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OPTIMIZATION PROBLEMS OF DISTRIBUTION OF RESOURCES

Optimization problems arise in many applied areas. So, they arise at the decision of a problem distribution of resources during creation of complex program, technical and other systems [1, 2].

There are lot of articles and books are devoted to the decision of optimization problems, in particular it is possible to note works [3-15]. Received by Gun and Takker necessary and sufficient conditions of an extremum in a problem finite-dimensional mathematical programming have played the big role in development of the theory of optimization and stimulated development of numerical methods. Pontrjagin L. and his pupils have formulated similar conditions for problems of the optimum control, served by a basis for the theory of optimum processes.

In applied areas, optimization problems frequently are multicriterial. From approaches to the decision of problems multicriterial optimization should allocate the following most widespread: optimization of hierarchical sequence of criteria; definition of set of not improved points; definitions of the decision based on this or that kind of the compromise.

As shows the analysis of various approaches of the decision of problems of multicriterial optimization, basically, known methods directly or indirectly are reduced to the decision of optimization problems with scalar criterion.

The most complex and difficultly formalizable stage of the decision of optimization problems is the choice of the approach and a method of the decision which would guarantee reception of the best decision. Therefore at the decision optimization problems the large value has presence of set of approaches and methods, each of which is most effective for the certain class, and an opportunity of operative replacement of approaches and methods at work on a computer.

The organization of work in a dialogue mode which allows to solve operatively tasks in view on a computer is expedient, to pass from one approach or a method to another, to choose the necessary circuit of the decision, to pass from one circuit to another, to correct parameters of the chosen circuit, the current point of search, etc.

At the decision of optimization problems usually it is necessary to collide with the functions of the purpose having the various physical nature and dimension. Criterial functions can accept values which frequently differ on some orders. There fore it is expedient to consider not set of criteria $\{f_1(x), f_2(x),..., f_k(x)\}$, and set of functions $\{L_1(f_1(x)), L_2(f_2(x)),..., L_k(f_k(x))\}$ equivalent to it where $L_i(f_i(x))$ – the monotonous transfor-mations resulting criterial functions $f_i(x)$ to a dimensionless kind and allowing to compare their numerical values among themselves.

Let's consider following optimization problem.

$$\min\{f_1(x), f_2(x),..., f_k(x)\},\ x \in W \subset E^n,$$
$$W = \{x \in E^n : \varphi_i(x) \geq 0,\ i = 1,2,...,m\}.$$

We assume, that area W not empty, limited. Functions $f_i(x)$, $\varphi_i(x)$ – are convex, continuously differentiated.

Search of the decision in considered optimization problem is carried out in two stages.

At the first stage the set of effective points is under construction

$$P_x = \{x \in W : \exists z \in W, \exists j \in \{1,...,k\},$$
$$f_i(z) \leq f_i(x), i \quad 1,...,k, f_j(z) < f_j(x)\}.$$

At the following stage one decision gets out of the received points.

Let's consider the approach combining two stages of decision-making, based on consecutive reception of the information on minimally possible values of functions $f_1(x), f_2(x),..., f_k(x)$ and use of one of methods of search of a point of allowable area.

Let on a step v of decision-making there was a point $x^v \in W$ in which criteria had values f_i^v $f_i^v = f_i(x^v)$, in particular, $i = 1,...,k$. at $v = 0$ in quality x^0 the any point of allowable area W with values of criteria $f_i^0 = f_i(x^0), i = 1,...,k$. Let's assume, that proceeding their purposes of decision-making in quality of minimally possible values of criteria are appointed $\bar{f}_i^v$, $i = 1,...,k$. Hence, further it is required to find even one point $x^{v+1} \in W^v$, where

$$W^v = \{x \in W : d_i^v(x) \quad \bar{f}_i^v - f_i(x) \geq 0, i = 1,...,k\}.$$

Let's define(determine) $x^{v+1} \in W^v$. Let $x^v \bar{\in} W^v$, otherwise it is possible to proceed to a step $v + 1$ and to carry out new purpose of criteria.

Let's consider sequence $\{x_t^v\}$, $t = 0,1,....$, $x_0^v = x^v$ which is points of a unconditional minimum of sequence of functions.

$$x_{t+1}^v = \lim_{\varepsilon \to 0} \arg\min_x \left\{ F_t(x, \bar{f}^v, \xi^t) \right.$$
$$\left. \sum_{i=1}^k \frac{1}{\bar{f}_i^v - f_i(x) + \xi_i^t} + \sum_{i=1}^m \frac{\varepsilon}{\varphi_i(x) + \beta_i^t} \right\}. \quad (1)$$
$$t = 0,1,...$$

Where ξ_i^t, ε_i^t, β_i^t are calculated as follows:

$$\xi_i^t = \begin{cases} f_i(x_t^v) - \bar{f}_i^v + \varepsilon_i^t,\ f(x_t^v) > \bar{f}^v, \\ 0,\ f(x_t^v) \leq \bar{f}^v, \end{cases} \quad (2)$$

$$\varepsilon_i^t = \|\nabla f_i(x_t^v)\| \varepsilon,\ \beta_i^t \quad \|\nabla \varphi_i(x_t^v)\| \varepsilon,\ \varepsilon > 0.$$

Let's accept $x^{v+1} = \lim_{t \to \infty} x_t^v$.

Let's consider $W_{\xi^t}^v = \{x \in W : d_i^v(x)$ $\bar{f}_i^v - f_i(x) + \xi_i^v \geq 0,\ i = 1,...,k\}$,

With border $R_{\xi^t}^v$. It is obvious, that $W^v \subseteq W_{\xi^t}^v$, $t = 0,1,...$. Positive sizes ε_i^t, β_j^t, $i = 1,...,k, j = 1,...,m$ are used with the purpose of performance $x_{t+1}^v \in D_{\xi^t}^v$. Taking into account specificity of function $F_t(x, \bar{f}^v, \xi^t)$, as index points for consecutive minimization it is possible to use x_t^v.

Let functions $f_i(x)$, $\varphi_j(x)$ – are convex, continuously differentiated. Exists $x_{t+1}^v \in W_{\xi^t}^v$, that

$$x_{t+1}^v = \arg\min_x F_t(x, \bar{f}^v, \xi^t).$$

Apparently, $W_{\xi^t}^v$ compactly. From a continuity of function $f_i(x)$, $\varphi_j(x)$ follows, that function $F_t(x, \bar{f}^v, \xi^t)$ is continuous inside $W_{\xi^t}^v$, and at $x \to z \in R_{\xi^t}^v$ and $x \in W_{\xi^t}^v$ function $F_t(x, \bar{f}^v, \xi^t)$

aspires to $+\infty$. Hence, function $F_t(x,\overline{f}^v,\xi^t)$ reaches a minimum of century $W^v_{\xi^t}$.

Let $f_i(x)$ – convex functions and ξ^t an any positive vector, then $x^v_{t+1} \in P_x$.

For convex functions $f_i(x)$ and any $\mu_i \geq 0,\ i=1,...,k$ it is carried out [15]

$$\arg\min_{x\in W}\left\{F(x;\mu,f): F(x;\mu,f)=\sum_{i=1}^{k}\mu_i f_i(x)\right\}\in P_x.$$

Let $\overline{\mu}_i=\left[f_i^v - f_i(x^v_{t+1})+\xi_i\right]^{-2}$, $i=1,...,k$. It is obvious, that $\overline{\mu}_i>0,\ i=1,...,k$. Then (1) coincides with realization of a method of the internal penalty concerning the decision of a problem $\min_{x\in W}F(x;\overline{\mu},f)$, hence, x^v_{t+1} there is an optimum decision of function $F(x;\overline{\mu},f)$ in allowable area *W*.

At practical realization of a method there is no necessity of carrying out in (1), at the fixed values ξ^t, optimization on *x* up to the end. It is possible to carry out calculation ξ^t according to (2) on each internal iteration at carrying out of unconditional minimization $F_t(x;\overline{f}^v,\xi^t)$. Thus, algorithms are combined external on calculation ξ^t and internal on carrying out of unconditional minimization $F_t(x;\overline{f}^v,\xi^t)$. It essentially raises efficiency of computing process.

If $W^v \neq \varnothing$, that $x^{v+1}\in W^v$, $x^{v+1}=\lim_{t\to\infty}x^v_t$

Thus, if the decision belongs W^v, there is an opportunity, at least, in enough its small vicinity to improve values of criteria. However in real practical problems as the decision that decision which satisfies to the certain criteria can be accepted. That is it is possible to make it as the optimum decision.

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Резюме

Құнарларды үлестіру көпкритерийлі оптимизациялық есептердің математикалық қамтамасыздығы қарастырылған. Есептердің шешілу әдістері ұсынылған.

Резюме

Статья посвящена разработке математического обеспечения решения многокритериальных оптимизационных задач распределения ресурсов. Предложен подход к решению рассматриваемых задач.

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