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S. N. BORANBAYEV

METHODS OF DEFINITION OF PARAMETERS MATHEMATICAL MODEL

(Представлена академиком НАН РК М. О. Отельбаевым)

Usually mathematical models are constructed in the form of the differential or integrated equations. Fitting and identification of models are follows further. Often, by virtue of step-type behavior of the information, at a stage of identification it is necessary to pass from the differential equations to certainly – different, from integrated – to the sums, etc. Then on the basis of known data it is necessary to estimate unknown parameters the equation of model. The present work is devoted to some questions of identification of model, development of corresponding methods and the software.

At identification of model it is possible to pass from variables standing in the equations of model, to other variables. It is possible to establish steady parities on available numbers of supervision between these variables. For example, in economic-mathematical models for exception of action of inflation in variables of model, believes [1-4]:

$$x(t) = x^n(t)/d^f(t),$$

Where $x^n(t)$ – observable a number of some variable, $d^f(t)$ – deflator. If a number of supervision $x^n(t)$ does not cause the big trust in the equations of model instead of $x^n(t)$ it is possible to use

$$x(t) = \mu x^n(t) + (1-\mu)x^e(t),$$

Where $0 \le \mu \le 1$, μ – a degree of trust to a number supervision $x^n(t)$, $x^e(t)$ – an expert estimation.

It is possible to use smoothing.

Linear smoothing:

$$x_p(t) = \sum_{s=-p}^{p} \alpha_s x^n(t+s) \quad \sum_{s=-p}^{p} \alpha = 1 \quad \alpha_s > 0.$$

Exponential smoothing:

$$x(t) = (x^n(t) - x^n(t-1))/(\ln x^n(t) - \ln x^n(t-1)).$$

Differential smoothing:

$$\frac{d^s x(t)}{dt^s} = \sum_{\theta=0}^{s} \alpha_\theta x^n(t-\theta).$$

Integrated smoothing:

$$\int_{t-\theta}^{t} x(t-s)q(s)ds = \sum_{i=0}^{\theta} \alpha_i x^n(t-i).$$

The smoothed derivative:

$$\frac{dx(t)}{dt_{(q)}} = \frac{\sum_{\theta=-q}^{q} \theta x^n(t+\theta)}{\sum_{\theta=-q}^{q} \theta^2}.$$

The smoothed trend:

$$\frac{1}{x(t)}\frac{dx(t)}{dt_{(q,s)}}=\frac{\sum_{k=0}^{q-1}x^n(t-k)}{\sum_{k=0}^{q-1}x^n(t-s-k)}-1.$$

It is possible to construct derivatives by means of trends. If the trend $x(t)=F(\alpha,t)$ well enough describing a number $x^n(t)$, then is found

$$\frac{dx^n(t)}{dt}\cong\frac{dF(\alpha,t)}{dt}.$$

Let's consider following mathematical model. Let the system of parities of model looks like:

$$y_i(t)=F_i(t,\overline{\alpha}_i,\overline{x}(t))\ ,\ i=1,...s\,.$$

On the basis of data about behavior y_i and $\overline{x}$ at the given kind of functions $F_i(t,\overline{\alpha}_i,\overline{x}(t))$ it is required to define factors α_i so that function $F_i(t,\overline{\alpha}_i,\overline{x}(t))$ well described behavior y_i.

Let a time number of supervision is $\{y_i\}$, $i=1,..M$ during the moments of time t_i, mistakes Δ_i of each supervision y_i are known. The kind of function $\varphi(\alpha,t)$ is postulated. We shall search for parameters α from a condition:

$$\min_{\alpha}\sum_{i=1}^{M}\rho^2(I_i,\varphi(\alpha,t_i)),$$

Where $I_i=[y_i-\Delta_i,y_i+\Delta_i]$.

Distance $\rho(I_i,\varphi(\alpha,t_i))$ from a piece I_i up to a point $\varphi(\alpha,t_i)$ we shall define as follows. We shall divide a piece I_i on N equal parts. Let y_{in} is n-th point of division. Then:

$$\rho^2(I_i,\varphi(\alpha,t_i))=\lim_{N\to\infty}\tfrac{1}{N}\sum_{n=0}^{N}(y_{in}-\varphi(\alpha,t_i))^2=$$

$$=\lim_{N\to\infty}\tfrac{1}{N}\sum_{n=0}^{N}(y_i-\Delta_i+2\tfrac{\Delta_i n}{N}-\varphi(\alpha,t_i))^2,$$

$$\rho^2(I_i,\varphi(\alpha,t_i))=(y_i-\varphi(\alpha,t_i))^2+\tfrac{2}{3}\Delta_i^2.$$

We faced to a problem:

$$\min_{\alpha}v(\alpha)=\min_{\alpha}\sum_{i=1}^{M}(y_i-\varphi(\alpha,t_i))^2.$$

It is a problem of unconditional minimization

$$v(\alpha)=\sum_{i=1}^{M}(y_i-\varphi(\alpha,t_i))^2.$$

The kind of function $v(\alpha)$ allows to state some reasons on ways of a finding of its points of a minimum and search of initial approach. If $\varphi(\alpha,t)$ twice continuously differentiated in the field of G function, then

$$\frac{\partial v(\alpha)}{\partial\alpha}=$$

$$=\sum_{i=1}^{M}2\frac{\partial\varphi(\alpha,t_i)}{\partial\alpha}(\varphi(\alpha,t_i)-y_i),$$

$$\frac{\partial^2 v(\alpha)}{\partial\alpha^2}=\sum_{i=1}^{M}2\left(\frac{\partial^2\varphi(\alpha,t_i)}{\partial\alpha^2}(\varphi(\alpha,t_i)-y_i)+\right.$$

$$\left.+\left(\frac{\partial\varphi(\alpha,t_i)}{\partial\alpha}\right)^2\right).$$

Hence, if in the field of G for all points $i=1,...,M$ the condition is done

$$\left(\frac{\partial\varphi(\alpha,t_i)}{\partial\alpha}\right)^2>\frac{\partial^2\varphi(\alpha,t_i)}{\partial\alpha^2}\big(y_i-\varphi(\alpha,t_i)\big).$$

function $v(\alpha)$ has in G a unique point of a minimum.

Let now $\varphi(\alpha,t)$ monotonous on α function, for the definiteness, increasing, and points y_i $i=1,...,M$, belong to set of values $\varphi(\alpha,t)$.

$$\frac{\partial v(\alpha)}{\partial\alpha}=\sum_{i=1}^{M}2\frac{\partial\varphi(\alpha,t_i)}{\partial\alpha}\big(\varphi(\alpha,t_i)-y_i\big),$$

$$\alpha_i^*:\ \frac{\partial\varphi(\alpha,t_i)}{\partial\alpha}\big(\varphi(\alpha,t_i)-y_i\big)\big|_{\alpha=\alpha_i^*}=0,$$

$$\alpha_j^*:\ \frac{\partial\varphi(\alpha,t_j)}{\partial\alpha}\big(\varphi(\alpha,t_j)-y_j\big)\big|_{\alpha=\alpha_j^*}=0.$$

The point α^* lays between α_i^* and α_j^*:

$$\alpha^*: \left(\frac{\partial \varphi(\alpha, t_i)}{\partial \alpha} (\varphi(\alpha, t_i) - y_i) + \right.$$

$$\left. + \frac{\partial \varphi(\alpha, t_j)}{\partial \alpha} (\varphi(\alpha, t_j) - y_j) \right) |_{\alpha = \alpha^*} = 0 .$$

It is possible to consider the sums of other members, etc. We can see, that local minims $v(\alpha)$ are located the close to each other, between roots of the equations

$$\alpha: \frac{\partial \varphi(\alpha, t_i)}{\partial \alpha} (\varphi(\alpha, t_i) - y_i) = 0 \; i = 1, ..., M .$$

As $\varphi(\alpha, t_i)$ it is monotonous on α these roots of all M of pieces, and if function $\varphi(\alpha, t_i)$ is chosen well, these roots are close to each other, i.e. local minima $v(\alpha)$ are located in rather limited area. It allows to construct effectively a grid on α and to find a minimum $v(\alpha)$.

The software package using methods of descent with constant step and the quickest descent [5], and also method of calculation of a grid is developed for the automation process of identification of model on some variables. As gradient methods have rather small speed of convergence near to a point of a minimum, the use of the following approach is more effective. The method of the quickest descent does a quantity of iterations; further descent is made with constant step. For search of initial approach the method of calculation of a grid on some variables is used: the grid only on some variables is set, further the problem of minimization in each point of this grid is solved. Then the best one gets out of sets of the received decisions. This decision will be initial approach for the more exact method.

By means of the software package calculations industrial functions for various regions were spent. Data from statistical collections were used.

In particular, calculations of function have been lead:

$$y(t) = ae^{Pt} \left[bK^{-\rho}(t) + (1-b)L^{-\rho}(t) \right]^{-\frac{\gamma}{\rho}} .$$

The point of initial approach has been received by calculation of a grid on factors b and ρ with the subsequent minimization on a and P, it was necessary $\gamma = 1$. Further methods of descent from the software package were applied.

LITERATURE

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Резюме

Математикалы модельдің параметрлерін анықтайтын әдістерді жасауға арналған. Математикалық модельдердің қасиеттерін қолданатын әдісі ұсынылады.

Резюме

Статья посвящена разработке методов определения параметров математической модели. Предлагается метод, использующий свойства математической модели.

Евразийский университет им. Л. Н. Гумилева

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