N E W S
OF THE NATIONAL ACADEMY OF SCIENCES OF THE REPUBLIC OF KAZAKHSTAN
SERIES OF GEOLOGY AND TECHNICAL SCIENCES
ISSN 2224-5278
Volume 1, Number 427 (2018), 127 – 135

UDC 539.3(043.3)

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AXISMETRIC PROBLEMS OF ELASTIC LAYER OSCILLATION LIMITED BY RIGID OR DEFORMED BOUNDRIES

Abstract. In the study of wave processes of plane and circular elements in deformable bodies, the concept of phase velocity is introduced as the rate of change of the phase medium. In the case of harmonic vibrations of cylindrical shell, the phase velocity is expressed in terms of the frequency of natural oscillations freely supported along the edges of the shell, and therefore, the investigation of waves in plane and circular elements has the most direct relation to the problem of determining the proper forms and vibration frequencies of shells of finite length. If the research is carried out in view of the rheological properties of the system or material, has environmental management systems, in general, exhibiting the rheological properties, the use of these methods is significantly hampered. In such cases, the influence of rheological parameters on the components of the complex phase velocity is studied for certain values of the vibration frequencies. Some axismetric problems of oscillation of elastic layer are limited by rigid or deformable boundaries under the influence of normal or rotational shearing stress. The solutions of the problems under consideration are obtained using integral transformations in coordinate or time.

Key words: elastic, axismetry, wave, oscillations, deformed, longitudinal, anisotropic medium, Laplace transform, anisotropic layer.

First we consider the problem for half-space under the assumption that the half-space $z > 0$ is an anisotropic medium with the axis of symmetry of the mechanical properties (axis) (axis z), and the surface of which is subjected to an impulse voltage at the time $\xi_{z0} = -f(r,t)$.

Because of the symmetry of the mechanical properties of the medium relative to the axis z of the unique nonzero component of the displacement vector, $U_0(r,z,t)$ only the stresses and ξ_{r0} and ξ_{z0} the ones determined by formulas

$$\xi_{r0} = C_{44}\left(\frac{\partial U_0}{\partial r} - \frac{U_0}{r}\right), \qquad \xi_{z0} = C_{66}\frac{\partial U_0}{\partial z} \tag{1}$$

The equation of motion reduces to one

$$\frac{\partial \xi_{r0}}{\partial r} + \frac{\partial \xi_{r0}}{\partial z} + \frac{2\xi_{r0}}{r} = \rho\frac{\partial^2 U_0}{\partial t^2} \tag{2}$$

Substituting the expressions for ξ_{r0} and ξ_{z0} from (1) into equation (2), we bring it to the form:

$$\frac{\partial^2 U_0}{\partial r^2} + \frac{1}{r}\frac{\partial U_0}{\partial r} - \frac{U_0}{r^2} + \gamma^2\frac{\partial^2 U_0}{\partial z^2} = \frac{1}{b^2}\frac{\partial^2 U_0}{\partial t^2} \tag{3}$$

where

$$b^2 = \frac{C_{44}}{\rho}; \ \gamma^2 = \frac{C_{66}}{C_{44}}$$

If the half-space is isotropic, then $\gamma = 1$ and $b = \sqrt{\frac{\mu}{\rho}}$.

The boundary conditions for U_0 has:

$$\xi_{z0} = -f(r,t) \quad \text{under } z = 0, \ t \geq 0 \tag{4}$$

$$U_0 \to 0 \quad \text{under } z \to \infty \tag{5}$$

The initial conditions of the problem are zero

$$U_0 = \frac{\partial U_0}{\partial t} = 0 \quad \text{under } t = 0 \tag{6}$$

The solution of equation (3) for the boundary (4) - (5) and the initial conditions (6) will be sought, by applying the Laplace transform. Suppose,

$$U(r,z,p) = \int_0^\infty U_0(r,z,t)e^{-pt}dt, \ \operatorname{Re} p > 0 \tag{7}$$

Definitely, for the function $U(r,z,p)$ we obtain the equation

$$\frac{\partial^2 U}{\partial r^2} + \frac{1}{r}\frac{\partial U}{\partial r} - \left(\frac{1}{r^2} + \frac{p^2}{b^2}\right)U + \gamma^2 \frac{\partial^2 U}{\partial z^2} = 0 \tag{8}$$

And U must satisfy the boundary conditions:

$$\frac{\partial U}{\partial z} = -\frac{f_0(r,p)}{C_{66}} \quad \text{under } z = 0, \ t > 0 \tag{9}$$

$$U_0 \to 0 \quad \text{under } z \to \infty \tag{10}$$

where

$$f_0(r,p) = \int_0^\infty (r,t)e^{-pt}dt.$$

The general solution of equation (5.6.8) is sought by the method of separation of variables (the Fourier method) and has the form:

$$U(r,z,p) = \int_0^\infty \alpha \left[A(\alpha,p)e^{-\frac{z}{\gamma}\sqrt{\alpha^2 + \frac{p^2}{b^2}}} + B(\alpha,p)e^{\frac{z}{\gamma}\sqrt{\alpha^2 + \frac{p^2}{b^2}}} \right] J_1(\alpha r)d\alpha. \tag{11}$$

where $A(\alpha,p)$ and $B(\alpha,p)$ are defined from boundary conditions (9)-(10) from the condition (10) follows that

$$B(\alpha,p) = 0 \tag{12}$$

Using boundary condition (9), for defining $A(\alpha,p)$ we'll take integral equation:

$$\int_0^\infty \alpha A(\alpha,p)\sqrt{\alpha^2 + \frac{p^2}{b^2}} J_1(\alpha r)d\alpha = \frac{\gamma}{C_{66}} f_0(r,p) \tag{13}$$

Let

$$f_0(r,p)=\int_0^\infty \alpha f_1(\alpha,p)J_1(\alpha r)d\alpha \tag{13}$$

Then

$$A(\alpha,p)=\frac{\gamma f_1(\alpha,p)}{C_{66}\sqrt{\alpha^2+\frac{p^2}{b^2}}} \tag{14}$$

Substituting expression (12) and (14) into formula (11), we obtain the following expression:

$$U(r,z,p)=\frac{\gamma}{C_{66}}\int_0^\infty \frac{\alpha f_1(\alpha,p)}{\sqrt{\alpha^2+\frac{p^2}{b^2}}}J_1(\alpha r)e^{-\frac{z}{\gamma}\sqrt{\alpha^2+\frac{p^2}{b^2}}}d\alpha \tag{15}$$

We consider the special case when

$$f_0(r,p)=\frac{\varphi_0(p)}{r} \tag{16}$$

In case (16) function

$$f_1(\alpha,p)=\frac{\varphi_0(p)}{\alpha}$$

and (15) has

$$U(r,z,p)=\frac{\gamma}{C_{66}}\int_0^\infty \frac{\varphi_0(p)}{\sqrt{\alpha^2+\frac{p^2}{b^2}}}e^{-\frac{z}{\gamma}\sqrt{\alpha^2+\frac{p^2}{b^2}}}J_1(\alpha r)d\alpha=\frac{1}{C_{66}}\varphi_0(p)I_{\frac{1}{2}}\left[\frac{p}{2b}\left(\sqrt{\frac{z^2}{\gamma^2}+r^2}-\frac{z}{\gamma}\right)\right]K_{\frac{1}{2}}\left[\frac{p}{2b}\left(\sqrt{\frac{z^2}{\gamma^2}+r^2}+\frac{z}{\gamma}\right)\right]$$

where $K_{\frac{1}{2}}, I_{\frac{1}{2}}$ Bessel functions of imaginary argument. Using the representations of the functions $I_{\frac{1}{2}}(\zeta)$ and $K_{\frac{1}{2}}(\zeta)$, for $U(r,z,p)$ we find that

$$U(r,z,p)=\frac{b\varphi_0(p)}{C_{66}rp}\left[e^{-\frac{pz}{\gamma a}}-e^{-\frac{p}{a}\sqrt{\frac{z^2}{\gamma^2}+r^2}}\right] \tag{17}$$

Paying attention to the expression (17) in p, for the required value $U_0(r,z,t)$, we'll take the expression

$$U_0(r,z,t)=\frac{b}{\gamma C_{66}r}\int_0^t f_1(t-\xi)\left[H\left(\xi-\frac{z}{\gamma b}\right)-H\left(\xi-\frac{1}{b}\sqrt{\frac{z^2}{\gamma^2}+r^2}\right)\right]d\xi \tag{18}$$

where

$$f(r,t)=\frac{f_1(t)}{r}.$$

The resulting expression $U_0(r,z,t)$ consists of two terms, the first term corresponding to plane wave propagating in half-space with velocity γb and parallel to the plane $z=0$, and the second term to the diffracted wave, which has the form of half-ellipsoid of revolution (hemisphere at $\gamma=1$) and in contact with plane wave on the rotation axis at $z=b\gamma t$.

Besides, from (18) follows, that $U_0(r,z,t)$ fades out from r as $1/r$.

If the acting function $f(r,t)$ is arbitrary, then we represent it in the form of Schlemmich series:

$$f(r,t)=\frac{1}{r}\sum_{j=0}^{\infty}a_j(t)J_0(jr)=\frac{f_r(t)}{r} \tag{19}$$

where

$$a_0(t)=\frac{1}{\pi}\int_0^{\pi}\left\{f_r(0,t)+U\int_0^1\frac{\frac{\partial f_r(\xi U,t)}{\partial r}}{\sqrt{1-\xi^2}}d\xi\right\}dU;$$

$$a_j(t)=\frac{2}{\pi}\int H\cos(jU)\left\{\int_0^1\frac{\frac{\partial f_r(\xi U,t)}{\partial r}}{\sqrt{1-\xi^2}}d\xi\right\}dU;\ j=1,2,... \tag{20}$$

For $f(r,t)$ type (19) function $f_1(\alpha,p)$, entering to the formula (15), is

$$f_1(\alpha,p)=\frac{1}{\alpha}\sum_{j=0}^{\infty}a_{j0}(p)H(\alpha-j) \tag{21}$$

where

$$a_{j0}(p)=\int_0^{\infty}a_j(p)e^{-pt}dt. \tag{22}$$

Following,

$$U(r,z,p)=\frac{\gamma}{C_{66}}\sum_{j=0}^{\infty}a_{j0}(p)\int_0^{\infty}\frac{e^{-\frac{z}{\gamma}\sqrt{\alpha^2+\frac{p^2}{b^2}}}}{\sqrt{\alpha^2+\frac{p^2}{b^2}}}J_1(\alpha r)d\alpha \tag{23}$$

Paying attention (23) on p and using theory about convolution, we'll take

$$U_0(r,z,p)=\frac{\gamma}{2C_{66}}\sum_{j=0}^{\infty}\int_g^t a_j(t-\xi)T_j(r,z,\xi)d\xi \tag{24}$$

where

$$T_j(r,z,\xi)=b\int_j^{\infty}J_1(\alpha r)J_0\left[\alpha\sqrt{b^2t^2-\frac{z^2}{\gamma^2}}\right]d\alpha \tag{25}$$

We generalize the problem for anisotropic layer of thickness h. where $z=h$ there can be two kinds of boundary conditions

1) $\tau_{z0}=-F(r,t)$ under $z=h$

2) $U_0=0$ under $z=h$

(26)

If $F(r,t)=0$, then condition (26) means, that the substance $z=h$ is free from stresses.

First of all, let's look at the problem, when $z=h$ there is boundary condition (26).

General decision has the type (11),

$$A(\alpha,p)=B(\alpha,p)+\frac{\gamma}{C_{66}}f_1(\alpha,p)$$

$$B(\alpha,p)=\frac{\gamma}{2C_{66}}\frac{f_1(\alpha,p)e^{-\frac{h}{\gamma}\sqrt{\alpha^2+\frac{p^2}{b^2}}}-f_2(\alpha,p)}{sh\left[\frac{h}{\gamma}\sqrt{\alpha^2+\frac{p^2}{b^2}}\right]} \tag{28}$$

where $f_1(\alpha,p)$ is defined from определяется из equations (13), $f_2(\alpha,p)$ from equations

$$F_0(r,p)=\int_0^\infty \alpha f_2(\alpha,p)J_1(\alpha r)d\alpha \tag{29}$$

Let

$$F_0(r,p)=0,\ \text{т.е.}\ f_2(\alpha,p)=0\ \text{и}\ f(r,t)=f_1(r,t).$$

Then

$$U(r,z,p)=\frac{1}{\gamma C_{66}}\int_0^\infty\frac{\varphi_0(p)}{\sqrt{\alpha^2+\frac{p^2}{b^2}}}\times\left[\frac{e^{-\frac{z-h}{\gamma}\sqrt{\alpha^2+\frac{p^2}{b^2}}}+e^{\frac{z-h}{\gamma}\sqrt{\alpha^2+\frac{p^2}{b^2}}}}{e^{\frac{h}{\gamma}\sqrt{\alpha^2+\frac{p^2}{b^2}}}-e^{-\frac{h}{\gamma}\sqrt{\alpha^2+\frac{p^2}{b^2}}}}\right]J_1(\alpha r)d\alpha$$

or

$$U(r,z,p)=\frac{1}{\gamma C_{66}}\sum_{n=0}^\infty\int_0^\infty\frac{\varphi_0(p)}{\sqrt{\alpha^2+\frac{p^2}{b^2}}}\times\left\{e^{-\frac{z+2nh}{\gamma}\sqrt{\alpha^2+\frac{p^2}{b^2}}}+e^{\frac{z-2(n+1)h}{\gamma}\sqrt{\alpha^2+\frac{p^2}{b^2}}}\right\}J_1(\alpha r)d\alpha \tag{30}$$

Calculating the quadratures in α (30) and then reversing with respect to p, for $U_0(r,z,t)$, we obtain the expression:

$$U_0(r,z,t)=\frac{b}{r\gamma C_{66}}\sum_{n=0}^{n_1}\left\{\int_0^t f_1(t-\xi)H\left[\xi-\frac{z+2nh}{\gamma b}\right]d\xi-\right.$$
$$\left.-\int_0^t f_1(t-\xi)H\left[\xi-\frac{1}{b}\sqrt{r^2+\left(\frac{z+2nh}{\gamma}\right)^2}\right]d\xi\right\}+$$
$$+\frac{b}{r\gamma C_{66}}\sum_{n=0}^{n_r}\left\{\int_0^t f_1(t-\xi)H\left[\xi+\frac{z-2(n+1)h}{\gamma b}\right]d\xi-\right.$$
$$\left.-\int_0^t f_1(t-\xi)H\left[\xi-\frac{1}{b}\sqrt{r^2+\left(\frac{z-2(n+1)h}{\gamma}\right)^2}\right]d\xi\right\} \tag{31}$$

where

$$n_1=\left[\frac{z+2nh}{\gamma bt}\right],\quad n_2=\left[\frac{-z+2(n+1)h}{\gamma bt}\right]$$

$[\xi]$ – the whole part of the number ξ.

Formula (31) contains all flat and diffracted incident and reflected waves.
Similarly, the problem for the layer is solved under the boundary condition (27) and we have

$$U_0(r,z,t)=\frac{b}{r\gamma C_{66}}\sum_{n=0}^{n_1}(-1)^n\left\{\int_0^t f_1(t-\xi)H\left[\xi\,\frac{z+2nh}{\gamma b}\right]d\xi-\right.$$
$$\left.-\int_0^t f_1(t-\xi)H\left[\xi-\frac{1}{b}\sqrt{r^2+\left(\frac{z+2nh}{\gamma}\right)^2}\right]d\xi\right\}-$$
$$-\frac{b}{r\gamma C_{66}}\sum_{n=0}^{\infty}(-1)^n\left\{\int_0^t f_1(t-\xi)H\left[\xi+\frac{z-2(n+1)h}{\gamma b}\right]d\xi-\right.$$
$$\left.-\int_0^t f_1(t-\xi)H\left[\xi-\frac{1}{b}\sqrt{r^2+\left(\frac{z-2(n+1)h}{\gamma}\right)^2}\right]d\xi\right\} \tag{32}$$

We consider the problem of the effect of pulse on half-space $f(r,t)$, when the function is given on bounded part of the surface $z=0$, e.g.

$$f(r,t)=\frac{f_1(t)}{r}\left[H(r)-H(r-r_0)\right] \tag{33}$$

Function $f(r,t)$ we can also imagine

$$f(r,t)=-f_1(t)\int_0^{\infty}\left[1-J_0(\alpha r)\right]J_1(\alpha r)d\alpha \tag{34}$$

Then for $U(r,z,p)$ has the expression

$$U(r,z,p)=\frac{\varphi_0(p)}{\gamma C_{66}}\int_0^{\infty}\frac{\left[1-J_0(\alpha r)\right]}{\sqrt{\alpha^2+\frac{p^2}{b^2}}}e^{-\frac{z}{\gamma}\sqrt{\alpha^2+\frac{p^2}{b^2}}}J_1(\alpha r)d\alpha \tag{35}$$

Paying attention to the expression (35) in *p*, we'll take

$$U_0(r,z,t)=\int_0^t f_1(t-\xi)N_0(r,z,\xi)d\xi \tag{36}$$

where

$$N_0(r,z,t)=N_{10}(r,z,t)-N_{20}(r,z,t) \tag{37}$$

then

$$N_{10}(r,z,t)=\frac{1}{\gamma r}H\left(r-\sqrt{b^2t^2-\frac{z^2}{\gamma^2}}\right);$$
$$N_{20}(r,z,t)=0$$

under

$$0<\sqrt{b^2t^2-\frac{z^2}{\gamma^2}}<r-r_0$$

or

$$0 < r_0 < r - \sqrt{b^2t^2 - \frac{z^2}{\gamma^2}}$$

$$N_{20}(r,z,t) = \frac{1}{\pi\gamma}\arccos\frac{b^2t^2 - \frac{z^2}{\gamma^2} - r^2 + r_0^2}{2r_0\sqrt{b^2t^2 - \frac{z^2}{\gamma^2}}}$$

under

$$|r - r_0| < \sqrt{b^2t^2 - \frac{z^2}{b^2}} < r_0 + r$$

or

$$\left| r - \sqrt{b^2t^2 - \frac{z^2}{b^2}} \right| < r_0 < r + \sqrt{b^2t^2 - \frac{z^2}{b^2}},$$

$$N_{20}(r,z,t) = 0$$

under

$$0 < \sqrt{b^2t^2 - \frac{z^2}{b^2}} < r_0 - r;\ r + r_0 < \sqrt{b^2t^2 - \frac{z^2}{b^2}} < \infty$$

or

$$0 < r_0 < \sqrt{b^2t^2 - \frac{z^2}{b^2}} - r;\ r + \sqrt{b^2t^2 - \frac{z^2}{b^2}} < r_0 < \infty$$

If instead of half-space we have layer of thickness h, then under the boundary condition (26)

$$U_0(r,z,t) = \sum_{n=0}^{n_1}\int_0^t f_1(t-\xi)N_{2n}(r,z,\xi)d\xi + \sum_{m=0}^{n_r}\int_0^t f_1(t-\xi)N_{2(m+1)}(r,z,\xi)d\xi \quad (38)$$

And then in boundary condition (28),

$$U_0(r,z,t) = \sum_{n=0}^{n_1}(-1)^n\int_0^t f_1(t-\xi)N_{2n}(r,z,\xi)d\xi - \sum_{m=0}^{n_r}(-1)^n\int_0^t f_1(t-\xi)N_{2(m+1)}(r,z,\xi)d\xi \quad (39)$$

where N_{2n} and $N_{2(m+1)}$ then, if z replaced by $z + 2nh$ and $-z + 2(m+1)h$, accordingly.

But now

$$f(r,t) = \frac{f_1(t)}{r}[H(r) - H(r - Dt)] \quad (40)$$

where D – rate of change of loading area.

In case (40) for half-space and $f_1(t) = H(t)p_0$

$$A(\alpha,p) = \frac{p_0\gamma}{C_{66}\sqrt{\alpha^2 + \frac{p^2}{b^2}}}\left[\frac{1}{p} - \frac{1}{\sqrt{\alpha^2D^2tp^2}}\right] \quad (41)$$

and

$$U_0(r,z,t)=\frac{b\gamma p_0}{C_{66}r}\left[\left(t-\frac{z}{\gamma b}\right)H\left(t-\frac{z}{\gamma b}\right)-\left(t-\sqrt{r^2+\frac{z^2}{\gamma^2 b^2}}\right)H\left(t-\sqrt{r^2+\frac{z^2}{\gamma^2 b^2}}\right)\right]-$$

$$-\frac{b\gamma p_0}{C_{66}\pi}\int_0^{\xi_1(r,z,t)}\arccos\frac{b^2\xi^2-\frac{z^2}{\gamma^2}-r^2+D^2(t-\xi)^2}{2D(t-\xi)\sqrt{b^2\xi^2-\frac{z^2}{\gamma^2}}} \qquad (42)$$

where $\xi_1(r,z,t)>0$ root of the equation:

$$\left[\left(b^2\xi_1^2-\frac{z^2}{\gamma^2}\right)-r^2+D^2(t-\xi_1)^2\right]^2=4D^2(t-\xi_1)^2\left(b^2\xi_1^2-\frac{z^2}{\gamma^2}\right)$$

Similarly, it is possible to solve the problem, when $z=0$ the displacement value is set U_0.

This work is devoted to the study of dynamics stability wave processes flat and circular elements and also flat of the class tasks of the impact loads on the moving surface of the layered elastic half-plane for nonlinear law depending on the deformation stresses.

When studying the stability of wave processes, some axismetric problems of oscillation of elastic layer are confined to hard or deformable boundaries under the influence of normal or rotational shear stresses. The solutions of the problems under consideration were obtained using integral transformations in coordinate or time.

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ҚАТТЫ НЕМЕСЕ ДЕФОРМАЦИЯЛАНАТЫН ШЕКТЕЛГЕН АУМАҚТАҒЫ СЕРПІМДІ ҚАБАТТЫҢ ОСЕСИММЕТРИЯЛЫҚ ТЕРБЕЛІС ЕСЕБІ

Аннотация. Деформацияланатын денелердегі жазық және дөңгелек элементтердің толқындық процестерін зерттеу кезінде, фазалық жылдамдық ұғымы фазалық ортаны өзгерту жылдамдығы ретінде енгізілген. Цилиндр қабықшасының гармониялық дірілі жағдайында фазалық жылдамдық қабықша шетіне еркін түрде бекіген өз тербелісінің жиілігі арқылы көрінеді, сондықтан жазық және дөңгелек элементтердегі толқындарды зерттеудің соңғы ұзындықтағы қабықша тербелісінің жиілігі мен өз формасын анықтау мәселесіне тікелей қатысы бар. Егер де зерттеуде қарастырылып отырған жүйе материалының реологиялық қасиеттерін немесе реологиялық қасиеттері көрінетін жалпы қоршаған орта жағдайын есепке ала отырып жүргізілсе, онда бұл тәсілдерді қолдану айтарлықтай қиындық тудырады. Мұндай жағдайларда діріл жиілігінің белгілі бір мәндері үшін реологиялық параметрлердің кешенді фазалық жылдамдық құрамдастарына әсері зерттеледі. Төменде қалыпты немесе айналмалы кернеулер әсерінен қатаң немесе деформацияланатын шекаралармен шектелетін серпімді қабаттың кейбір осьтік симметриялық есептері қарастырылған. Қарастырылған мәселелердің шешімдері координата немесе уақыт бойынша интегралдық түрлендірулерді қолдану арқылы алынған.

Түйін сөздер: серпімді, осесимметрия, толқын, тербеліс, деформацияланатын, көлбеу, анизотропты орта, Лапласа түрлендіруі, анизотропты қабат.

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ОСЕСИММЕТРИЧНЫЕ ЗАДАЧИ КОЛЕБАНИЯ УПРУГОГО СЛОЯ ОГРАНИЧЕНЫЕ ЖЕСТКИМИ ИЛИ ДЕФОРМИРУЕМЫМИ ГРАНИЦАМИ

Аннотация. При исследованиях волновых процессов плоских и круговых элементов в деформируемых телах вводится понятие фазовой скорости, как скорости изменения фазовой среды. В случае гармонических колебаний цилиндрической оболочки фазовая скорость выражается через частоту собственных колебаний свободно опертой по краям оболочки, и поэтому, исследование волн в плоских и круговых элементах имеет самое прямое отношение к проблеме определения собственных форм и частот колебаний оболочек конечной длины. Если исследования проводятся с учетом реологических свойств материала рассматриваемой системы или, имеется окружающая систему среда, также в общем случае, проявляющая реологические свойства, использование этих способов значительно затруднено. В таких случаях изучается влияние реологических параметров на составляющие комплексной фазовой скорости при определенных значениях частот колебаний. Ниже рассматриваются некоторые осесимметричные задачи колебания упругого слоя ограниченые жесткими или деформируемыми границами при воздействии на него нормального или вращательного касательного напряжения. Решения рассматриваемых задач получены с использованием интегральных преобразований по координате или по времени.

Ключевые слова: упругий, осесимметрия, волна, колебания, деформируемый, продольный, анизотропная среда, преобразование Лапласа, анизотропный слой.

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