda_j(\sigma)<0$, $\forall \sigma \in R^m$ or

b) $\lambda_j(\sigma)=0$, $\forall \sigma \in R^m$ or

c) $\lambda_j(\sigma)>0$, $\forall \sigma \in R^m$.

4^0. Separation of eigenvalues:

a) for $j \neq l$ $\lambda_j(\sigma) \neq \lambda_l(\sigma)$, $\forall \sigma \in R^m$ or

b) for $j \neq l$ $\lambda_j(\sigma) = \lambda_l(\sigma)$, $\forall \sigma \in R^m$,

i.e. for each value j the eigenvalue $\lambda_j(\sigma)$ has constant multiplicity $k_j = const$ for all $\sigma \in R^m$.

5^0. Each of the sets $\mathrm{Re}\{\lambda_j(\sigma)\}$ and $\mathrm{Im}\{\lambda_j(\sigma)\}$ has properties 1^0-4^0.

The properties 1^0-5^0 are briefly called Λ-properties of the matrix $A(\sigma)$.

It is obvious that characteristic matrix $\lambda E - A(\sigma) = H(\lambda,\sigma)$ has for all $\sigma \in R^m$ constant rank n and its invariant λ-polynomials $i_1(\lambda,\sigma)$, ..., $i_n(\lambda,\sigma)$ such that, starting with the second, they are a divisor of the previous one, $i_1(\lambda,\sigma)$, ..., $i_r(\lambda,\sigma)$ are polynomials of degree greater than zero with respect to λ and

$$i_{r+1}(\lambda,\sigma)=...=i_n(\lambda,\sigma)=1.$$

Then the characteristic matrix $H(\lambda,\sigma)$ is represented by relations

$$H(\lambda,\sigma)=P(\lambda,\sigma)diag[i_1(\lambda,\sigma),...,i_r(\lambda,\sigma),1,...,1]Q(\lambda,\sigma), \tag{3}$$

where $P(\lambda,\sigma)$ and $Q(\lambda,\sigma)$ are non-singular $n \times n$-matrices that λ- polynomials are with independent of λ determinants $\det P(\lambda,\sigma)=p(\sigma)\neq 0$ and $\det Q(\lambda,\sigma)=q(\sigma)\neq 0$.

Companion matrices of invariant polynomials

$$i_j(\lambda,\sigma)=\lambda^{n_j}-\alpha_{j1}(\sigma)\lambda^{n_j-1}-...-\alpha_{jn_j}(\sigma),\ j=\overline{1,r},\ n_1+...+n_r=n$$

denote by

$$A_j^*(\sigma)=\begin{pmatrix} 0 & 1 & 0 & ... & 0 \\ 0 & 0 & 1 & ... & 0 \\ ... & ... & ... & ... & ... \\ 0 & 0 & 0 & ... & 1 \\ \alpha_{jn_j}(\sigma) & \alpha_{jn_j-1}(\sigma) & \alpha_{jn_j-2}(\sigma) & ... & \alpha_{j1}(\sigma) \end{pmatrix},\ j=\overline{1,r}. \tag{4}$$

It is obvious that the representation (3) can be obtained on the basis of elementary transformations known from theory of λ-matrices [12] under which properties of multiperiodicity and continuous differentiability in σ for matrices participating in relation (3) are preserved. Consequently, these properties also hold for the matrices (4).

In [13], the condition of equivalence of matrix $\lambda E - A(\sigma)$ to the matrix with one invariant λ-polynomials are established and theorem of reducibility of the matrix $A(\sigma)$ to the Jordan normal form by multiperiodic continuously differentiable non-singular transformation matrix is proved.

Moreover, system (1) was equivalent to one equation with higher order D_e operator with companion matrix of the form (4).

In this article we raise the question about investigating the reducibility of system (1) to the D_e-system with the matrix of Jordan normal form, when the matrix (2) with several invariant polynomials satisfies the conditions 1^0-5^0.

In other words, in [13] we consider D_e-system, which is equivalent to one D_e-equation of order n, and in this case, by (3), our system (1) breaks up into r linear D_e-equations of orders n_1, ..., n_r $(n_1 + n_2 + ... + n_r = n)$. The essence of the problem is to reduce this general D_e-system (1) to D_e-system with matrix $J(\sigma)$ of the Jordan canonical normal form, where Λ-properties of matrix $A(\sigma)$ are essential value.

When raising the question, it is obvious that this study is adjacent to the studies [14-19].

To solve the problem posed, we use the true normal form $A^*(\sigma)$ of matrix $A(\sigma)$, which are related by a similarity relation

$$A^*(\sigma) = L^{-1}(\sigma)A(\sigma)L(\sigma). \tag{5}$$

The relation (5) to be a result of the representation (3), where

$$A^*(\sigma) = diag\left[A_1^*(\sigma),...,A_r^*(\sigma)\right]$$

with diagonal elements of the form (4), $L(\sigma)$ is a non-singular continuously differentiable ω-periodic matrix:

$$L(\sigma + k\omega) = L(\sigma) \in C_\sigma^{(e)}(R^m),\ \forall k \in Z^m. \tag{6}$$

Relations (5)-(6), as well as (3)-(4) are obtained on the basis of methods of the theory of equivalent transformations of polynomial matrices for which smoothness and multiperiodicity of the matrices are saved.

Further, in view of (5), (6) and the change

$$x = L(\sigma)z,\ \det L(\sigma) \neq 0,\ L(\sigma + k\omega) = L(\sigma),\ k \in Z^m \tag{7}$$

system (1) is reducible to the system

$$D_e z = A^*(\sigma)z, \tag{8}$$

which is equivalent to the system of subsystems

$$D_e z_j = A_j^*(\sigma)z_j, \quad (8_j)$$

where $A_j^*(\sigma)$ has the form (4), $j = \overline{1,r}$, $z = (z_1,...,z_r)$.

In the case of the known elementary divisors of matrix $A(\sigma)$ the system (1), and, consequently, the system (8) can be reduced to an even simpler form.

Indeed, in view of (2) and Λ-properties of the matrix $A(\sigma)$, we have full information about its eigenvalues. Hence, it exists a non-singular, really smooth ω-periodic matrix of the transformation $\tilde{L}(\sigma)$ such that

$$\tilde{A}(\sigma) = \tilde{L}^{-1}(\sigma)A(\sigma)\tilde{L}(\sigma), \quad (\tilde{5})$$

where $\widetilde{A}(\sigma)=diag\left[\widetilde{A}_1(\sigma),...,\widetilde{A}_l(\sigma)\right]$ is the second true form of matrix $A(\sigma)$, $\widetilde{A}(\sigma)$ have the form (4), in which the coefficients of the degree are non-zero elements of the last row

$$(\lambda-\lambda_j(\sigma))^{n_j}=\lambda^n+\beta_{j1}(\sigma)\lambda^{n-1}+\beta_{j2}(\sigma)\lambda^{n-2}+...+\beta_{jn_j}(\sigma),$$

which are an elementary divisor of the characteristic matrix (3). We write the properties of matrix $\widetilde{L}(\sigma)$ in the form

$$\widetilde{L}(\sigma+k\omega)=\widetilde{L}(\sigma)\in C_\sigma^{(e)}(R^m),\ k\in Z^m,\ \widetilde{L}(\sigma)\neq 0.\ (\widetilde{6})$$

Here, the eigenvalues $\lambda_j(\sigma)$, $j=\overline{1,n}$ are assumed to be real-valued.

Then, by the relations ($\widetilde{5}$), ($\widetilde{6}$) and the change

$$x=\widetilde{L}(\sigma)\widetilde{z}\ \ (\widetilde{7})$$

system (1) can be represented in the form

$$D_e\widetilde{z}=\widetilde{A}(\sigma)\widetilde{z},\ (\widetilde{8})$$

which consists from l subsystems

$$D_e\widetilde{z}_\rho=\widetilde{A}_\rho(\sigma)\widetilde{z}_\rho,\ (\widetilde{8}_\rho)$$

where $\rho=\overline{1,l}$, $\widetilde{z}_\rho=\left(\widetilde{z}_{\rho 1},...,\widetilde{z}_{\rho n_\rho}\right)$, $n_1+...+n_l=n$, $\widetilde{z}=(\widetilde{z}_1,...,\widetilde{z}_l)$.

Next, we should consider the reduction of system (1) to system with Jordan canonical form.

In the case of simple roots of matrices $A_j^*(\sigma)$:

$$\lambda_{ji}(\sigma)\neq\lambda_{jk}(\sigma),\ \sigma\in R^m,\ (i\neq k)$$

of the characteristic equation

$$\det\left[\lambda E-A_j(\sigma)\right]=0,\ i,k\in\overline{1,n_j},\ j=\overline{1,r},\ n_1+...+n_r=n$$

it is not difficult to verify that the Vandermonde matrix of the form

$$B_j(\sigma)=\begin{pmatrix}1 & 1 & ... & 1\\ \lambda_{j1}(\sigma) & \lambda_{j2}(\sigma) & ... & \lambda_{jn_j}(\sigma)\\ \lambda_{j1}^2(\sigma) & \lambda_{j2}^2(\sigma) & ... & \lambda_{jn_j}^2(\sigma)\\ ... & ... & ... & ...\\ \lambda_{j1}^{n_j-1}(\sigma) & \lambda_{j2}^{n_j-1}(\sigma) & ... & \lambda_{jn_j}^{n_j-1}(\sigma)\end{pmatrix}$$

satisfies the matrix equation

$$A_j(\sigma)B_j(\sigma)=B_j(\sigma)J_j(\sigma),$$

where $J_j(\sigma)=diag\left[\lambda_{j1}(\sigma),...,\lambda_{jn_j}(\sigma)\right]$ and also

$$\det B_j(\sigma)=\prod_{n_j\geq i>k\geq 1}\left(\lambda_{ji}(\sigma)-\lambda_{jk}(\sigma)\right)\neq 0.$$

Consequently, in this case the system (8_j) under conditions (2) and 1^0-5^0 is reducible to the Jordan canonical form

$$D_e y_j=J_j(\sigma)y_j\ \ (9_j)$$

by non-singular linear transformation

$$z_j=B_j(\sigma)y_j,\ \det B_j(\sigma)\neq 0,\ B_j(\sigma+k\omega)=B_j(\sigma)\in C_\sigma^{(e)}(R^m),\ k\in Z^m,\ (8_j^*)$$

where $j=\overline{1,r}$, $\sigma \in R^m$.

Then for $\sigma \in R^m$ the transformation

$$z=B(\sigma)y,\ \det B(\sigma)\neq 0,\ B(\sigma+k\omega)=B(\sigma)\in C_\sigma^{(e)}(R^m),\ k\in Z^m, \quad (8^*)$$

leads system (8) to the D_e-system of Jordan canonical form

$$D_e y = J(\sigma)y, \quad (9)$$

where $B(\sigma)=diag[B_1(\sigma),...,B_r(\sigma)]$, $J(\sigma)=diag[J_1(\sigma),...,J_r(\sigma)]$, $y=(y_1,...,y_r)$.

In the case of multiple elementary divisors of matrix $A(\sigma)$ will be necessary to use its second normal form $\widetilde{A}(\sigma)$ from system ($\widetilde{8}$) and its subsystems ($\widetilde{8}_\rho$) with matrices $\widetilde{A}_\rho(\sigma)$.

To reduce the matrix $\widetilde{A}_\rho(\sigma)$ to the Jordan normal form $J_\rho=\lambda_\rho E_\rho+I_\rho$ with identity matrix E_ρ and first off-diagonal oblique range I_ρ it is necessary to construct matrix $T_\rho(\sigma)$ with elements

$$t_{ij}^{(\rho)}(\sigma)=\begin{cases}\sum\limits_{k=1}^{j} C_{i-1}^{j-1}\lambda_\rho^{i-k}(\sigma), & j\le i,\\ \sum\limits_{k=1}^{i} C_{i-1}^{j-1}\lambda_\rho^{i-k}(\sigma)=b_{ii}, & j>i,\end{cases}$$

where C_i^j is total number of combinations of i in a total of j.

The reader will have no difficulty in verifying that [20]

$$A_\rho(\sigma)T_\rho(\sigma)=T_\rho(\sigma)J_\rho(\sigma)$$

and also $\det T_\rho(\sigma)=1$.

Then the change

$$\widetilde{z}_\rho = T_\rho(\sigma)\widetilde{y}_\rho$$

leads the system ($\widetilde{8}_\rho$) to the system

$$D_e\widetilde{y}_\rho = J_\rho(\sigma)\widetilde{y}_\rho$$

with a Jordan cage $J_\rho=\lambda_\rho E_\rho+I_\rho$.

Consequently, the change

$$\widetilde{z}=T(\sigma)y \quad (\widetilde{8}^*)$$

system (8) leads to the system (9) of the Jordan normal form, where $T(\sigma)=diag[T_1(\sigma),...,T_l(\sigma)]$ is non-singular ω-periodic, smooth transformation matrix.

In the case of complex eigenvalues, as can be seen from structures of matrices $T_\rho(\sigma)$ and $J_\rho(\sigma)$, matrices $T(\sigma)$ and $J(\sigma)$ are complex-valued. In view the condition 5^0 its real and imaginary parts are distinguished without any special difficulties for all $\sigma\in R^m$.

Thus, by transformations (6)-($\widetilde{6}$), (7)-($\widetilde{7}$) and (8^*)-($\widetilde{8}^*$) non-singular linear change

$$x=L^*(\sigma)y \quad (1^*)$$

leads the D_e-system (1) to the D_e-system (9) with Jordan matrix $J(\sigma)$. The matrix $L^*(\sigma)$ is transformation matrix $L^*(\sigma)=L(\sigma)B(\sigma)$ and it has properties

$$\det L^*(\sigma)\neq 0,\ L^*(\sigma+k\omega)=L^*(\sigma),\ k\in Z^m.\ (1^{**})$$

We call system (9) the Jordan canonical D_e-system of system (1).

We formulate the main result in the form of the following theorem.

Theorem. Let the matrix $A(\sigma)$ possessing the property (2) has eigenvalues $\lambda_j(\sigma)$, $j=\overline{1,n}$, satisfying the conditions 1^0-5^0. Then the system (1) can be reduced to the Jordan canonical D_e-system (9) by linear transformation (1^*)-(1^{**}).

As an application of the theorem proved, we consider D_e-system of triangular type

$$\begin{cases} D_e x = A_{11}(\sigma)x, \\ D_e y = A_{21}(\sigma)x + A_{22}(\sigma)y, \end{cases} \qquad (10)$$

where x is n_1-vector-function, y is n_2-vector-function, $A_{11}(\sigma)$, $A_{21}(\sigma)$ and $A_{22}(\sigma)$ are multiperiodic with ω-vector-period, smooth in $\sigma\in R^m$ matrices of order $n_1\times n_1$, $n_2\times n_2$, $n_{21}=n_2\times n_1$.

We suppose that the block matrix

$$A(\sigma)=\begin{pmatrix} A_{11}(\sigma) & O \\ A_{21}(\sigma) & A_{22}(\sigma)\end{pmatrix} \qquad (11)$$

satisfies the condition

$$A(\sigma+k\omega)=A(\sigma)\in C_\sigma^{(e)}(R^m),\ k\in Z^m\ (11^*)$$

where O is zero block. The diagonal blocks $A_{11}(\sigma)$ and $A_{22}(\sigma)$ have Λ-properties, therefore, these blocks have Jordan forms

$$J_j(\sigma)=L_j^{-1}(\sigma)A_{jj}(\sigma)L_j(\sigma),\ j=1,2\ (12^*)$$

with non-singular ω-periodic and smooth matrices

$$L_j(\sigma+k\omega)=L_j(\sigma)\in C_\sigma^{(e)}(R^m),\ \det L_j(\sigma)\neq 0,\ k\in Z^m,\ j=1,2.\ (12^{**})$$

Then by theorem linear non-singular ω-periodic, smooth in $\sigma\in R^m$ transformation of form

$$\begin{cases} x = L_1(\sigma)u, \\ y = L_2(\sigma)v \end{cases} \qquad (12)$$

leads system (10) to a linear system

$$\begin{cases} D_e u = J_1(\sigma)u, \\ D_e v = B(\sigma)u + J_2(\sigma)v \end{cases} \qquad (13)$$

with diagonal blocks $J_1(\sigma)$ and $J_2(\sigma)$ of the Jordan canonical form, where

$$B(\sigma)=L_2^{-1}(\sigma)A_{21}(\sigma)L_1(\sigma)$$

is smooth, ω-periodic in $\sigma\in R^m$ $n_2\times n_1$-matrix.

It is obvious that the system (13) has more convenient form in comparison with the system (10) for integration and qualitative investigation.

The system of form (13) can be called the semi-canonical form of the triangular system (10).

Thus, we can give the following corollary to theorem proved.

Corollary. Let triangular matrix (11) satisfying the condition (11^*) has Λ-properties. Then the system (10) by transformation (12)- (12^*)-(12^{**}) is reduced to the semi-canonical D_e-system (13).

In conclusion, we note that the problem posed of studies we have used the methods of [20].

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СЫЗЫҚТЫ БІРТЕКТІ D_e -ЖҮЙЕЛЕРДІ ЖОРДАНДЫҚ КАНОНДЫҚ ТҮРГЕ КЕЛТІРУ

Аннотация. Мақалада көп периодты тұрақты матрицамен және диагональ бойынша дифференциалдау операторымен сызықты біртекті жүйенің канондық түрге келтірілуі жөнінде теорема дәлелденген. Алынған нәтижелер негізінде D_e-сызықты теңдеулер жүйесінің (θ, ω, ω)-периодты шешімінің бар және жалғыз болуының шартын зертеп және шешімнің құрылымын анықтауға болады. Бірінші ретті дербес туындылы теңдеулердің сызықты жүйелерінің периодты шешімдерін зерттеу кезінде айнымалы элементті матрицаларды ыңғайлы түрге келтірілу қажеттілігі туындайды. Осы байланыста [1-2] жұмыстарының нәтижелерін және [3-5] монографияларында оларға түсіндірмелерді ескереміз. Негізгі бөлімі бірдей бірінші ретті дербес туындылы D_e-теңдеулер жүйесінің көп периодты шешімдерінің сұрақтарын зерттеу [6-7] енбектерінен бастау алатыны белгілі. Олардың негізінде кейбір әрі қарай сапалы зерттеулері [8-11] жұмыстарында жалғастырған.

Кілт сөздер: сызықты біртекті жүйе, дифференциалдық оператор, жордандық канондық түрі, көп периодты матрица, негізгі диагональ, вектор-период.

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ПРИВЕДЕНИЕ ЛИНЕЙНЫХ ОДНОРОДНЫХ D_e -СИСТЕМ К ЖОРДАНОВОМУ КАНОНИЧЕСКОМУ ВИДУ

Аннотация. В заметке доказана теорема о приводимости к каноническому виду линейной однородной системы с оператором дифференцирования по диагонали и многопериодической матрицей постоянной на диагонали. На основе полученных результатов можно выяснить структуры решений и исследовать условия существования и единственности (θ, ω, ω) -периодического решения линейной D_e-системы уравнений. При исследовании периодических решений линейных систем уравнений в частных производных первого порядка возникает необходимость приведения матриц с переменными элементами к удобному виду. В этой связи отметим результаты работ [1-2] и комментарии к ним в монографиях [3-5]. Известно, что исследование вопросов многопериодических решений систем D_e-уравнений в частных производных первого порядка с одинаковой главной частью берет свое начало в трудах [6-7]. На их основе дальнейшие некоторые качественные исследования продолжены в работах [8-11].

Ключевые слова: линейная однородная система, дифференциальный оператор, жордановый канони-ческий вид, многопериодическая матрица, главная диагональ, вектор-период.

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