УДК 530.145.6

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DETERMINATION OF CONSTITUENT MASS OF QUARKS IN THE FIELD CORRELATOR METHOD

In the framework of the field correlator method, the mass spectrum of the mesons consisting of the light-heavy quarks with orbital and radial excitations is determined. Dependence of the constituent mass of a free state is derived. When quarks are light, then the difference of constituent and current masses of quarks is greater than current masses of quarks, and when quarks are heavy than the difference of these masses is insignificant.

1. Introduction

The description of the mass spectrum of hadrons is one of the fundamental problems of strong interactions. On the basis experimental data for the u- and d- quarks this is in the ranges 4-8.5 MeV, and for the s- quark is 80-155 MeV and for the c- quark is 1-1.4 GeV (for details see Ref. [1]). At present, many-potential and nonpotential models exist which investigate the property and dynamics of hadrons (for example see Ref. [2]-[4]). Most of the models for the description of the property of hadrons consisting of light quarks use the constituent mass of quarks as free parameters and for the u- and d- quarks this is in the range of 150-220.5 MeV. Thus, the difference between constituent and current masses of light quarks is very large, but this difference for the c- and b- quarks is not large. Then we face such problems as whether we can determine the dependence of constituent masses on current masses and how to explain that the difference between constituent masses and current masses for the light quarks is very large but for heavy quarks is small. The answers to these questions give us a possibility to understand the formation mechanism of relativistic bound state and also to explain the transition dynamics of confinment to deconfinment phase of hadrons.

In the framework of various approaches the constituent mass dependence of the current quark mass and of the basic quantities of relativistic bound states is considered. At present, the dependence of constituent mass on current mass of quarks is determined in the framework of the sum rule approach [5] using gauge-invariant representation of condensate of quark field at high momentum transfer or at a short distance in Euclidean space. However, to explain many phenomena which appear at large distances this method is not sufficient [6].

The dependence of the constituent quark mass on the current quark mass and quark condensate is also considered [8] in the framework of the QCD instant on vacuum model [7]. However, the confinement is absent in an instant on vacuum model.

In [9], using the Fock - Feynman - Schwinger representation one of the unique methods of consideration of nonferturbative characteristics of interaction was suggested for description of properties of relativistic bound states (detailed see Ref. [10]). This method is based on using a gauge – invariant Green function for white objects and the transformation matrix can be represented in the form of a functional integral [11]. Father, this method was improved and successfully applied for description of hadrons and glueball mass spectrum [12]. The main moment of this method is the calculation of the functional integral. Certainly, this integral is not evaluated in a general way, but one can calculate it only using some physical suggestions. One of the alternative methods for calculation of the functional integral is suggested in Ref. [13] and the glueball mass are determined. In this work, were calculated the mass spectrum of mesons consisting of light–heavy quarks with orbital and radial excited states and determined the dependence of the constituent mass on the current mass of quarks.

2. The mass spectrum of the bound state

In this section, we will present one of the alternative methods of the bound-state mass determination in the framework of the Field Correlator Method (FCM) [9, 10] with nonperturbative and relativistic character of the interaction taken into account. Let us consider an interaction between two charged scalar particles in the external gauge field. We assume that these particles constitute a bound state. Let us determine the mass of a bound state by investigating the asymptotic behavior of the polarization loop function for a charged scalar particle in the external gauge field. The polarization loop function for a scalar loop particle can be written as

$$\Pi(x-y)=\left\langle G_{m_1}(x,y|A)G^*_{m_2}(y,x|A)\right\rangle_A . \quad (2.1)$$

Here the averaging over the external gauge field $A_\alpha(x)$ is performed. The green function $G_m(y,x|A)$ for the scalar particle in the external gauge field is determined from the equation

$$\left[\left(i\frac{\partial}{\partial x_\alpha}+\frac{g}{c\hbar}A_\alpha(x)\right)^2+\frac{c^2}{\hbar^2}m^2\right]G(x,y|A_a)=\delta(x-y), \quad (2.2)$$

where m is the mass of the scalar particle, and g is the coupling constant. In averaging over the external gauge field $A_\alpha(x)$, let us consider only the lowest order or only or only the two-point Gauss correlator

$$\left\langle \exp\left\{i\int dx A_\alpha(x)J_\alpha(x)\right\}\right\rangle_A=\exp\left\{-\frac{1}{2}\iint dxdy J_\alpha D_{\alpha\beta}(x-y)J_\beta(y)\right\}. \quad (2.3)$$

Where $J_\alpha(x)$ is the real current. The propagator of the gauge field has the following from:

$$D_{\alpha\beta}(x-y)=\left\langle A_\alpha(x)A_\beta(y)\right\rangle_A . \quad (2.4)$$

From (2.1) one can see that for determination of the loop function one needs to determine the Green function. The solution of (2.2) can be represented us a functional integral in the following way (for details see Ref. [14]).

$$G(x,y|A)=\int_0^\infty\frac{ds}{(4\pi s)^2}\exp\left\{-sm^2-\frac{(x-y)^2}{4s}\right\}\cdot\int d\sigma_B\exp\left\{ig\int_0^\infty d\xi\frac{\partial z_\alpha(\xi)}{\partial\xi}A_\alpha(\xi)\right\}, \quad (2.5)$$

where the following notation is used:

$$Z_\alpha(\xi)=(x-y)_\alpha\xi+y_\alpha-2\sqrt{s}B_\alpha(\xi);$$

$$d\sigma_\beta=N\delta B\exp\left\{-\frac{1}{2}\int_0^1 d\xi B^2(\xi)\right\}, \quad (2.6)$$

with the normalization

$$B_\alpha(0)=B_\alpha(1)\quad 0;\qquad =\int d\sigma_\beta\quad 1.$$

where N is the normalization constant. Substituting (2.5) into (2.1) and averaging over the external gauge field $A_\alpha(x)$, one can obtain for the loop function

$$\Pi(x)=\int_0^\infty\int_0^\infty\frac{d\mu_1 d\mu_2}{(8x\pi^2)^2}\exp\left\{-\frac{|x|}{2}\left(\mu_1+\frac{m_1^2}{\mu_1}\right)-\frac{|x|}{2}\left(\mu_2+\frac{m_2^2}{\mu_2}\right)\right\}J(\mu_1,\mu_2). \quad (2.7)$$

Here

$$J(\mu_1\mu_2)=N_1N_2\int\int\delta\vec{r}_1\delta\vec{r}_2\exp\left\{-\frac{1}{2}\int_0^x d\tau\left(\mu_1\dot{\vec{r}}_1^{\,2}(\tau)+\mu_2\dot{\vec{r}}_2^{\,2}(\tau)\right)\right\}\exp\left\{-W_{1,1}+2W_{1,2}-W_{2,2}\right\} \quad (2.8)$$

and the following notation is used:

$$W_{i,j}=(-1)^{i+j}\frac{g^2}{2}\int_0^x\int d\tau_1 d\tau_2 \dot{Z}_{\alpha}^{(i)}(\tau_1)D_{\alpha\beta}\left(Z^{(i)}(\tau_1)-Z^{(j)}(\tau_2)\right)\dot{Z}_{\beta}^{(j)}(\tau_2). \tag{2.9}$$

We have the loop-function for the scalar particles with masses m_1 and m_2, and the interaction can be performed with exchange of gauge field. This interaction of constituent particles has two parts: first, the exchange interactions of constituent particles and the contributions of these interactions determined by $W_{1,2}$ and second, the interactions of constituent particles by themselves such as the self-energy diagram contributions and represented by $W_{1,1}$ and $W_{2,2}$ (for details see Ref.[15]). In particular, $W_{1,1}$ and $W_{2,2}$ define nonpotential interactions, and $W_{1,2}$ defines potential interactions with nonlocal nature. On the other hand, the functional integral introduced in (2.8) is analogous to the Feynman trajectory integral for the motion of two particles with masses μ_1 and μ_2 in the nonrelativistic quantum mechanics [16]. The interaction between these particles is described by expression (2.9) which contains both the potential and nonpotential parts. The mass of the bound state is usually defined through the loop function in the following way:

$$M=-\lim_{|x-y|\to\infty}\frac{\ln(\Pi(x-y))}{|x-y|}. \tag{2.10}$$

From (2.10) is follows that knowing the loop function one can determine the mass of the bound state as well. However, the functional integrals in (2.7) and in (2.8) cannot be evaluated in a general way. According to (2.10), one needs to derive the loop function in asymptotics. Let the functional integral in (2.8) be defined in the $|x-y|\to\infty$ limit in the following way:

$$\lim_{|x|\to\infty}J(\mu_1,\mu_2)=\exp\{-xE(\mu_1,\mu_2)\}. \tag{2.11}$$

where $E(\mu)$ is a value depending only on μ_1 and μ_2 and on the coupling constant g. In this approximation the integral in (2.10) is evaluated by the saddle-point technique and, hence, for the bound state mass we obtain

$$M=\sqrt{m_1^2-2\mu^2E'(\mu)}+\sqrt{m_2^2-2\mu^2E'(\mu)}+\mu E'(\mu)+E(\mu). \tag{2.12}$$

The parameter μ can be determined from the equation

$$\frac{1}{\mu}=\frac{1}{\mu_1}+\frac{1}{\mu_2}\equiv\frac{1}{\sqrt{m_1^2-2\mu^2E'(\mu)}}+\frac{1}{\sqrt{m_2^2-2\mu^2E'(\mu)}}, \tag{2.13}$$

where the following designation is used:

$$E'(\mu)=\partial E(\mu)/\partial\mu.$$

From (2.12) and (2.13) we see that if we determine $E(\mu)$, then we can define the mass and constituent mass bound state. We will consider the parameters μ_1 and μ_2 as masses of the constituent particles in the bound state. These masses differ from m_1 and m_2 which represent the masses of a free state. In the description on the mass spectrum of the relativistic bound state the constituent mass, which differs from the mass of initially free particle, is usually introduced. Particularly, when describing the hadron mass spectrum, the masses of the constituent and current quark are introduced. The quantity $E(\mu)$ is defined as an eigenvalues of the interaction Hamiltonian with the potentials in (2.9).

3. Determination of the mass spectrum of mesons consisting of light-heavy quarks

3.1. The energy spectrum of linear potential

In the FCM assume that the confinement of color objects can be explained only within the framework of nonpertubation theory, and it is connected with the initiation linear potential or qark-antiquark string as a result of nonperturbative interaction gluons. Let us determine $E(\mu)$ the energy spectrum of linear potential with orbital and radial excitations. The Schrodinger equation (SE) can be rewritten in the form:

$$\left[\frac{1}{2\mu}\vec{P}^2 + \sigma\, r\right]\Psi = E(\mu)\Psi\,, \tag{3.1}$$

where σ is string tension. The energy spectrum and wave function are determined from SE in the framework of the oscillator representation (OR) method [13, 17]. First of all, make the transformation of canonical variables leading to the Gaussian asymptotics in the d- dimensional auxiliary space R^d, and the interaction Hamiltonian is represented in the normal form creation a^+ and annihilation a operators (for details see Ref.[17]). We can rewrite the Hamiltonian (3.1) in the form

$$H = H_0 + \varepsilon_0(E) + H_I\,, \tag{3.2}$$

where H_0 is the Hamiltonian of the free oscillator

$$H_0 = \omega(a_j^{+} a_j)\,, \tag{3.3}$$

and ε_0 (E) is the ground-state energy in R^d

$$\varepsilon_0(\mathrm{E}) = \frac{d}{4}\omega + \frac{4\rho^2\mu E\Gamma\left(\frac{d}{2}+2\rho-1\right)}{\omega^{2\rho-1}\Gamma\left(\frac{d}{2}\right)} - \frac{4\rho^2\mu\sigma\Gamma\left(\frac{d}{2}+3\rho-1\right)}{\omega^{3\rho-1}\Gamma\left(\frac{d}{2}\right)}; \tag{3.4}$$

here H_I is the interaction Hamiltonian represented in the normal form

$$H_I = \int_0^\infty dx \int\left(\frac{d\eta}{\sqrt{\pi}}\right)^d \cdot e^{-\eta^2(1+x)} : e_2^{-i\sqrt{x\omega}(q\eta)} : \left\{-\frac{4\rho^2\mu}{\omega^{2\rho-1}}\cdot\frac{Ex^{-2\rho}}{\Gamma(1-2\rho)} + \frac{4\rho^2\mu\sigma x^{-3\rho}}{\omega^{3\rho-1}\Gamma(1-3\rho)}\right\}, \tag{3.5}$$

and the notation: $e_2^{-x} = e^{-x} - 1 + x - \frac{x^2}{2}$ is used, :*: is the symbol of the normal ordering, and η_j, q_j are vectors in the d-dimensional auxiliary space R^d, the d-dimension of auxiliary space defined as

$$d = 2 + 2\rho + 4\rho\ell\,, \tag{3.6}$$

here ρ is the variational parameter connected with asymptotic behavior of the wave function (for details see Ref.[17]). Let us determine the energy spectrum with orbital and radial excitations. In the OR the wave functions with radial excitation are defined as

$$|n_r\rangle = C_{n_r}\left(a^+a^+\right)^{n_r}|0\rangle, \tag{3.7}$$

where C_{n_r} is the normalization constant determined as

$$C_{n_r} = \left[\frac{\Gamma(d/2)}{4^{n_r} n_r!\Gamma(d/2+n_r)}\right]^{1/2}, \tag{3.8}$$

and the energy spectrum in R^d determined in the following way:

$$\varepsilon_{n_r}(E) = \langle n_r|H|n_r\rangle = \varepsilon_0(E) + 2n_r\omega + \langle n_r|H_I|n_r\rangle\,. \tag{3.9}$$

The matrix element $\langle n_r|H_I|n_r\rangle$ is represented in the form

$$\langle n_r|H_I|n_r\rangle = -\frac{4\rho^2\mu E}{\omega^{2\rho-1}}\cdot\frac{\Gamma\left(\frac{d}{2}+2\rho-1\right)}{\Gamma\left(\frac{d}{2}\right)}\widetilde{B} + \frac{4\rho^2\mu\sigma}{\omega^{3\rho-1}}\cdot\frac{\Gamma\left(\frac{d}{2}+3\rho-1\right)}{\Gamma\left(\frac{d}{2}\right)}\widetilde{C}\,. \tag{3.10}$$

The calculational details of the matrix element (3.10) are given in Appendices A, and the parameters $\widetilde{B}$ and $\widetilde{C}$ are represented in (A.7) and (A.8), respectively. In the OR the energy spectrum of the initial systems and oscillator frequency are determined from the system of two equations [17]:

$$\begin{cases} \varepsilon(E) = 0 \\ \dfrac{\partial \varepsilon_0(E)}{\partial \omega} = 0 \end{cases}. \tag{3.11}$$

Taking into account (3.4) and (3.9), from (3.11) we determine the energy spectrum with orbital and radial excitations, and then the mass and constituent mass defined from (2.12) and (2.13), respectively. After some simple calculations for the bound state mass we obtain

$$M(\ell, n_r) = \sqrt{\sigma}\left(\sqrt{\mu_0 s^2} + \sqrt{\xi^2 + \mu_0 s^2} + \frac{s^2}{\sqrt{\mu_0}}\right), \tag{3.12}$$

and for the constituent mass of quarks we have

$$\mu_2(\ell, n_r) = \sqrt{\sigma \mu_0 s^2}, \quad \mu_2(\ell, n_r) = \sqrt{\sigma(\xi^2 + \mu_0 s^2)}, \tag{3.13}$$

also, the energy spectrum can be written as follows:

$$E(\ell, n_r) = \frac{3}{2}\min_{\rho}\left(\frac{s^2\sqrt{\sigma}}{\sqrt{\mu_0}}\right), \tag{3.14}$$

where the following notation is used:

$$\xi^2 = \frac{m_q^2}{\sigma}; \qquad \mu_0 = \frac{1}{2}\frac{s^4 - 2\xi^2 s}{2s^3 - \xi^2} + \sqrt{\frac{(s^4 - 2\xi^2 s)^2}{4(2s^3 - 2\xi^2)^2} + \frac{s^2\xi^2}{2s^3 - \xi^2}};$$

$$s^2 = \frac{2}{3}\left[\frac{\Gamma^2(4\rho + 2\rho\ell)\Gamma(2 + \rho + 2\rho\ell)}{4\rho^2\mu^3 D_1 \cdot \Gamma(3\rho + 2\rho\ell)}\right]^{\frac{1}{3}}\left(\frac{D_2 D_1 + 2D_3}{2}\right). \tag{3.15}$$

Here

$$D_1 = \frac{\rho + (3\rho - 1)\widetilde{B} - (2\rho - 1)\widetilde{C}}{\rho + 2(2\rho - 1)\cdot D_n + \frac{1}{2}\widetilde{B}}, \quad D_n = \frac{n_r}{1 + \rho + 2\rho\ell}, \tag{3.16}$$

$$D_2 = (1 + 4\cdot D_n)\cdot\frac{1}{1 + \widetilde{B}}, \quad D_3 = \frac{1 + \widetilde{C}}{1 + \widetilde{B}}.$$

Thus, the mass spectrum of mesons consisting of light-heavy quarks with orbital and radial excitations is analytically determined for the linear potential. In this case, we assume that

$$\frac{m_a}{\sqrt{\sigma}} \cong \frac{m_d}{\sqrt{\sigma}} << 1, \quad \xi = \frac{m_s}{\sqrt{\sigma}} \neq 0, \tag{3.17}$$

where m_u and m_d is the current mass light quarks, and m_s is the current mass s quark.

3.2. The energy spectrum Cornell potential

Let us determine the energy spectrum for the Cornell potential with orbital and radial excitations. The SE can be rewritten in the form

$$\left[\frac{1}{2\mu}P^2+\sigma\cdot r-\frac{4}{3}\frac{\alpha_s}{r}\right]\Psi=E(\mu)\Psi. \tag{3.18}$$

According to the OR, the interaction Hamiltonian is represented in the correct form in d dimensional auxiliary space R^d. In this case, the ε_0 (E)–energy spectrum ground state in R^d is represented as

$$\varepsilon_0(E)=\frac{d\omega}{4}+\frac{4\rho^2\mu\sigma}{\omega^{3\rho-1}}\frac{\Gamma\left(d/2+3\rho-1\right)}{\Gamma\left(d/2\right)}-\frac{4\rho^2\mu E}{\omega^{2\rho-1}}\frac{\Gamma\left(d/2+2\rho-1\right)}{\Gamma\left(d/2\right)}-\frac{16\alpha_s\rho^2\mu}{3\omega^{\rho-1}}\frac{\Gamma\left(d/2+\rho-1\right)}{\Gamma\left(d/2\right)}, \tag{3.19}$$

and the interaction Hamiltonian equals

$$H_I=\int_0^\infty dx\int\left(\frac{d\eta}{\sqrt{\pi}}\right)^d \exp\{-\eta^2(1+x)\}:e_2^{-2i\sqrt{x\omega}}:\times$$
$$\times\left[-\frac{4\rho^2\mu\sigma}{\omega^{3\rho-1}}\frac{x^{-3\rho}}{\Gamma(1-3\rho)}-\frac{4\rho^2\mu E}{\omega^{2\rho-1}}\frac{x^{-2\rho}}{\Gamma(1-2\rho)}-\frac{16\alpha_s\rho^2\mu}{3\omega^{\rho-1}}\frac{x^{-\rho}}{\Gamma(1-\rho)}\right]. \tag{3.20}$$

After some calculations for the energy spectrum of the initial systems with orbital and radial excitations we obtain

$$\frac{E(\ell,n_r)}{\sqrt{\sigma}}=\min_\rho\left[\frac{Z^2}{8x\rho^2}\frac{\Gamma(2+\rho+2\rho\ell)}{\Gamma(3\rho+2\rho\ell)}\cdot\frac{1+4\cdot D_n}{1+\tilde{B}}+\frac{1}{Z}\cdot\Gamma(4\rho+2\rho\ell)\Gamma(3\rho+2\rho\ell)\times\right.$$
$$\left.\times\frac{1+\tilde{C}}{1+\tilde{B}}-\frac{4\alpha_s Z}{3}\cdot\Gamma(2\rho+2\rho\ell)\cdot\Gamma(3\rho+2\rho\ell)\cdot\frac{1+\tilde{D}}{1+\tilde{B}}\right]; \tag{3.21}$$

the parameter z is determined by the equations

$$Z^3-Z^2x\frac{16\alpha_s\rho^2}{3}\frac{\Gamma(2\rho+2\rho\ell)}{\Gamma(2+\rho+2\rho\ell)}\frac{\left[\rho+(2\rho-1)\tilde{D}-(\rho-1)\tilde{B}\right]}{\left[\rho+(2\rho-1)D_n+\frac{1}{2}\tilde{B}\right]}-$$
$$-4x\rho^2\times\frac{\Gamma(4\rho+2\rho\ell)}{\Gamma(2+\rho+2\rho\ell)}\frac{\left[\rho+(3\rho-1)\tilde{B}-(2\rho-1)\tilde{C}\right]}{\left[\rho+(2\rho-1)\cdot D_n+\frac{1}{2}\tilde{B}\right]}=0. \tag{3.22}$$

Taking into account (3.21) and (3.22) from (2.13) we obtain for the constituent mass of quarks:

$$\mu_1(\ell,n_r)=\sqrt{\sigma}\sqrt{-2x^2\frac{d}{dx}\left(\frac{E}{\sqrt{\sigma}}\right)},$$

$$\mu_2(\ell,n_r)=\sqrt{\sigma}\sqrt{\xi^2-2x^2\frac{d}{dx}\left(\frac{E}{\sqrt{\sigma}}\right)}. \tag{3.23}$$

Then the parameter x is defined from the equation

$$1-\frac{1}{\sqrt{-2\frac{d}{dx}\left(\frac{E}{\sqrt{\sigma}}\right)}}-\frac{x}{\sqrt{\xi^2-2x^2\frac{d}{dx}\left(\frac{E}{\sqrt{\sigma}}\right)}}=0\,. \qquad (3.24)$$

For linear and Cornell potentials the dependence of spin-averaged masses $\widetilde{M}(n_r,\ell)$ (for details see Ref.[18]) of ℓ and n_r is given in Fig. 1. The Regge slope $\alpha_L'(0)$ and the intercept $\alpha_L(0)$ of the Regge L-trajectory for $\widetilde{M}(n_r,\ell)$ are calculated with the value of the parameters: $\alpha_S = 0.39$ is the coupling constant,

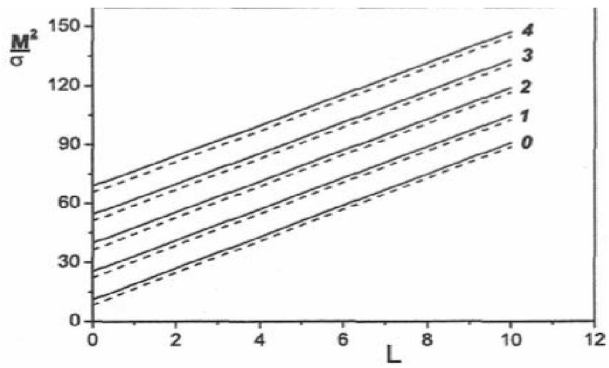

Fig. 1. The dependence of square spin-averaged masses of mesons consisting of light-heavy quarks from ℓ at the values $n_r = 0,\cdots 4$ and $m_s = 0.155GeV, \alpha_S = 0.39, \sigma = 0.19GeV^2$. The solid line for the linear potential, and dashed for the Cornell potential, respectively

$\sigma = 0.19GeV^2$ is the string tension and $m_s = 0.155GeV$ is the s - quark current mass. The Regge L-trajectory for the linear potential is linear. However, for the Cornell potential the Regge slope with growth of ℓ increases, in particular at $\ell = 1, \alpha_L' = 0.629GeV^{-2}$, and at $\ell = 5, \alpha_L' = 0.656GeV^{-2}$, so the trajectory is not linear. This behaviors of the Regge L-trajectory quality is in agreement with the experimental results [19] for the K- meson Regge trajectory.

3.3. Determine the dependence of constituent masses on current mass

From (3.23) we see that the mass of constituent particles depends on m and E, so in our approach the dependence of the constituent mass on the current mass m and the quantum numbers ℓ and n_r determined. At the values of the parameters $\alpha_s = 0.39$, $\sigma = 0.19GeV^2$ from (3.23) we determine the dependence of $\mu_q \equiv \mu_2$ on m for the case of Cornell potential. The numerical results are represented in Fig. 2. From Fig.2 we can see that when m is small, then the difference between constituent and current masses of quarks is greater than the current mass, and with increasing m this difference of masses decreases. The relation $\Delta = (\mu_q - m)/m$ is determined at any values of current m. Our results: at $m = 0.155GeV$ the relations equal Δ=1.942, so the difference is twice greater than the current mass, this is m=0.45GeV this is 0.44, and in the case m=1.5GeV equals 0.095; so the difference between constituent and current masses is 10.5 smaller than the current mass. Thus, our results show that if the value of m is in the range of mass of light quarks, then is the difference between constituent and current masses very great, but m equals of heavy, then this difference is very small.

Fig. 3 represents the dependence of constituent masses of light quarks on current masses of heavy quarks for the ground state and first orbital excitations. With increasing current masses of heavy quarks the constituent masses of light quarks also increase, but after m > 3GeV go to saturation. However, constituent

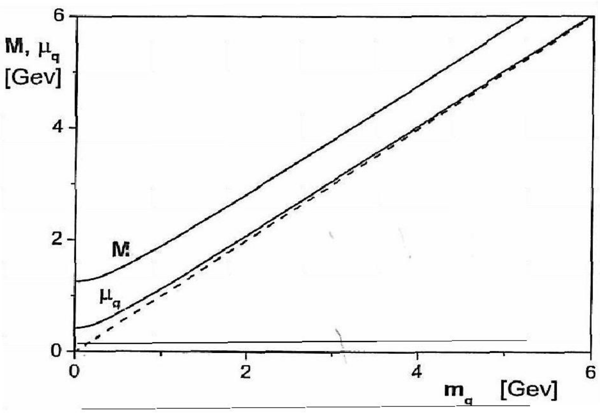

Fig. 2. The dependence of constituent masses of quarks and mass of meson on the current mass of quarks for the ground state at the value $\alpha_s = 0.39$ and $\sigma = 0.19GeV^2$

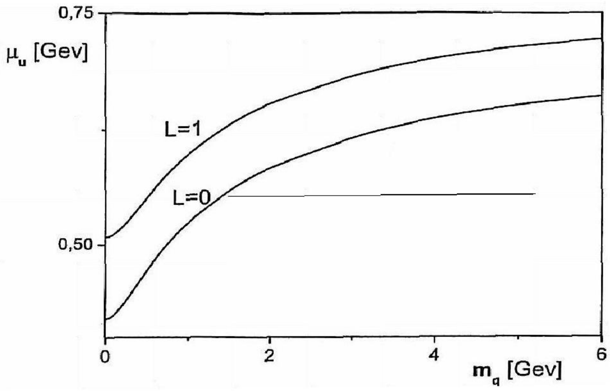

Fig. 3. The dependence of constituent mass of light quarks on the current mass heavy quark with $\ell = 0$, $\ell = 1$ and at the value $\alpha_s = 0.39$ and $\sigma = 0.19GeV^2$

masses of heavy quarks and the masses of bound states with increase in current masses of heavy quark increase.

On the basis of the received results it is possible to conclude:

– Within the framework of FCM, the mass spectrum mesons consisting of light-heavy quarks with orbital and radial excitations are determined. The Regge slope and the intercept of the Regge L-trajectory for the spin-averaged masses are determined. At small $\ell \le 5$ the Regge trajectory for mesons consisting of light and heavy quarks is nonlinear.

– Dependence of the constituent mass of quarks on the current mass of quarks is determined. Our results show that when quarks are light, then the difference between the constituent and current masses of quarks is greater than the current masses of quarks but if quarks are heavy as c and b, then the difference of these masses is insignificant.

Appendix A

Let us give some calculational details of the matrix element $\langle n_r | H_I | n_r \rangle$:

$$\langle n_r | H_I | n_r \rangle = \int_0^\infty dx \left\{ -\frac{4\rho^2 \mu E}{\omega^{2\rho-1}} \cdot \frac{x^{-2\rho}}{\Gamma(1-2\rho)} + \frac{4\rho^2 \mu\sigma}{\omega^{3\rho-1}} \cdot \frac{x^{-3\rho}}{\Gamma(1-3\rho)} \right\} \times$$
$$\times \left\langle n_r \left| \cdot \int \left(\frac{d\eta_1}{\sqrt{\pi}} \right)^d \cdot e^{-\eta^2(1+x)} : e_2^{-2i\sqrt{x\omega}(q\eta)} : \right| n_r \right\rangle \quad \text{(A.1)}$$

For calculation of this matrix element, we very often treat the following term:

$$T_n(x) = \int \left(\frac{d\eta}{\sqrt{\pi}} \right)^d \cdot e^{-\eta^2(1+x)} \langle n | : e_r^{-2i\sqrt{x\omega}(q\eta)} : | n \rangle . \quad \text{(A.2)}$$

Taking into account Eq.(3.7) and relations

$$\left(a^+ a^+\right)^n = (-1)^n \frac{d^n}{d\beta^n} e^{-\beta\left(a^+ a^+\right)} \Bigg|_{\beta=0} \qquad (-1)^n \frac{d^n}{d\beta^n} \int \left(\frac{d\xi}{\sqrt{\pi}} \right)^d e^{-\xi - 2i\sqrt{\beta}\left(a^+ \xi\right)} \Bigg|_{\beta=0} , \quad \text{(A.3)}$$

and using the following representations:

$$e_2^{-i(k\mp\vec{a})} = P_v e^{-iv(k\mp\vec{a})},$$

where P_v is the operator defined according to the following rules:

$$P_\upsilon const = 0,\ \ P_\upsilon \upsilon^n = 0,\ \ n \le 2;\ P_\upsilon \upsilon^n = 1\ \ n \succ 2;$$

after some manipulations from (A.2) we have:

$$T_n(x) == P_v C_n^2 \frac{d^{2n}}{d\alpha^n \beta^n} \int \left(\frac{d\eta}{\sqrt{\pi}^2}\right)^d \iint \left(\frac{d\xi_1}{\sqrt{\pi}}\right)^d \left(\frac{d\xi_2}{\sqrt{\pi}}\right)^d e^{-\eta^2(1+x)-\xi_1^2-\xi_2^2}$$

$$\times \left\langle 0 \left| e^{-2i\sqrt{\alpha}(a\xi_1)} \cdot e_r^{-iv\sqrt{2x}(a^+\eta)} e_r^{-iv\sqrt{2x}(a\eta)} e^{-2i\sqrt{\beta}(a^+\xi_{21})} \right| 0 \right\rangle \Big|_{\beta,\alpha=0}. \tag{A.4}$$

Finally, we obtain

$$T_n(k) = \sum_{k=2}^{2n} \sum_{S=0}^{n} (-1)^k \frac{x^k}{(1+x)^{k+d/2}} \frac{\Gamma(1+n)}{\Gamma(n+d/2)} \cdot \frac{2^{2S-k}}{\Gamma(n-S+1)} \cdot \frac{\Gamma(k+n-S+d/2)}{\Gamma^2(k-S+1)\Gamma(2S-k+1)}. \tag{A.5}$$

Taking into account (A.4) and (A.5), after integration over x and some simplification (A.1) we have:

$$\langle n_r | H_I | n_r \rangle = -\frac{4\rho^2 E\mu}{\omega^{2\rho-1}} \frac{\Gamma(d/2+2\rho-1)}{\Gamma(d/2)} \tilde{B} + \frac{4\rho^2\sigma\mu}{\omega^{3\rho-1}} \frac{\Gamma(d/2+3\rho-1)}{\Gamma(d/2)} \tilde{C}, \tag{A.6}$$

here

$$\tilde{B} = -\frac{\Gamma(1+n_r)}{\Gamma(n_r+d/2)} \cdot \frac{\Gamma(d/2)}{\Gamma(1-2\rho)} \cdot \sum_{k=2}^{2n_r} (-1)^k A_{n_r}(k) \frac{\Gamma(1+k-2\rho)}{\Gamma(k+d/2)} \tag{A.7}$$

and

$$\tilde{C} = -\frac{\Gamma(1+n_r)}{\Gamma(n_r+d/2)} \cdot \frac{\Gamma(d/2)}{\Gamma(1-3\rho)} \cdot \sum_{k=2}^{2n_r} (-1)^k A_{n_r}(k) \frac{\Gamma(1+k-3\rho)}{\Gamma(k+d/2)}, \tag{A.8}$$

here

$$A_{n_r}(k) = \sum_{s=2}^{n_r} \frac{2^{2S-k}}{\Gamma(n_r-S+1)} \cdot \frac{\Gamma(k+n_r-S+d/2)}{\Gamma(k-S+1)\Gamma(2S-k+1)}. \tag{A.9}$$

Using these relations represented in (A.6)-(A.9), we arrive at expressions for the energy spectrum of the linear potentials. Analogously, we can derive the matrix element $\langle n_r | H_I | n_r \rangle$ for the Hamiltonian (3.20) with the Cornell potential:

$$\langle n_r | H_I | n_r \rangle = -\frac{4\rho^2 E\mu}{\omega^{2\rho-1}} \frac{\Gamma(d/2+2\rho-1)}{\Gamma(d/2)} \tilde{B} + \frac{4\rho^2\sigma\mu}{\omega^{3\rho-1}} \times$$

$$\times \frac{\Gamma(d/2+\rho-1)}{\Gamma(d/2)} \tilde{C} - \frac{16\rho^2\alpha_s\mu}{3\omega^{\rho-1}} \frac{\Gamma(d/2+\rho-1)}{\Gamma(d/2)} \tilde{D}, \tag{A.10}$$

where

$$\widetilde{D}=-\frac{\Gamma(1+n_r)}{\Gamma(n_r+d/2)}\cdot\frac{\Gamma(d/2)}{\Gamma(1-\rho)}\cdot\sum_{k=2}^{2n_r}(-1)^k A_{n_r}(k)\frac{\Gamma(1+k-\rho)}{\Gamma(k+d/2)}. \tag{A.11}$$

Using (A.10) we determine the energy spectrum for the Cornell potential with orbital and radial excitations.

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Резюме

Өрістік коррелятор әдісі шеңберінде орбиталдық және радиалдық қозған күй кезіндегі жеңіл және ауыр кварктардан тұратын мезондардың массалық спектрі анықталған. Конституэнттік массаның еркін күй массасынан тәуелділігі анықталған. Сонымен бірге, егер кварктар жеңіл болса, онда кварктардың бастапқы және конституэнттік массаларының айырмашылығы кварктардың бастапқы массасынан көп есе артық болады, ал кварктар ауыр болса, онда бұл массалардың айырмашылығы аз болатыны көрсетілген.

Резюме

В рамках метода полевых корреляторов определен массовый спектр орбитальных и радиальных возбужденных состояний мезонов, состоящих из легко-тяжелых кварков. Определена зависимость конституэнтной массы от массы свободного состояния. Показано, что когда кварки являются легкими, разность конституэнтных и исходных масс кварков оказывается в несколько раз больше, чем исходные массы кварков, если кварки являются тяжелыми, то разность этих масс незначительно.

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Поступила 1.10.07г.