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NUMERICAL ANALYSIS OF THE CHARACTER OF EFFLUX OF HIGHLY VISCOUS FLUIDS FROM THE "NARROW CHANNEL"

Abstract. Based on the mathematical model of the efflux of igneous materials we have formulated a mathematical problem of the efflux of high-viscosity fluids from the so-called "narrow channel". A new approach to the solution of a quasi linear parabolic equation with a modified domain of integration is proposed. An algorithm and program for solving the problem have been developed and a numerical analysis of a special case has been carried out. The outcome of the solution has been presented in the form of graphs of the sought-for function which determines the free surface of the flown out viscous fluid.

Key words: igneous substance, high-viscosity fluid, quasi-linear parabolic equation, algorithm and program for solving the problem.

1. Introduction. One of the problems arising in the model studies of tectonic processes in the Earth's interior is a quantitative analysis of the movements of igneous materials rising from its lower layers. This article is devoted to a numerical analysis of the effluxprocess of high-viscosity fluidsfrom the so-called "narrow channel" with the help of the developed computer program. The outcome of this work is of theoretical and practical interest for researchers of tectonic processes in the Earth's interior.

Statement of the problem. According to one of the hypotheses on the processes taking place in the upper mantle, it is believed that the heated igneous substancesrise up the so-called "narrow channels" which leads to a hydrodynamic instability due to the interaction with the overlying denser asthenosphere substances [1-4]. For model studies of this process it is assumed that the movement of igneous substances is regarded as movement of a highly viscous fluid at very low Reynolds numbers [5-8]. A mathematical model of the process was obtained and published by the authors in article [9]. On the basis of this mathematical model a mathematical problem has been formulatedin which it is required to develop a computer program to solve the resulting mathematical problem and carry out a numerical experiment to analyzethe efflux process of high-viscosity fluidfrom the "narrow channel" [10].

Formulation of the mathematical problem. It is required to solve the following two equations:

$$\frac{\partial\xi}{\partial t}=\frac{ER}{3}\cdot\frac{\partial}{\partial x}(\xi^3\cdot\frac{\partial\xi}{\partial x})+\begin{cases} y(x,t),\ if\ x\in[-1,1], \\ 0,\ if\ x\notin[-1,1], \end{cases} \quad (1)$$

$$\int_0^{p(t)}\xi(x,t)\cdot dx=\int_0^t\int_0^r y(x,t)\cdot dxdt, \quad (2)$$

for the initial conditions

$$t=0,\quad \xi(x,0)=0,\quad p(0)=r, \quad (3)$$

andthe boundary conditions

$$x = 0, \quad \frac{\partial \xi(0,t)}{\partial x} = 0, \tag{4}$$

$$x = p(t), \quad \xi[p(t),t] = 0. \tag{5}$$

This problem is solved in the following domain with variable boundaries:

$$\{0 \le x \le p(t),\ 0 \le t \le 1\}.$$

The problem (1) - (5) is solved to determine two unknown functions: $\xi(x,t)$ and $p(t)$.

The physical nature of these functions wasdescribed in article [3], and it is given here. Function $\xi(x,t)$ determines the boundary of the rising high-viscosity fluid, function $p(t)$ determines the change ofthe intersection point between the boundary and the horizontal axis where $\xi(x,t) = 0$.

Here, the equation (1) is a nonhomogeneous quasi-linear parabolic equation, and the equation (2) is an integral equation. All variables and parameters in the equations, initial and boundary conditions are dimensionless quantities.

2. Method.

The essence of the method of solving the problem. It is obvious that the formulated mathematical problem (1) - (5) can not be solved analytically, so we have to use a numerical method for the solution. To solve the quasi-linear parabolic equation (1) we can use the method of finite difference. For the numerical solution of the equation (1), the function must be prescribedwhich is in its right hand side $y(x,t)$. This function can be prescribed, it determines the velocity of the fluid from the "narrow channel". If it is defined, the right side of the equation (2) can also be calculated.

Let the right-hand side of the equation (2) be expressed as the following function:

$$S(t) = \int_0^t \int_0^r y(x,\alpha)dxd\alpha\ . \tag{6}$$

Then the equation (2) is written as follows:

$$\int_0^{p(t)} \xi(x,t)dx = S(t). \tag{7}$$

Here we have two problems:

- Firstly, the domain of integration of the equation (1) is a variable, therefore, it needs to be taken into account when using the finite difference method;

- Secondly, in the equation (7), the upper limit of the integral is an unknown function $p(t)$ while its right-hand side can be calculated if this function $y(x,t)$ is prescribed.

To solve these problems we propose a new approach the essence of which is given here. For a certain moment of time t it is believed that the value of the function $p(t)$ is known. Then we solve the equation (1) using the finite difference method for a certain short period of time$[t, t+\Delta t]$.. Here Δt – quite a small value.

So, for the fixed value $p(t)$ we numerically solve the equation (1) in the domain

$$\{0 \le x \le p(t);\quad [t, t+\Delta t]\}.$$

Computational scheme for solving differential equations. Let us now consider a problem of equations in mathematical physics where a differential partial equation (1) with the initial conditions (3) and the boundary conditions (4,5) is solved. Equation (1) is quasi-linear, so to solve this equation a finite difference method is selected. "Currently, the finite difference method is the only method to effectively solve quasilinear equations" [11].

According to the rules of the finite difference method we first select partition steps for corresponding arguments: for variable t -τ, and for variable$x$ - h. In the future we consider discrete values of all variables included in the original equation (1), with the initial conditions (3) and the boundary conditions (4,5). We introduce the following notations:

$$u_{ij} = \xi(x_i, t_j),\ x_i = h\cdot(i-1),\ t_j = \tau\cdot j,\ i = 1,2,3,\ldots,n+1,\ j = 1,2,3,\ldots,m;$$
$$n = p_j / h,\ \ m = 1/\tau,\ \ p_j = p(t_j). \tag{8}$$

One can write the following formulas which replace the derivatives of the sought-for function with finite difference equations:

$$\frac{\partial\xi}{\partial t} \approx \frac{u_{ij+1} - u_{ij}}{\tau} \tag{9}$$

Use of this formula to replace the first time derivative results in implicit computational scheme [11].

$$\frac{\partial}{\partial x}\left(\xi^3\frac{\partial\xi}{\partial x}\right) \approx \frac{1}{h^2}\left\{\left(\frac{u_{ij+1} + u_{i+1\,j+1}}{2}\right)^3 \cdot \left(u_{i+1\,j+1} - u_{ij+1}\right) - \left(\frac{u_{i-1\,j+1} + u_{ij+1}}{2}\right)^3 \cdot \left(u_{ij+1} - u_{i-1\,j+1}\right)\right\}.$$

(10)\

In the formula (10) we use the following substitutions:

$$\xi(x_i, t_{j+1}) \approx \frac{u_{ij+1} + u_{i-1j+1}}{2};$$
$$\xi(x_{i+1}, t_{j+1}) \approx \frac{u_{ij+1} + u_{i+1j+1}}{2};$$
$$\frac{\partial\xi(x_i, t_{j+1})}{\partial x} \approx \frac{u_{ij+1} - u_{i-1j+1}}{h};$$
$$\frac{\partial\xi(x_{i+1}, t_{j+1})}{\partial x} \approx \frac{u_{i+1j+1} - u_{ij+1}}{h};$$

The formula available in the right-hand side of equation (1) is replaced by the following expression, the values of which are considered to be known:

$$y_{ij} = y(x_i, t_j). \tag{11}$$

Substitution of formulas (9) - (11) into equation (1) allows to obtain the following formula for the unknown discrete valuesu_{ij} of the sought–for function:

$$u_{ij+1} = u_{ij} + \frac{ER\cdot\tau}{3\cdot h^2}\left\{\left(\frac{u_{ij+1} + u_{i+1\,j+1}}{2}\right)^3 \cdot \left(u_{i+1\,j+1} - u_{ij+1}\right) - \left(\frac{u_{i-1\,j+1} + u_{ij+1}}{2}\right)^3 \cdot \left(u_{ij+1} - u_{i-1\,j+1}\right)\right\} +$$
$$\begin{cases} y_{ij}, \text{если } 1 \le i \le k, \\ 0, \text{если } k+1 < i \le n. \end{cases}$$

(12)

Here $k = \frac{1}{h}$ - number of points where the function $y(x,t)$ is different from zero: $i = 0,1,2,\ldots,n-1$; $j = 1,2,\ldots,m$. It should be noted that the parameter$n$ is a variable, depending on time$t$. Its value can be fixed; it may be quite large. Then the problem is solved in the initial moments of time and for that part of the interval, where $x > p(t)$.

Based on the initial conditions (3) and taking into account the introduced notations it will be rewritten in the following discrete form:

$$u_{i0} = 0,\quad i = 1,2,3,\ldots,n. \tag{13}$$

The boundary conditions (4,5) determine the values of the sought-for function at the boundaries of the domain of integration, and will be written in the form of the following discrete formulas:

$$u_{0j} = u_{1j},\quad u_{nj} = 0,\quad j = 1,2,3,\ldots,m. \tag{14}$$

So, instead of a quasi-linear partial differential equation (1), with initial (3) and boundary (4,5) conditions, we obtain a system of nonlinear algebraic equations (12) - (14) with respect to the discrete values of the sought-for function u_{ij} . The solution to this system of algebraic equations allows usto obtain a numerical solution to the problem stated in this section.

Note that this system of equations is performed for a fixed value of variable t or j.

Formula (12) is a system of nonlinear algebraic equations which requires the iteration method [11] to solve it.

To apply the iteration method for solving the system of equations it is expedient to transform it and bring it to the following standard form:

$$A_i \cdot z_{i-1} - C_i \cdot z_i + A_{i+1} \cdot z_{i+1} = -F_i. \tag{15}$$

where i = 1,2, ...,n-1; and, as noted above, the parameter n may be a variable value.

In this formula (15), the following notations are introduced:

$$u_{i-1j+1} = z_{i-1}, u_{ij+1} = z_i , u_{i+1j+1} = z_{i+1}$$

$$A_i = \frac{ER}{3}\frac{\tau}{h^2}\left(\frac{z_{i-1}+z_i}{2}\right)^3;$$

$$A_{i+1} = \frac{ER}{3}\frac{\tau}{h^2}\left(\frac{z_i+z_{i+1}}{2}\right)^3; \tag{16}$$

$$C_i = 1 + A_i + A_{i+1}; \quad F_i = -z_i - \begin{cases} y_{j+1}, & if\ 0 \le i \le k, \\ 0, & if\ k+1 \le i \le n; \end{cases}$$

This system of equations (15) is a system of linear algebraic equations for the discrete values of the sought-for function for the new iteration, and its coefficients are determined by the values of the same function in the previous iteration. The peculiarity of the system matrix (16) allows us to use the well-known sweep method to solve this matrix[11].

The numerical values of the second unknown function in this problem $p(t)$ is determined by the recurrence formula (28), and, each time after each iteration at that.

Sweep method of solving a system of algebraic equations. In each iteration for solving a system of algebraic equations (15) the sweep method is used. According to this method the solution of equations (15) is sought in the following form:

$$z_i = \alpha_{i+1} \cdot z_{i+1} + \beta_{i+1}. \tag{17}$$

Here $\alpha_i$andβ_i–sweep coefficients, which are determined by the recurrence formulas:

$$\alpha_{i+1} = \frac{A_{i+1}}{1 + A_{i+1} + A_i \cdot (1-\alpha_i)};$$
$$\beta_{i+1} = \frac{F_i + A_i \cdot \beta_i}{1 + A_{i+1} + A_i \cdot (1-\alpha_i)}. \tag{18}$$

Thus, for any point in time the values $(z_i, i = 1,2,3, \dots, n)$ are determined of the sought-for function by the sweep method.

After determining these values of function $\{z_i\}$ for a particular value t (or j) we can be determine the value of the function $p_j = p(t_j)$.

Use of the iterations method. All of the above formulas are for one iteration, i.e. all calculations related to the formulas above occur within a single iteration. Now we need to determine the formula to implement the iterative process itself.

Let's first consider the application of an iterative process for solving a system of algebraic equations (12). To describe the iterative process that takes place during solving this problem, we must make a notation which should show the values of the sought-for parameters in the two adjacent iterations: in the previous and current one. Here, the following notations are introduced:

$z1_i$ - The value of the sought-for function in the previous iteration;

z_i - The value of the same function in the current iteration.

Then the formula (2.38), determining the coefficients of the system of algebraic equations (2.37) will be written in the following form:

$$A_i = a \cdot (\frac{z1_{i-1}+z1}{2})^3; \quad A_{i+1} = a \cdot (\frac{z1_i + z1_{i+1}}{2})^3$$

$$F_i = z1_i + \begin{cases} y_{j+1}, \ if\ 0 \le i \le k; \\ 0, if\ k+1 < i \le n; \end{cases} \tag{19}$$

where $a = \frac{ER}{3} \cdot \frac{\tau}{h^2}$ - constant value, $k = \frac{1}{h}$, $n = \frac{p_j}{h}$.

Use of the iterations method requires verification after each iteration for the accuracy of the results. To verify for the accuracy of the calculations and the ending of the iterative process usually this condition is used:

$$\max | z_i - z1_i | < \varepsilon \text{ for } 1 \le i \le n. \tag{20}$$

In this condition (20) $\varepsilon > 0$ is considered an anticipatorily prescribed small number; it determines the accuracy of the calculations, therefore, the accuracy of the solution. After this condition is attained the iterative process terminates, and transition to the next value of parameter j or moment in time t is implemented.

Consider a formula that determines the change in the right (and left) border of the domain:

$$p_{j+1} = p_j + \frac{2 \cdot (S_{j+1} - S_j)}{z_i + z1_i}, \tag{21}$$

where $j = 0,1,2,\ldots, m$,

$$S_j = \int_0^{j\tau} \int_0^r y(x,t) dx dt. \tag{22}$$

If the integrand $y(x,t)$ is prescribed then calculation of the integral (22) is not difficult. We can use one of the numerical integration methods.

At the initial time, the value of the function $p(t)$ is determined, it is equal to$r$. Its following values are determined by the recurrence formula (21). Here there is a condition (2), which must be the condition forcompleti on of the iterative process.

So, all the formulas for determining the discrete values of the sought-for functions $\xi(x,t)$ and $p(t)$ have been determined. The values of the first of them are marked u_{ij} (or z_i), and the second value p_j.

3. Algorithmic implementation.

The algorithm for solving the problem. The main component of a computer model of any problem is the software. Therefore, at first a program for the numerical solution of the problem should be developed. To develop a program an algorithm for solving this problem should be drawn up.

Consider an algorithm for solving the problem, which is being developed on the basis of the computational scheme proposed in the previous section of this paper. The algorithm of the proposed computational process will have the following stages:

Stage 1. Input the initial values of the sought-for function z_i for moment of time t= 0 i.e. when j = 0 (zero iteration); here we assign to the sought-for functions $p(t)$ и $z = \xi(x,t)$ their initial values: $p(0) = 1$; $z_i = 0_i$, $i = 0,1,2,\ldots, n$. Here the following option is possible when it is assumed that the number of subdivision points $n$is set. Then step h along the coordinate x will be determined from the formula $h = \frac{p(t)}{n}$ for each time point t. We can use another option when h is constant, and the number of points of subdivision of the variables interval $[0, p(t)]$ will be determined by the formula $n = \frac{p(t)}{h}$. Here we use the initial conditions of the problem (2.35). Simple cycle for parameter (cycle counter) i will be used here. Also, here in this cycle, the argument x values are calculated according to the formula $x_i = i \cdot h$, $i = 0,1,2, \dots, n$.

Stage 2. Onset f the cycle in parameter j i.e. with respect to time t.

Stage 3. Onset of the iterative process. To set up the iterative process we consider two values of the sought-for functions for one and the same time point and for one and the same point along variable x. One value is considered to be computed in the previous iteration $z1_i$ and another value z_i computed in the current iteration. Before performing each iteration, the computed value of the function in the previous iteration will be considered $z1_i$, and the new computed value z_i. Therefore, before performing a new iteration we assign the values obtained of the function z_i to the variables $z1_i$. This assigning process is carried out as a cycle in parameter i according to the formula $z1_i = z_i$, $i = 1,2, \dots, n$.

Stage 4. Here coefficient values A_i, F_i of the system of algebraic equations (15) according to the formulas (16) must be computed. The computation of these formulas will be carried out in the form of a cycle in parameter i.

Stage 5. Direct sweep. In the beginning the first sweep coefficients α_1 and β_1 are computed according to the formulas: $\alpha_1 = 1$, $\beta_1 = 0$. And then, according to the recurrence formulas (18) the other sweep coefficients α_i and β_i are computed. These computations are performed with the help of the cycle in parameter $i = 1,2,3, \dots, n-1$.

Stage 6. Reverse sweep. At first the sought-for function is assigned a value on the right boundary i.e. formula $z_n = 0$ is performed. Then, according to the formula (17) the values of the sought-for function for the rest of the points are determined i.e. for $i = n-1, n-2, \dots, 1$. This is done using the cycle in parameter i.

Stage 7. Verification of the condition of accuracy (20). For this purpose we consider the maximum absolute value from the difference of values of the sought-for function for two iterations. To determine this value we first use the cycle computes the absolute difference between these values according to the formula:

$$\gamma_i = | z_i - z1_i |, i = 0,1,2, \dots, n.$$

As a result we will get an array of positive numbers $\{\gamma_i\}$. To determine the maximum element $\max\{\gamma_i\}$ in this array a cycle is composed whose parameter (counter) is ***i***. After this we make a comparison of $\max\{\gamma_i\}$ with a predetermined small number of $\varepsilon > 0$. If the condition of accuracy $\max\{\gamma_i\} < \varepsilon$ is not fulfilled then the iterative process continues. For this purpose transition to Stage 3 is made. If this condition of accuracy is satisfied, then the iterative process is completed for a given value j and then we move to the next value of parameter j i.e. to the new value t.

Stage 8. Before moving to the next stage in parameter j (to Stage 2) first value n is computed. And for this purpose a new value of the function $p(t)$ is determined. To compute the new value of this function this formula is used:

$$p_{j+1} = p_j + \frac{2 \cdot (S_{j+1} - S_j)}{z_i + z1_i}. \tag{23}$$

Next, the value of n is determined. To determine the value of this parameter equation (2) is used. Use of this equality is possible when velocity of the fluid from the channel is prescribed.

Stage 9. Displaying the results of solving the problem. We print out the values of the elements of the following sets:

- $\{z_i\}$, $i = 1,2,\ldots,n$, for any time point t (or j);
- $\{p_j\}$, $j = 1,2,\ldots,m$.

Using the notation introduced earlier (8) and (16) we can obtain the discrete values of the sought-for function $\xi(x,t)$:

$$\xi(x_i, t_j) = z_i, \text{ for } j = 1,2,3,\ldots,m.$$

Thus, an algorithm for solving the problem discussed in the previous section is proposed, for which a solution method is selected. Now a program for solving the problem for its special case scan be developed and a numerical experiment can be conducted.

4. Special Case.

The case where the efflux velocity is constant. For specific implementation of the proposed algorithm a specific form of the function*y (t)* must be prescribed. For this case, consider an option when the efflux velocity from the fissure is constant, i.e. $y(t) - const.$ Without limiting the generality, in this case, we can assume that $y(t) = 1$, and the formula (3.17) can be rewritten as follows:

$$S_j = \frac{2}{3}\cdot\tau\cdot j, \quad j = 1,2,3,\ldots,m. \tag{24}$$

On the other hand, the value S_j may be approximately equal to the following sum:

$$S_j = h\cdot\sum_1^n z_i\ . \tag{25}$$

From the formula (2) we conclude that following equation must be satisfied:

$$h\cdot\sum_{i=1}^n z_i = \frac{2}{3}\cdot\tau\cdot j. \tag{26}$$

This formula (26) is the condition for determining the value of the unknown parameter n. Determination of values of parameter n (number of nodal points in the grid) is also performed with the iterative method.

In this problem, the only parameter that determines the nature of the process in question parameter *ER.* Therefore, solution of the problem will be carried out for different values of this parameter. Analysis of the results of the numerical solution of this problem can be performed for each value of this parameter. Values for this parameter are selected on the basis of the data available in the geological literature [1-8].

5. Investigation Results.

The results of the numerical experiment. Based on the developed program a numerical experiment has been conducted for different options of this problem. Computations were performed for the following values of the dimensionless parameter *ER:* 0.1; 0.5; 1; 10.

The results are shown in graphs of the sought-for function for different timepoints t . In this case, with all the values being the same of other parameters characterizing the viscous fluid, the dynamic viscosity coefficient will have a relatively high value for small values *ER*. For large values of the dimensionless parameter dynamic viscosity coefficient will be relatively small. This is due to the fact that in the formula which determines the parameter *ER* the dynamic viscosity coefficient is in the denominator i.e. this parameter is inversely proportional to the viscosity coefficient.

The computations were performed for the following values of the parameters of the problem: $h = 0{,}01$; $\tau = 0{,}0001$; $\varepsilon = 0{,}0001$; $n = 300$.

5.1. Computational results.

The computation results for different time points and for different values of the dimensionless parameter *ER* are presented . Here are shown graphs of the sought-for function $z = \xi(x,t)$ for different points and different time points.

In this case, the dynamic viscosity coefficient is considered relatively large; a flow of viscous fluid that has leaked from the fissure, does not spread in the horizontal direction, it behaves as a rigid body. A graphical representation of the results shown in Figure 1.

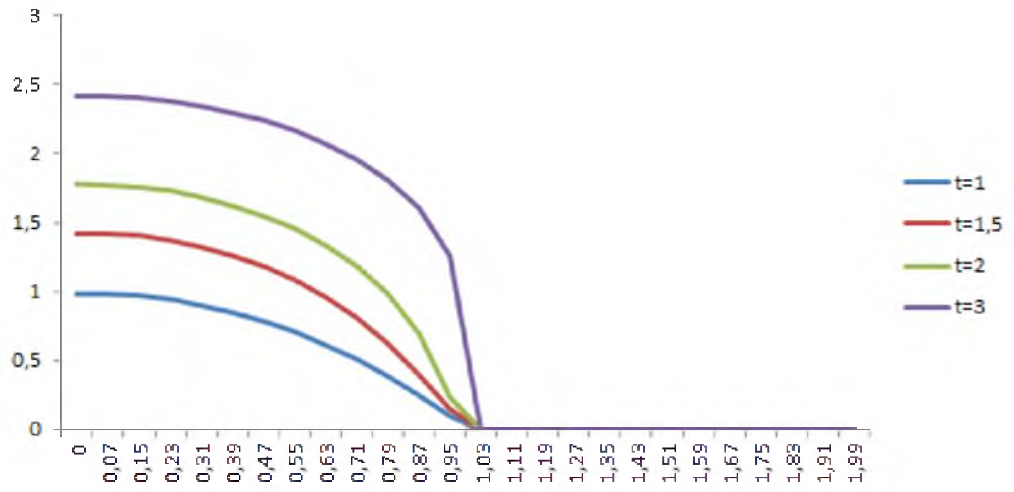

Figure 1 – The positions of the free fluid boundary for ER = 0,1

In this case, after a certain time the fluid begins to spread in the horizontal direction (the last column of the table). This process is shown in Figure 2. At time point $t = 3$ the horizontal boundary of the fluid, the point of intersection of the free surface of fluid with the horizontal surface, moves to a certain distance. In this case, at time point $t = 3$ the boundary is at the point $x = 1{,}2$.

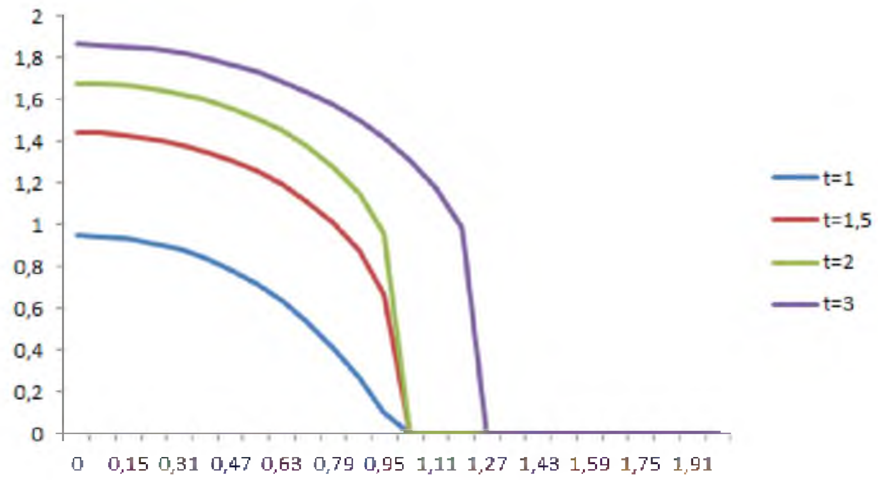

Figure 2 – The positions of the free fluid boundary for ER = 0,5

Figure 2 shows that the boundary of the fluid is moved in the horizontal direction (for $t = 3$). There is some spreading of the fluid in the horizontal direction. Compared with the previous case, when $ER = 0{,}1$, the effect of the viscosity coefficient is obvious.

A graphical representation of the outcome for parameter $ER = 1$ is shown in Figure 3. In this case the spreading of fluid in the horizontal direction is more visible than in the previous cases. At time point $t = 2$ the horizontal boundary change occurs, and at time point $t = 3$ the border is already at the point $x = 1{,}5$.

Where $ER = 10$ the fluid begins to spread already at time point $t = 1{,}5$. The horizontal boundary is at point $x = 2{,}25$.

A graphical representation of the table data shown in Figure 4.

It shows the change dynamics in the free boundary of the out flown fluid depending on the time. For the parameter value $ER = 10$ when the viscosity of the fluid is relatively low, over time it begins spreading on the horizontal surface. Figure 1 shows only the right-hand part of the area in question and its borders (for $x \geq 0$). And its left-hand part (for $x \leq 0$) is the mirror image of the right-hand part.

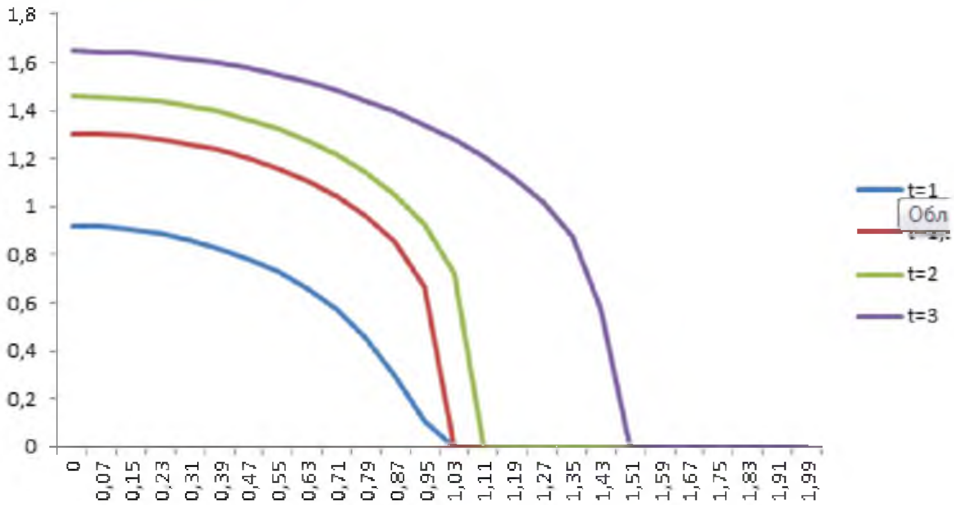

Figure 3 – The positions of the free fluid boundary for *ER* = 1

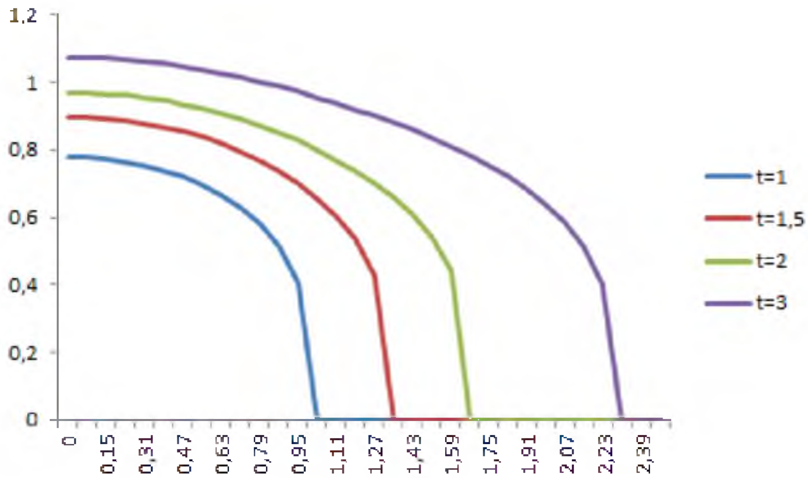

Figure 4 – The positions of the border $z = \xi(x,t)$ for *ER* = 10

6. Data Comparison. When comparing the data, we can be convinced that a lower viscosity fluid spreads in the horizontal direction more rapidly than the higher viscosity fluid.

For a comparative analysis it is appropriate to consider raising to the highest of the free surface of the fluid i.e. comparison for the various cases the maximum values of the function $z = \xi(x,t)$.

For this purpose a selection is made with the largest values of this function (Table) for different values of the parameter ER. For comparative analysis here are given the data for different time points.

Comparison of the data in Table 1 shows that the liquid with a higher dynamic coefficient of viscosity rises higher than the liquid with lower viscosity. This means that the viscous fluid with a large dynamic viscosity behaves as a "solid" to a certain point of time.

Further evidence that the lower viscosity fluid spreads in the horizontal direction more rapidly than higher viscosity liquid. This conclusion is derived from the comparison of the position of the flow-out boundary for different values of ER.

Here are the results of computations for time interval $0 \le t \le 3$. In the case where ER = 0.1, i.e. when the dynamic fluid viscosity coefficient is a large number, the horizontal flow-out boundary remains unchanged. For ER = 0.5 change in this boundary begins after $t \ge 2{,}20$, for ER = 1 after $t \ge 1{,}80$ and ER = 10 after $t \ge 1{,}05$. To compare these results Figure 5 shows graphs of the function $z = \xi(x,t)$ for different values of parameter *ER*.

The greatest values of the function $z = \xi(x,t)$

T	ER = 10	ER = 1	ER = 0.5	ER = 0.1
0.50	0.4704	0.4958	0.4977	0.4994
1.00	0.7771	0.9178	0.9497	0.9868
1.05	0.7966	0.9520	0.9889	1.0336
1.10	0.8120	0.9847	1.0269	1.0797
1.25	0.8494	1.2078	1.3330	1.2144
1.50	0.8969	1.3042	1.4421	1.4238
1.80	0.9429	1.3929	1.5563	1.6471
1.85	0.9495	1.4137	1.5725	1.6813
1.90	0.9560	1.4316	1.5877	1.7146
2.00	0.9694	1.4581	1.6156	1.7789
2.20	0.9937	1.5061	1.6812	2.1379
2.25	0.9992	1.5162	1.6953	2.1595

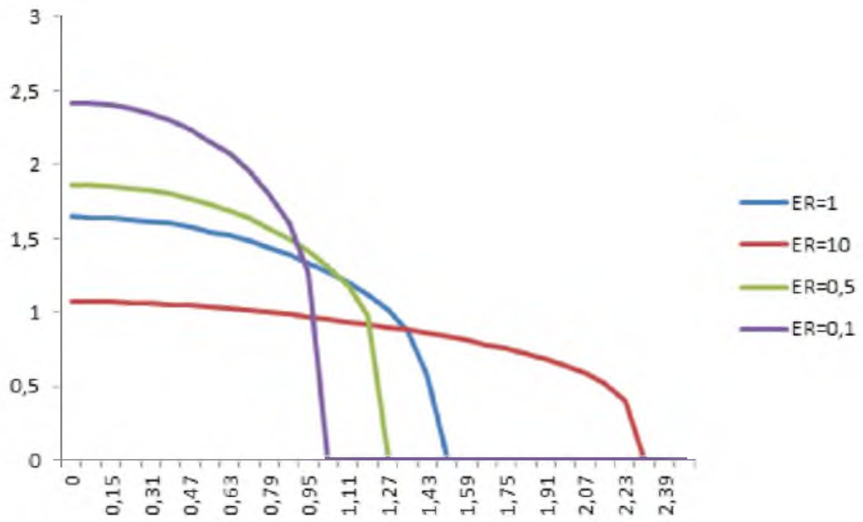

Figure 5 – Positions of free fluid boundary at time point t =3 for different values of *ER*

7. Conclusion. Analysis of the solutions of the problem for the dimensionless parameter values discussed here is primarily concerned with the dynamic coefficient of viscosity. Obviously the same results can be used to analyze in other parameters, e.g. density of earth formations or geometric dimensions and others. However, the values of these parameters are determined by other research methods. Information about them is available in the literature. Determination of the viscous properties of the Earth's rocks is considered fairly complicated and hypotheses are used in many studies. Existing data in the literature are determined through modeling studies of tectonic processes.

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АСА ТҰТҚЫРЛЫ СҰЙЫҚТЫҢ ЖІҢІШКЕ АРНАДАН АҒЫП ШЫҒУЫНЫҢ МАТЕМАТИКАЛЫҚ МОДЕЛІ

Аннотация. Мақала ішкі тектоникалық проңестердің әсерімен «жіңішке арна» арқылы жогары көтерілетін қыздырылган магматикалық заттардың қозагалысын зерттеуге арнлаған математикалық модель құрастыруга арналган есеп қойылған. Интегралды аймагы «өзгерін отыратын» шарт кою арқылы парабола типті квазисызықты теңдеуді шешудің жаңа жолы ұсынылған. Есептің алгоритмі құрылып бағдарламасы жасалған және нақты жагдайда сандық талдау жүргізілген. Есептің шешімінің нәтижесі жердің астынан аса тұтқұрлы сұйықтықтың«жіңішке арна» арқылыжердің бетіне көтеріліп шыққан қозгалысының функция графигі түрінде көрсетілген.

Тірек сөздер: магмалық заттар, аса тұтқұрлы сұйықтық, парабола типті квазисызықты теңдеу, алгоритм және есепті шешу бағдарламасы.

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ЧИСЛЕННЫЙ АНАЛИЗ ХАРАКТЕРА ИСТЕЧЕНИЯ СИЛЬНОВЯЗКОЙ ЖИДКОСТИ ИЗ «УЗКОҒО КАНАЛА»

Аннотация. Поставлена математическая задача об истечении сильновязкой жидкости из т.н. «узкого канала» на основе математической модели истечения магматических веществ. Предложен новый подход к решению квазилинейного уравнения параболического типа при условии изменения области интегрирования. Разработаны алгоритм и программа решения задачи, проведен численный анализ для частного случая. Результаты решения задачи представлены в виде графиков искомой функции, определяющей свободную поверхность вытекшей вязкой жидкости.

Ключевые слова: магматические вещества, сильновязкая жидкость, квазилинейное уравнение параболического типа, алгоритм и программа решения задачи.

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