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B. Sinchev[1], A. M. Mukhanova[2]

[1]International University of Information Technology, Almaty,Kazakhstan
[2]Almaty Technological University, Almaty,Kazakhstan
E-mail:sinchev@mail.ru, nuraksulu72@mail.ru

THE DESIGN OF UNIQUE MECHANISMS AND MACHINES. I.

Abstract. This work is devoted to the development of numerical methods of calculating the kinematic and dynamic parameters for the design of a new mechanism of class IV based on equations kinematics and dynamics. Kinematic and dynamic models of these mechanisms are described by differential-algebraic equations. These equations are singular differential equations. For the first time, we consider the analysis of dynamics and kinematics for mechanisms of high class in a joint formulation. A cardinal breakthrough is possible in the transition of mechanisms of lower class to high class to design new mechanisms and machines. A new concept based on Runge-Kutta method is offered for mechanisms of high class aimed at solving kinematic - dynamic model. Methods, algorithms and programs are developed to define the position and the speed of links for mechanism of class IV at any given accuracy. A numerical experiment has shown a huge advantage of mechanism of class IV to design a new mechanism for mechanical engineering and robotics.

Keywords: kinematics, dynamics, mechanism of high class, mechanical engineering, robotics.

Introduction. The modern theory of analysis and synthesis for lever mechanisms of lower class (class II) is complete and theirfeatures are studies and used in practice. Therefore, a significant breakthrough in the design of new machinery and equipment is possible in the way of upgrading the facility.

The exploration of mechanisms of high class was started with well-known Assyria's work [1]. Academicians Artobolevsky I.I. [2], Zholdasbekov U.A. [3] and a lot of other scientists made their contribution into the further development of the theory of those mechanisms. In these works, the benefits of mechanisms of high class compared with mechanisms of lower class (class II) were established based on geometric and approximate methods.

The kinematic-dynamic model for mechanism of highclass is described by differential-algebraic equations (DAE).Some questions of the investigation of DAE are given in [4-6].

The complete analysis of mechanisms of high class involves the simultaneous solution of kinematic and dynamic problems. According to the classical theory of mechanisms and machines,a separate solution of complex problems is considered. Planar lever mechanism of class IV as the basic mechanism out of all mechanisms of high class and in much the same way a four-link mechanism out of mechanisms of lower class II are going to be studied.

This work is devoted to the development of a new calculation theory for the underlying mechanism of class IV done based on the theory of differential equations.

In the second part of this work, we consider singular differential-algebraic equations and a power analysis of these mechanisms.

The proposed methods, algorithms and the held computational experiment made possibilities to identify the unique characteristics of the above mentioned mechanism of class IV and their use in various scientific and technological fields (in the mechanisms like departure chassis of an aircraft, load-lifting cranes, presses, robots and other devices).

The common model for mechanism of high class. Suppose the mechanism of high class (MHC) consists of n moving links. Numbers of output links vary from 1 to n-m, and the number of input ones varies from n-m +1 to n, m is the number of input links directly related to drive units (Figure 1). The proposed mechanism of the numbering units enables the use of vector-matrix notation in solving various equations in kinematics and dynamics.

Then the general model for mechanism of class IV (n=6, m=2) is represented in the form:

– differential equations of this mechanism's dynamics having two degrees of freedom (two generalized coordinates) are:

$$L''_{\dot{\overline{\varphi_k}}\dot{\overline{\varphi_k}}}(\vec{\varphi},\dot{\vec{\varphi}})\ddot{\vec{\varphi}}_k + L''_{\dot{\overline{\varphi_k}}\overline{\varphi_k}}(\vec{\varphi},\dot{\vec{\varphi}})\dot{\vec{\varphi}}_k + L'_{\overline{\varphi_k}}(\vec{\varphi},\dot{\vec{\varphi}}) = Q_k, \tag{1}$$

and having the initial conditions

$$\bar{\varphi}_k(t_0) = \bar{\varphi}_k^0, \dot{\bar{\varphi}}_k(t_0) = \dot{\varphi}_k^0, k = 5,6 \tag{2}$$

equations of kinematics in the differential form are

$$\begin{pmatrix}\dot{\varphi}_1\\ \dot{\varphi}_2\\ \dot{\varphi}_3\\ \dot{\varphi}_4\end{pmatrix} = \begin{pmatrix}a_{11} & a_{12} & a_{13} & a_{14}\\ a_{21} & a_{22} & a_{23} & a_{24}\\ a_{31} & a_{32} & a_{33} & a_{34}\\ a_{41} & a_{42} & a_{43} & a_{44}\end{pmatrix}^{-1} \begin{pmatrix}b_{11} & b_{12}\\ b_{21} & b_{22}\\ b_{31} & b_{32}\\ b_{41} & b_{42}\end{pmatrix}\begin{pmatrix}\dot{\varphi}_6\\ \dot{\varphi}_5\end{pmatrix} \tag{3}$$

where $a_{11} = l_1 y_1$, $a_{12} = l_2 y_2$, $a_{13} = l_3 y_3$, $a_{14} = 0, a_{21} = l_1 x_1$, $a_{22} = l_2 x_2$, $a_{23} = l_3 x_3$, $a_{24} = 0$, $a_{31} = l_1^* y_1^*$, $a_{32} = 0$, $a_{33} = l_3^* y_3^*$, $a_{34} = l_4 y_4, a_{41} = l_1^* x_1^*$, $a_{42} = 0$, $a_{43} = l_3^* x_3^*$, $a_{44} = l_4 x_4$, $b_{11} = l_6 y_6$, $b_{12} = -l_5 y_5$, $b_{21} = l_6 x_6$, $b_{22} = -l_5 x_5$, $b_{31} = b_{11}$, $b_{32} = b_{12}$, $b_{41} = b_{21}$, $b_{42} = b_{22}$,

$$A = \begin{pmatrix}a_{11} & a_{12} & a_{13} & a_{14}\\ a_{21} & a_{22} & a_{23} & a_{24}\\ a_{31} & a_{32} & a_{33} & a_{34}\\ a_{41} & a_{42} & a_{43} & a_{44}\end{pmatrix}.$$

Here, we introduce the following notation: $\bar{\varphi}, \dot{\bar{\varphi}}$ are the vectors of the angular coordinates and velocity of the input links 5 and 6; $x_i = cos\varphi_i, y_i = sin\varphi_i$, φ_i- are the rectangular and angular coordinates of output links (i=1,2,3,4); $x_1^* = \cos(\varphi_1 \pm \alpha_1)$, $y_1^* = \sin(\varphi_1 \pm \alpha_1), x_3^* = \cos(\varphi_3 \pm \alpha_3)$, $y_3^* = \sin(\varphi_3 \pm \alpha_3)$ – are the rectangular coordinates of basic links 1 and 3, Q_5, Q_6 are coerced moments to input (leading) links. L is Lagrange function (Lagrangian),

$$L = K - P = \frac{1}{2}\dot{\vec{q}}' H(\vec{q})\dot{\vec{q}} - \bar{m}'\bar{h} * g,$$

where $\bar{m}$ are vectors of masses of mechanism's moving links, $\bar{m}, \bar{h}$ are vectors of dimension n, $\bar{h} = \bar{l} sin\bar{\varphi}$, $\bar{l}$ is the vector of moving links for the mechanism of class IV, which is determined by the equations of kinematics, $H(\vec{q})$ is the order matrix nxn of the whole mechanism's kinetic energy, g isa gravitational acceleration. According to S. Koenig theory we define that for mechanism of class IV the Lagrangian $L = K - P$ which consists of kinetic and potential energies.

Task 1. Solve the general model (1) - (3) for mechanism of class IV using Runge-Kutta's method for $|A| \neq 0$.

The solution of the problem allows determining position and speed of all linksfor this mechanism in the time interval $T = t_1 - t_0$. However, even in the determination of the Lagrange operator of type II (complete calculation of the system's coefficients (1)) you cannot apply Runge-Kutta's method to the joint decision of systems of differential equations (1) and (3), since initial conditions (initial values of angular coordinates of output units 1-4) are unknown in the system (3). In the beginning, we have to find initial conditions

$$\varphi_1(t_0), \varphi_2(t_0), \varphi_3(t_0), \varphi_4(t_0) \tag{4}$$

starting with kinematic(trigonometric) equations:

$$\begin{aligned}
&l_1 \cos\varphi_1 + l_2 \cos\varphi_2 + l_3 \cos\varphi_3 + l_5 \cos\varphi_5 - l_6 \cos\varphi_6 - l_0 \cos\varphi_0 = 0\\
&l_1 \sin\varphi_1 + l_2 \sin\varphi_2 + l_3 \sin\varphi_3 + l_5 \sin\varphi_5 - l_6 \sin\varphi_6 - l_0 \sin\varphi_0 = 0\\
&l_1^* \cos(\varphi_1 - \alpha_1) - l_3^* \cos(\varphi_3 + \alpha_3) + l_4 \cos\varphi_4 + l_5 \cos\varphi_5 - l_6 \cos\varphi_6 - l_0 \cos\varphi_0 = 0\\
&l_1^* \sin(\varphi_1 - \alpha_1) - l_3^* \sin(\varphi_3 + \alpha_3) + l_4 \sin\varphi_4 + l_5 \sin\varphi_5 - l_6 \sin\varphi_6 - l_0 \sin\varphi_0 = 0
\end{aligned} \tag{5}$$

Thus, a general model (1), (3) of mechanism of class IV can be represented by a system of differential equations of order n + 2m in the normal form of Cauchy:

$$\dot{x} = \vec{f}(x,t) \tag{6}$$

Having the initial conditions

$$x(t_0) = x_0\ , \tag{7}$$

where $\vec{f}(x,t)$ is the known vector-function of dimension n + 2m, $x_1 = \varphi_1, x_2 = \varphi_2, x_3 = \varphi_3, x_4 = \varphi_4\ , x_5 = \varphi_5, x_6 = \varphi_6, x_7 = \dot{\varphi}_5, x_8 = \dot{\varphi}_6, n = 4, m = 2$.

The equations of the system (6) are represented by differential equations (3) of the kinematics and dynamics (1) in normal form of Cauchy. The initial conditions (9) of the system of differential equations (6) are equal to initial conditions (4) and (2).In these differential equations (6) the massand inertia characteristics of mechanism of class IV are taken into account.Finally, the positions and velocities of all the moving links of mechanism of class IV are defined addressing the system of differential equations (6) having the initial conditions (7) using Runge-Kutta's method. At the same time, the solutions must satisfy the equations of kinematics (5), otherwise we have to use Runge-Kutta's method with a variable step. This is a brief essence of the proposed method of immersion.

The kinematic method for the mechanism of class IV. We go to the determination of initial values of angular coordinates (4) of output links 1-4 when initial values of angular coordinates (3) of input links 5 and 6 for the mechanism of class IV are given. We just note that the differential equations (3) are derived from the trigonometric equations (5) describing the positions of links for mechanism of class IV.

Task 2. Define the initial values of the angular coordinates of output links 1-4 of mechanism of class IV at the initial value of the angular coordinate input links 5 and 6.

Excluding the angular coordinates φ_2 and $\varphi_4$the system (5) takes the form:

$$\begin{aligned} &A_1x_1 + B_1y_1 = C_1 \\ &A_2x_1 + B_2y_1 = C_2 \\ &x_1^2 + y_1^2 = 1, \end{aligned} \tag{8}$$

where

$$x_1^* = x_1\cos\alpha_1 + y_1\sin\alpha_1, \qquad y_1^* = x_1\sin\alpha_1 - y_1\cos\alpha_1,$$

$$x_3^* = x_3\cos\alpha_3 - y_3\sin\alpha_3, \qquad y_3^* = x_3\sin\alpha_3 + y_3\cos\alpha_3,$$

$$A_1 = 2(-l_3x_3 + l_5x_5 - l_6x_6 - l_0x_0)l_1,\ B_1 = 2(-l_3y_3 + l_5y_5 - l_6y_6 - l_0y_0)l_1$$

$$\begin{aligned} C_1 &= l_2^2 - l_1^2 - l_3^2 - (l_5x_5 - l_6x_6 - l_0x_0)^2 - (l_5y_5 - l_6y_6 - l_0y_0)^2 + \\ &+ 2l_3x_3(l_5x_5 - l_6x_6 - l_0x_0) + 2l_3y_3(l_5y_5 - l_6y_6 - l_0y_0), \end{aligned}$$

$$A_2 = 2(-l_3^*x_3 + l_5x_5 - l_6x_6 - l_0x_0)l_1^*\cos\alpha_1 - 2(-l_3^*y_3 + l_5y_5 - l_6y_6 - l_0y_0)l_1^*\sin\alpha_1,$$

$$B_2 = 2(-l_3^*x_3 + l_5x_5 - l_6x_6 - l_0x_0)l_1^*\sin\alpha_1 + 2(-l_3^*y_3 + l_5y_5 - l_6y_6 - l_0y_0)l_1^*\cos\alpha_1,$$

$$\begin{aligned} C_2 &= l_4^2 - l_1^{*2} - l_3^{*2} - (l_5x_5 - l_6x_6 - l_0x_0)^2 - (l_5y_5 - l_6y_6 - l_0y_0)^2 + \\ &+ 2l_3^*(l_5x_5 - l_6x_6 - l_0x_0)x_3^* + 2l_3^*(l_5y_5 - l_6y_6 - l_0y_0)y_3^*. \end{aligned}$$

The originality of the approach [7] is to switch from the system (8) after an application of the universal substitution to the system of quadratic equations:

$$\begin{aligned} &(A_1 + C_1)u^2 - 2B_1u + C_1 - A_1 = 0 \\ &(A_2 + C_2)u^2 - 2B_2u + C_2 - A_2 = 0. \end{aligned} \tag{9}$$

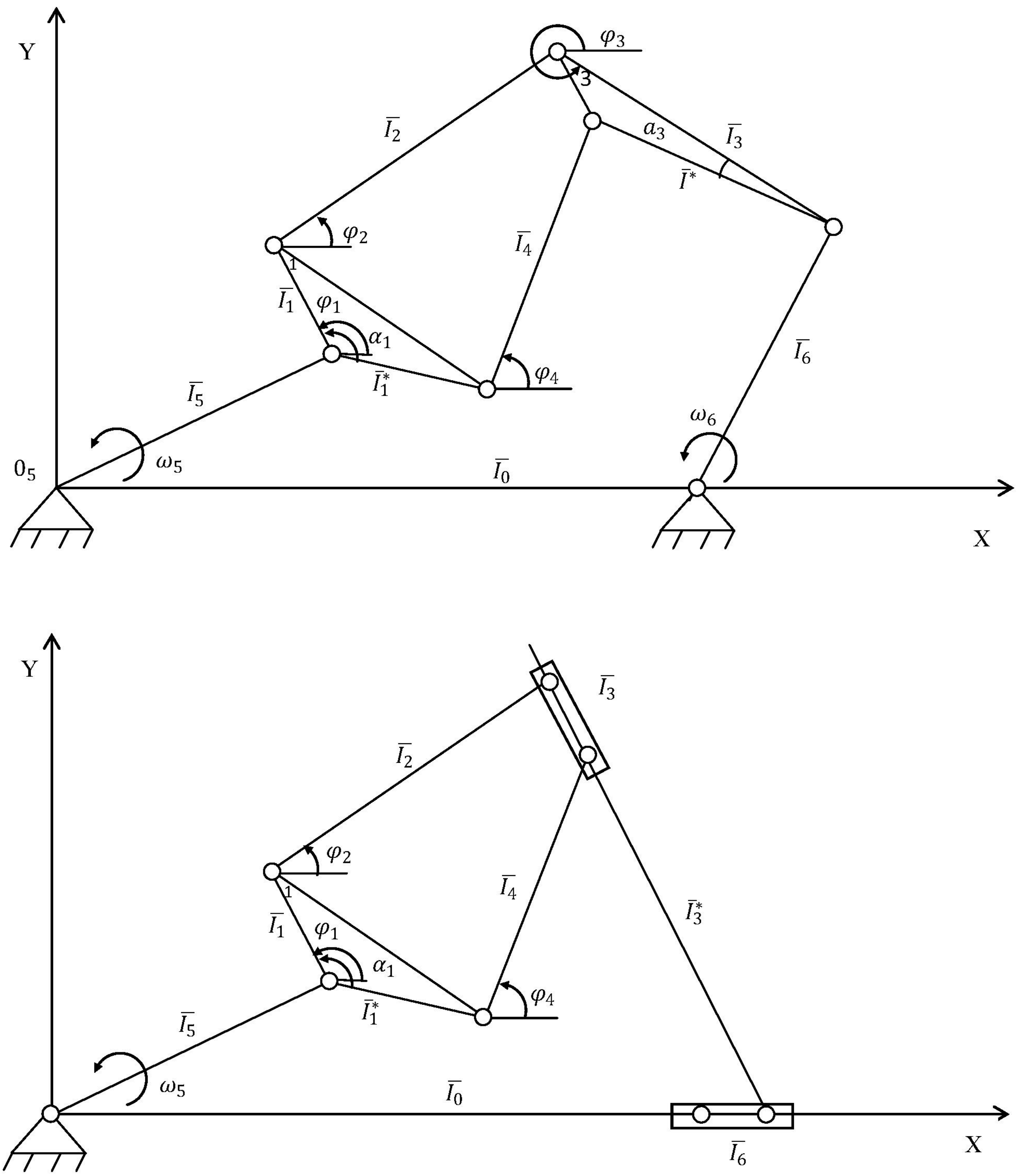
Initially, we must determine the real solution of the nonlinear system (9). For this purpose, we define the discriminants of the equations (9)

$$D_1 = A_1^2 + B_1^2 - C_1^2,\ D_2 = A_2^2 + B_2^2 - C_2^2$$

and conditions of the system solutions' reality (8)

$$D_1 > 0, D_2 > 0, \text{or} D_1 \geq 0, D_2 > 0, \text{or} D_1 > 0, D_2 \geq 0.$$

Discriminants are algebraic inequalities of the fourth degree regarding the variable $x_3 = cos\varphi_3$ or $y_3 = sin\varphi_3$. Therefore, it is easy to determine this variable's exact ranges based on higher algebra.



Mechanism of class IV having rotational and serial pairs

The common algorithm for solving the dynamic and kinematic problems of the mechanism of class IV. Fundamentals of the above offered theory have formed the basis of algorithms and programs determining dynamic and kinematic parameters of the mechanism of class IV.

Step 1. Input of data:$\varphi(t_0) = \varphi_5^0$,

$$l_0, l_1, l_2, l_3, l_4, l_5, l_1^*, l_3^*, \alpha_1, \alpha_3, l_6 = 0.$$

Step 2. Determination of the initial conditions $\varphi_1(t_0), \varphi_2(t_0), \varphi_3(t_0), \varphi_4(t_0)$on the basis of the method of kinematics (10)-(12)

Step 3.Start Runge-Kutta's method to determine all angular coordinates and speeds for the mechanism of class IV:

$$\varphi_1(t_0), \varphi_2(t_0), \varphi_3(t_0), \varphi_4(t_0)\text{and}\dot{\varphi}_1(t_0), \dot{\varphi}_2(t_0), \dot{\varphi}_3(t_0), \dot{\varphi}_4(t_0)$$

Step 4. Output of results.

We found the position, speed of links and reaction forces in joints for mechanism of class IV on the basis of the developed programs.

Discussion of results. Initially, we represent the scientific value of the developed analytical methods for calculating kinematic and dynamic parameters of mechanism of high class. For the first time necessary and sufficient conditions for the existence of real solutions of the kinematic and dynamic general model describing the position, speed and inertia characteristics of all moving parts of the mechanism of class IV have been found.

Kinematic and dynamic tasks are reduced to the study of non-linear system of ordinary differential equations. Today, the generalized force - control (moment-control) can be determined on the basis of control theory. In the final analysis all these problems are solved by the Runge-Kutta's method. Power analysis makes it possible to calculate the strength of the mechanism of class IV parameters and to select the power for the desired drive unit. Thus, a fresh approach to make a simultaneous solution of kinematic, and dynamic problems of the mechanism of class IV is offered. The obtained results are easy to apply to mechanisms of high class with a larger number of closed contours. The importance of the fundamental scientific result presenting in brief the theory of mechanisms of high class liesin its practical application. The question always arises: how to use the theory actually proposed in a compressed form, and the unique properties of these mechanisms in the design of new devices and machines.

For the first time the programs on the Delphi's language calculate position, velocity inthe mechanism of class IV with any desired accuracy.

The results of program on the position and velocity of links for the mechanism of class IV

Initial data: L_0 =10cm, L_1 =2cm, L_1^* =2cm, L_2 =8,268cm, L_3 =5cm, L_3^* =5cm, L_4 =5,9133cm, L_5 =4cm,L_6 =0, α_1 =60^0, α_3 =30^0, φ_0 =0, $\varepsilon = 0{,}001$, φ_{5min} =70^0, φ_{5max} =105^0, φ_{3min} =70^0, φ_{3max} =100^0, h =0,05^0, $\dot{\varphi}_5$=0,01rad/s.

The program defines values of the angular coordinates of all remaining links at the possible ranges of variationφ_{3min} =70$^0 \leq \varphi_3 \leq \varphi_{3max}$ =100^0, $\varphi_{5min} = 70^0 \leq \varphi_5 \leq \varphi_{5max}$ =105^0. Separately we give the initial values of the angular coordinate of input link 5 φ_5 =90,0499999999989^0 and of output link3 φ_3=90,0499999999989^0 in accordance with which the movement of the mechanism starts. If we change the angular coordinate φ_5 of input link 5 90,0499999999989^0 to 90,2499999999988^0

The position and velocity of change for output links 1-4 are listed in table 1and table 2.

Table 1 – Angular coordinate of links formechanism of class IV

φ_1	φ_2	φ_3	φ_4	φ_5
30.151212399452	-0.031675872130	90.0499999999989	12.928351949062	90.0499999999989
30.153170114784	-0.032123943447	90.0999999999989	12.907880756584	90.0999999999989
30.155201340299	-0.032593556243	90.1499999999989	12.887379206970	90.1499999999989
30.155693019626	-0.032636365534	90.1499999999989	12.879818673972	90.1999999999989
30.157800302286	-0.033107168760	90.1999999999989	12.859325664381	90.2499999999988

Table 2 – Velocityof links for mechanism of class IV

$\dot{\varphi}_1$	$\dot{\varphi}_2$	$\dot{\varphi}_3$	$\dot{\varphi}_4$
0,085501549459072	-0,0179259921115466	0,0241681781581009	-0,00757057156281362
0,0853804108253469	-0,0179268654072137	0,024138568162871	-0,00755344825048743
0,0852599951693558	-0,0179278751364108	0,0241091893026834	-0,00753641949441759
0,0852777526589473	-0,0179964249228941	0,0241643846455302	-0,00753728917467803
0,0852777526589473	-0,0179974850000152	0,0241350541017292	-0,00752029545151407

The computational experiment indicated that the mechanism of class IV has broad functionality. These mechanisms can be used in various fields of science and technics. These mechanisms can be used in the design of robots and cranes with high lifting capacity, mechanisms of departure chassis of a fighter aircraft landing at high speed on aircraft carriers, heavy transport and passenger aircrafts landing on the airfield, presses and hydraulic hammers at a significant rate of working body movements, and etc.

Conclusion. For the first time we make the following conclusions:

– a new direction is suggested in simultaneous calculation of kinematics and dynamics for mechanisms of high class on the basis of the differential-algebraic equations;

– methods and algorithms for determining the position and velocity of links in the mechanism of class IV are developed;

– a program for calculation of kinematic and dynamic parameters in the mechanism of class IV with any given accuracy is written.

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Б. Синчев[1], А. М. Муханова[2]

[1]Международный университет информационных технологий, Алматы, Казахстан,
[2]Алматинский технологический университет, Алматы, Казахстан

ПРОЕКТИРОВАНИЕ УНИКАЛЬНЫХ МЕХАНИЗМОВ И МАШИН. I.

Аннотация. Эта работа посвящена разработке численных методов расчета кинематических и динамических параметров для проектирования механизма IV класса на основе уравнений кинематики и динамики. Кинематические и динамические модели этих механизмов описываются дифференциально-алгебраическими уравнениями. Эти уравнения относятся к сингулярным дифференциальным уравнениям. Впервые мы рассматриваем анализ динамики и кинематики механизмов высоких классов в совместной формулировке. Происходит кардинальный прорыв при переходе от механизма низшего класса к высокому классу для проектирования новых механизмов и машин. Предлагается новая концепция, основанная на методе Рунге-Кутта для механизмов высокого класса, направленная на решение кинематико-динамической модели. Разработаны методы, алгоритмы и программы для определения положения и скорости звеньев механизма класса IV с любой заданной точностью. Численный эксперимент показал огромное преимущество механизма IV класса для разработки нового механизма для машиностроения и робототехники.

Ключевые слова: кинематика, динамика, механизм высокого класса, машиностроение, робототехника.

Б. Синчев[1], А. М. Муханова[2]

[1]Халықаралық ақпараттық технологиялар университеті, Алматы, Қазақстан,
[2]Алматы технологиялық университеті, Алматы, Қазақстан

БІРЕГЕЙ МЕХАНИЗМДЕРДІ ЖӘНЕ МАШИНАЛАРДЫ ЖОБАЛАУ. І.

Аннотация. Жұмыс кинематика мен динамиканың теңдеулеріне негізделген IV класс жаңа механизмін жасау үшін кинематикалық және динамикалық параметрлерін есептеудің сандық әдістерін әзірлеуге арналған. Бұл механизмдердің кинематикалық және динамикалық үлгілері дифференциалдық алгебралық теңдеулермен сипатталады. Бұл теңдеулер сингулярлы дифференциалдық теңдеулерге жатады. Бірлескен тұжырымдамада жоғары деңгейлі механизмдердің динамикасын және кинематика анализін алғаш рет қарастырамыз. Жаңа механизмдер мен машиналарды жобалау үшін төменгі сынып механизмінен жоғары сыныпқа көшудің түбегейлі серпілісі бар.Кинематикалық-динамикалық модельді шешуге бағытталған жоғары дәрежелі механизмдер үшін Runge-Kutta әдісіне негізделген жаңа тұжырымдама ұсынылды. Сыныптың IV тетігінің кез-келген дәлдікпен байланыстарын және жылдамдығын анықтау әдістері, алгоритмдері және бағдарламалары әзірленді. Сандық эксперимент инженерлік және робототехниканың жаңа механизмін жасау үшін IV класты механизмнің үлкен артықшылығын көрсетті.

Түйінді сөздер: кинематика, динамика, жоғары дәрежелі механизм, машина жасау, робототехника.

Сведения об авторах:

Синчев Бахтгерей – д.т.н., профессор, Международный университет информационных технологий, e-mail: sinchev@mail.ru

Муханова Аксулу Мухамбеткалиевна – к.т.н., и.о. доцента, Алматинский технологический университет, e-mail: nuraksulu72@mail.ru