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PHYSICAL ASPECTS OF STRANGE ATTRACTORS IN LORENTZ'S EQUATIONS

Abstract

In deriving of the nonlinear differential equations, describing mathematical geoengineering models, unjustified simplifications are made, which yield the results inadequate to the phenomenon under consideration. As examples the Lorentz's and Burger's nonlinear equations are analyses.

Keywods: Lorentz's attractors , stochasticity.

Кілт сөздер: Лоренц аттракторлары, стохастикалық.

Ключевые слова: Аттракторы Лоренца, стохастический.

It is well-known, that at the description of the physical phenomena is used the certain mathematical approach. Moreover, approaches are put in pawn in the process of physical value measurements. However we can always predict consequences of physical experiments with known, beforehand set, accuracy. The last is connected with the determinacy of the mathematical equations describing considered systems. If mathematical models of problems are described by certain systems of the nonlinear differential equations results of calculations can be not determined, they are stochastic. Stochasticity contains in the nature of the nonlinear differential equations, and are well-known. The examples of such equations Edward N. Lo-rentz's equations are typical. In the given work we wish to pay attention to the restrictions containing the attempt of the adequate description of the nature processes in mathematical models. For example, in the mathematical model for true weather forecasting in the set place of globe, have been receive, as was mentioned, Lorentz's having not determined decisions the known differential equations - so-called strange attractors Lorentz [1]. Lorentz was surprised that his equations, with enough simple and general assump-tions, mathematically well describe process of the turbulence beginning in the nature – stochasticity. The phenomenon essence lays in the nature of the received nonlinear differential equations when under known initial (boundary) conditions and the set parameters of problems, not determined, stochastic decisions always turn out. We will more in detail consider the Lorentz's system, which describes all processes which are passing in a liquid layer (or gas, for example the atmosphere), warmed up from below. We will admit, that on the top border constant temperature T_0, and on bottom border $T_0 + \Delta T$ is supported. As heating gas or a liquid is easier than cold, so at a difference of temperatures convection current in the environment arises. So we have the distributed system in which condition is characterized by fields of velocity distribution, density and temperature: (x, y, z, t), ρ(x, y, z, t), T (x, y, z, t).

Such system is described by Nave-Stokes equation:

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v}\Delta)\vec{v} = -\frac{1}{\rho}\nabla p + \nabla^2\vec{v} + g, \tag{1}$$

together with the equations of continuity, heat conductivity and condition. Here t – time, $\vec{V} = (V^1, \dots, V^n)$ – vector field of velocities. Unknown pressure p and $\vec{V}$ are time functions t and coordinates $x \in \Omega$, where $\Omega \in R^n$, n=2,3-plane or three-dimensional area in which gas or a liquid moves.

Further we will present unknown field in the form of expansion coefficient on some full system of basic functions. We will construct expansion on basis of trigonometrical functions in a kind:

$$\sin m\alpha x * \sin n\beta y, \quad \sin m\alpha x * \cos n\beta y, \quad \cos m\alpha x * \sin n\beta y, \tag{2}$$

where β = π/h, α = π/l = πa/h, m and n – integers.

For factors expansions U_{mn} and V_{mn} current function can receive infinite system of the equations. It turns out in the event that to consider Lorentz's model only essential and distinct from zero only terms with U_{11}, V_{11}, V_{02} and conveniently them will designate accordingly through X, At and Z.

$$u = -X\beta\sin\alpha x\cos\beta y, \tag{3}$$

$$v = X\alpha\cos\alpha x\sin\beta y, \tag{4}$$

$$\theta = Y\cos\alpha x\sin\beta y - Z\sin 2\beta y. \tag{5}$$

Having substituted them in the equations:

$$\left(\llbracket(u\rrbracket_y - v_x\right)_t = -uu_{xy} - vu_{yy} + uv_{xx} + vv_{xy} - \gamma g\theta_x \quad + v\left(u_{xxy} + u_{yyy} - v_{xyy} - v_{xxx}\right)$$
(6)

$$\theta_t = -(\theta u)_x - (\theta v)_y + h^{-1}\Delta T v + k\left(\theta_{xx} + \theta_{yy}\right), \tag{7}$$

$$u_x + v_y = 0 . \tag{8}$$

Let's receive following equations:

$$\dot{X} = \frac{2\gamma g Y\alpha}{\alpha^2 - \beta^2} - v(\alpha^2 - \beta^2)X,$$

$$\dot{Y} = \frac{\alpha\Delta T}{h}X - k(\alpha^2 - \beta^2)Y - \alpha\beta XZ. \tag{9}$$

$$\dot{Z} = \frac{1}{2} XY\beta\alpha - 4Z\beta^2 k.$$

Let's enter some constant factors, and write dynamic variables in the forms: X=Ax, Y=By, Z=Cz, t=D τ.

$$\dot{X} = \frac{\alpha\gamma g}{\alpha^2+\beta^2}\frac{BD}{A}y - Dv(\alpha^2+\beta^2)x,$$

$$\dot{Y} = \frac{\alpha\Delta T}{h}\frac{AD}{B}x - Dk(\alpha^2+\beta^2)y - \frac{ACD}{B}\alpha\beta xz,$$

$$\dot{Z} = -4k\beta^2 Dz + \frac{ABD}{2C}\beta\alpha xy.$$

Input some constant designation:

$$D = k^{-1}(\alpha^2+\beta^2)^{-1}, \quad \frac{\alpha\gamma g}{\alpha^2+\beta^2}\frac{BD}{A} = \frac{v}{k}, \quad \frac{ACD}{B}\alpha\beta = 1, \quad \frac{ABD}{C}\alpha\beta = 1.$$

Let's find, entering dimensionless parameters:

$$a = \frac{v}{k}, \quad b = \frac{4\beta^2}{\alpha^2+\beta^2} = \frac{4}{1+\alpha^2}, \quad r = \frac{\alpha^2\beta\gamma\Delta T}{kvh(\alpha^2+\beta^2)^3}$$

It is possible to write down the equation (9) in the form in which they have been written down by Lorentz:

$$\dot{x} = \sigma(y - x),$$

$$\dot{y} = x(r - z) - y, \tag{10}$$

$$\dot{z} = -bz - zy.$$

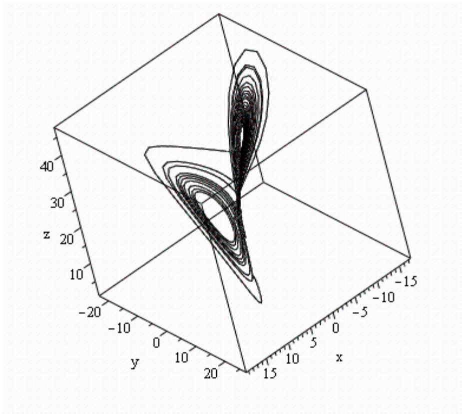
However, assumptions more adequate to physical models at the deducting the Lorenz equations slightly alter the equations and their decisions. The common character of such tendencies for the nonlinear differential equations applied in various areas of natural sciences [2] had been reviewing.

Let's compare numerical decisions (Figure 1–3) of the Lorenz equations:

$$\frac{d}{dt}x(t) + \sigma\left(x(t) - y(t)\right) = 0,$$

$$\frac{d}{dt}y(t) - rx(t) + x(t)z(t) = 0, \tag{11}$$

$$\frac{d}{dt}z(t) + bz(t) - x(t)y(t) = 0.$$



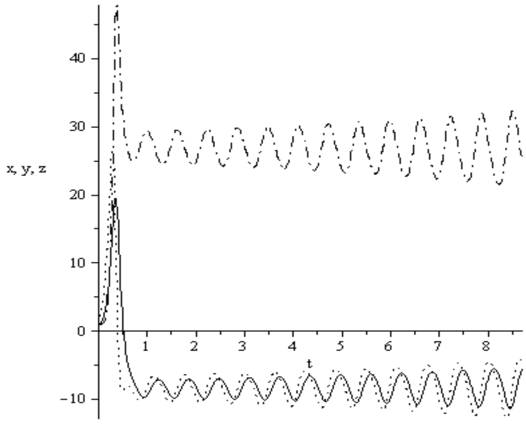


Figure 3

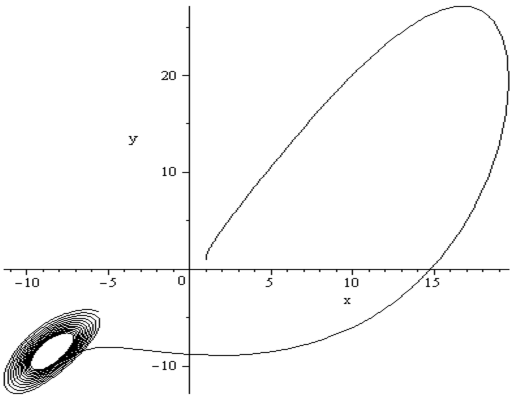
Figure 4

With our modified equations:

$$\frac{d}{d\tau_i}x(\tau_i)+\sigma\left(x(\tau_i)-y(\tau_i)\right)=0,$$

$$\frac{d}{d\tau_i}y(\tau_i)-rx(t)+x(t)\left(1-\left(\sum_{i=1}^{\infty}\sin(\tau_i)\right)\right)z(\tau_i)+y(\tau_i)=0, \tag{12}$$

$$\frac{d}{d\tau_i}z(\tau_i)+b\left(1-\left(\sum_{i=1}^{\infty}\sin(\tau_i)\right)\right)z(\tau_i)-x(\tau_i)y(\tau_i)=0.$$



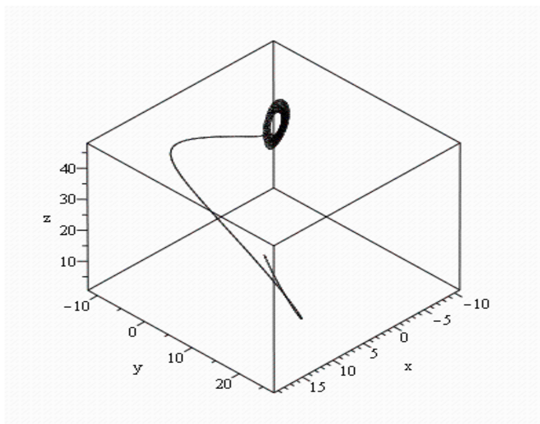
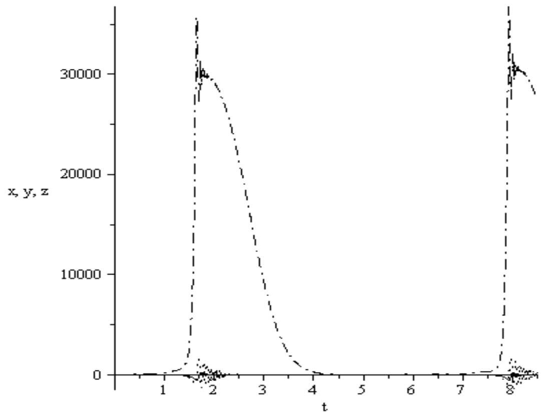


Figure 5

Figure 6



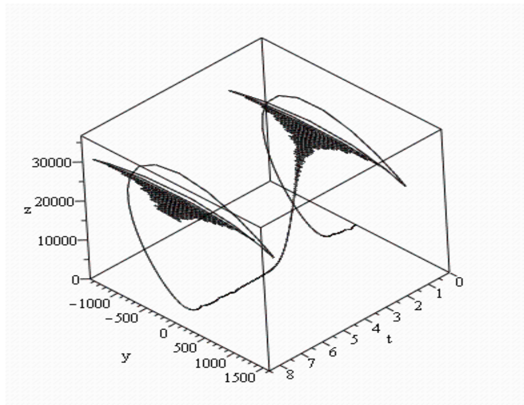


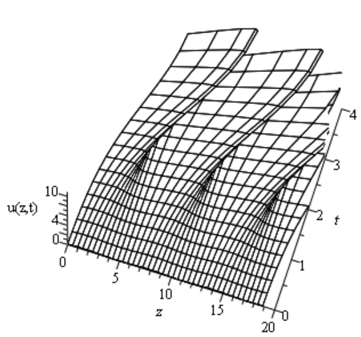
Figure 7 Figure 8

These equations identically pass in Lorentz's equations (10) or (11) when internal brackets of the equations with sum expression are insignificant. In other cases corresponding to more real mathematical models, numerical decisions (Figure 4–8) give distinctive from Lorentz's classical decisions.

Here we consider in detail the Lorentz's equations because they are unreasonably often used in models of many scientific problems. As a following example of an essential divergence of mathematical model with geophysical systems, we will consider the modified nonlinear wave equation describing, for example, a consequence of strong earthquake or oceanic waves-murderers [3].

$$\frac{\partial}{\partial t}u(z,t)+(1+\sin(t))*u(z,t)*\left(\frac{\partial}{\partial t}u(z,t)\right)-u(z,t)=0, \quad (13)$$

$$\frac{\partial}{\partial t}u(z,t)+(1+\sin(z))*u(z,t)*\left(\frac{\partial}{\partial t}u(z,t)\right)-u(z,t)=0, \quad (14)$$



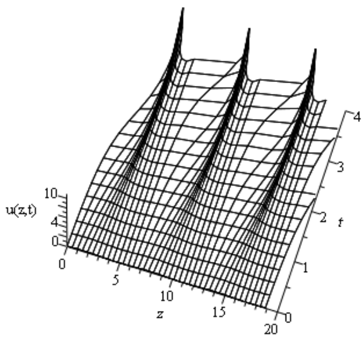


Figure 9 Figure 10

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Резюме

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ЛОРЕНЦ ТЕҢДЕУЛЕРІНДЕГІ ТҰРАҚСЫЗ АТТРАКТОРЛАРДЫҢ

ФИЗИКАЛЫҚ ТҰРҒЫДАН ЗЕРТТЕЛУІЛЕРІ

Жасалынған жұмыста геоинженерияның математикалық моделін сипаттайтын бейсызық теңдеулер тобы қарастырылған. Қарастырылған теңдеулер белгілі шекаралық жағдайда шешімдері алынған Лоренц және Бургерс теңдеулері. Теңдеулер толық шешілген және белгілі физикалық мағынаға ие. Дегенмен осы теңдеу-лерді шешу барысында негізсіз жеңілдетулер енгізіліп, соның салдарынан теңдеулердің шешімдері дұрыс табылмаған. Осы тұрғыда Лоренц теңдеулеріндегі орнықсыз аттракторладың пайда болуы зерттеліп, тең-деулерге бірнеше өзгерістер енгізілді. Алынған нәтиже Лоренц теңдеулерінің нәтижесінен біршама басқа.

Кілт сөздер: Лоренц аттракторлары, стохастикалық.

Резюме

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ФИЗИЧЕСКИЕ АСПЕТЫ СТРАННЫХ АТТРАКТОРОВ УРАВНЕНИИ ЛОРЕНЦА

В работе анализируются физические аспекты возникновение странных аттракторов в уравнениях Ло-ренца. Отмечается, что при выводе нелинейных дифференциалных уравнений, описываюших математи-ческие геоинженерные модели, делаются необоснованные упрощения, которые, в конечном счете, дают неодекватный процессу результат. В связи с этим проанализированы модифицированные нелинейные уравнения Лоренца.

Ключевые слова: аттракторы Лоренца, стохастический.

Поступила 5.07.2013г.