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(2+1)-DIMENSIONAL GENERALIZATIONS OF THE KORTEWEG-DE VRIES EQUATIONS

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Multidimensional nonlinear soliton equations are an object of the intensive researches in recent years. They are universal mathematical models because they describe different physical situations. The author presents the method of the construction of new solidly two-dimensional soliton A1-A14 and AI-AXII equations on the given H1-H5 and HI-HIV bilinear forms. The A1-A14 and AI-AXII equations are (2+1)-dimensional generalizations of the Korteweg-de Vries equation. The H1-H5 and HI-HIV bilinear forms are (2+1)-dimensional generalizations of the classical bilinear form of Hirota.

I. Introduction. The classical Korteweg-de Vries equation or (1+1)-dimensional equations of KdV is called universal mathematical model [1-4]. It describes the different physical phenomena in various environments. In 1895 the equation of KdV appeared as the description of waves on water. In 1966 of Washimi M. and Taniuti T. deduced the equation of KdV studying poorly nonlinear ion-acoustic waves of compression in plasma. In 1969 of Gardner C.S. and Morikawa G.M. received the equation of KdV investigating weak hydromagnetic long waves in plasma.

The two-dimensional generalizations of the equation of KdV also possess the universality nature. For example, Kadomtsev-Petviashvili's equation was found for poorly nonlinear long waves in dispersing environments [5]. Tappert and Varma, and Narayanamurti and Varma received KP equation studying thermal impulses in firm bodies [6, 7]. Kako and Rowlands received KP equation for two-dimensional distribution ion-acoustic solitons [8].

Today we know that scientists study solitons in oceans (a tsunami, vortical solitons), in firm crystal bodies (a dislocation, domain walls), in magnetic materials (solitons in ferromagnetics, electromagnetic solitons), in fiber light guides (optical soliton, soliton networks), in the atmosphere of Earth and other planets (soliton Rossbi or a red spot of Jupiter), in galaxies (black holes), in live organisms (nervous impulses) and others. Therefore studying of multidimensional universal models such as the equation of KdV represents a great interest.

Later it was constructed the equation Veselov-Novikov as two-dimensional integrated expansion KdV equation [9]. Nizhnik L.P. proposed the spatial two-dimensional modified KdV equation [10]. The equation was constructed for a given pair of Lax or auxiliary of the linear system. Myrzakulov R. M. deduced (2+1)-dimensional complex mKdV equation from the spin UM-LXVII model.

In 2001, the author found (2+1)-, (3+1)-and (4+1)-dimensional KdV equations, different from the above listed models. In 2004, the author has formulated a method of constructing new multidimensional generalizations of the equation of KdV [11-15]. Higher hierarchies subsidiary linear systems for these equations were built. It proves the integrability of the equations and allows to solve them using a method of a return problem of dispersion [16-18]. In this article the author presents new (2+1)-dimensional analogues of the Korteweg-de Vries which were found by her over the years. Each equation is associated with a (2+1)-dimensional bilinear form, which allows to solve this equation using the Hirota's method.

II. Method of the construction of the (2+1)-dimensional soliton A1-A14 equations by the given (2+1)-dimensional H1-H5 bilinear forms. It is known that soliton equations have bilinear forms. We consider the (2+1)-dimensional bilinear form

$$\left(D_x D_t + D_x^m D_y^n\right)(\varphi\cdot\varphi)=0, \tag{1}$$

Where $m+n=4,\ m,n=\overline{0,4},\ \varphi=\varphi(x,y,t)$ is an adequately smooth complex-valued function,

$$D_x D_t(\varphi\cdot\varphi)=2\left(\varphi_{xt}\varphi-\varphi_x\varphi_t\right),$$

$$D_x^m D_y^n(\varphi\cdot\varphi)=\left(\partial_x-\partial_{x'}\right)^m\left(\partial_y-\partial_{y'}\right)^n\varphi(x,y,t)\varphi(x',y',t')\Big|_{x'=x,y'=y,t'=t}$$

The form (1) contains five (2+1)-dimensional bilinear form, which we call *the H1-H5 bilinear forms.*

The ***H1 form*** is $\left(D_x D_t + D_x^2 D_y^2\right)(\varphi \cdot \varphi) = 0$.

The ***H2 form*** is $\left(D_x D_t + D_x^3 D_y\right)(\varphi \cdot \varphi) = 0$.

The ***H3 form*** is $\left(D_x D_t + D_x D_y^3\right)(\varphi \cdot \varphi) = 0$.

The ***H4 form*** is $\left(D_x D_t + D_y^4\right)(\varphi \cdot \varphi) = 0$.

The ***H5 form*** is $\left(D_x D_t + D_x^4\right)(\varphi \cdot \varphi) = 0$.

Theorem 1. The H1-H5 bilinear forms are complex solidly two-dimensional generalizations of the (1+1)-dimensional bilinear form of Hirota

$$\left(D_x D_t + D_x^4\right)(f \cdot f) = 0, \tag{2}$$

where $f = f(x,t)$ is an adequately smooth real function.

Proof. We show that a linear transformation

$$x = a_{11}x' + a_{12}y', \quad y = a_{21}x' + a_{22}y', \quad t = t' \tag{3}$$

Driving the H1-H5 bilinear forms into the bilinear form (2). We find the partial derivatives of the function $\varphi = \varphi(x,y,t)$ and take $\varphi = g(x',t')$, where $g = g(x',t')$ is an adequately smooth real function. Then $g_y = 0$. We put it into the H1-H5 bilinear forms. We obtain

$$\frac{a_{22}}{|A|}2\left(g_{x't'}g - g_{x'}g_{t'}\right) + \frac{a_{22}^2 a_{12}^2}{|A|^4}2\left(g_{x'x'x'x'}g - 4g_{x'x'x'}g_{x'} + 3g_{x'x'}^2\right) = 0, \tag{4}$$

$$\frac{a_{22}}{|A|}2\left(g_{x't'}g - g_{x'}g_{t'}\right) - \frac{a_{22}^3 a_{12}}{|A|^4}2\left(g_{x'x'x'x'}g - 4g_{x'x'x'}g_{x'} + 3g_{x'x'}^2\right) = 0, \tag{5}$$

$$\frac{a_{22}}{|A|}2\left(g_{x't'}g - g_{x'}g_{t'}\right) - \frac{a_{22} a_{12}^3}{|A|^4}2\left(g_{x'x'x'x'}g - 4g_{x'x'x'}g_{x'} + 3g_{x'x'}^2\right) = 0, \tag{6}$$

$$\frac{a_{22}}{|A|}2\left(g_{x't'}g - g_{x'}g_{t'}\right) + \frac{a_{12}^4}{|A|^4}2\left(g_{x'x'x'x'}g - 4g_{x'x'x'}g_{x'} + 3g_{x'x'}^2\right) = 0, \tag{7}$$

$$\frac{a_{22}}{|A|}2\left(g_{x't'}g - g_{x'}g_{t'}\right) + \frac{a_{22}^4}{|A|^4}2\left(g_{x'x'x'x'}g - 4g_{x'x'x'}g_{x'} + 3g_{x'x'}^2\right) = 0. \tag{8}$$

We compare the equations (4)-(8) with the bilinear form (2). We have $g = f$ and

$$\frac{a_{22}}{|A|} = \frac{a_{22}^2 a_{12}^2}{|A|^4} = 1, \quad \frac{a_{22}}{|A|} = -\frac{a_{22}^3 a_{12}}{|A|^4} = 1,$$

$$\frac{a_{22}}{|A|} = \frac{a_{22} a_{12}^3}{|A|^4} = 1, \quad \frac{a_{22}}{|A|} = -\frac{a_{12}^4}{|A|^4} = 1, \quad \frac{a_{22}}{|A|} = -\frac{a_{22}^4}{|A|^4} = 1,$$

where $|A| = a_{11}a_{22} - a_{12}a_{21}$. Hence the H1 and H4 bilinear forms coincide with the bilinear form (2) in case of $a_{11} + a_{21} = 1$, $a_{22} + a_{12} = 1$ or $a_{11} - a_{21} = 1$, $a_{22} - a_{12} = 1$. The H2 and H3 bilinear forms coincide with the bilinear form (2) in case of $a_{11} + a_{21} = 1$, $a_{22} + a_{12} = 1$. The H5 bilinear form coincide with the bilinear form (2) in case of $a_{22} = \dfrac{a_{12}a_{21}}{a_{11} - 1}$. The theorem 1 is proved.

We consider the (2+1)-dimensional nonlinear soliton equations, which we call *the A1-A14 equations.*

The ***A1 equation*** is

$$\psi_t + \psi_{xyy} + 2\left[\psi^2\right]_y + \left[UV\right]_y = 0,$$

$$V_x = \psi_y, \; U_y = \psi_x, \; \psi = 2(\ln\varphi)_{xy}.$$

The **A2** ***equation*** is

$$\psi_t + \psi_{xxy} + 3\left[\psi U\right]_y = 0,$$

$$U_y = \psi_x, \; \psi = 2(\ln\varphi)_{xy}.$$

The *A3 equation* is
$$\psi_t + \psi_{yyy} + 3[\psi V]_y = 0,$$
$$V_x = \psi_y,\ \psi = 2(\ln\varphi)_{xy}.$$
The *A4 equation* is
$$\psi_t + V_{yyy} + 3[V^2]_y = 0,$$
$$V_x = \psi_y,\ \psi = 2(\ln\varphi)_{xy}.$$
The *A5 equation* is
$$\psi_t + \psi_{xyy} + 2[V^2]_x + [\psi W]_x = 0,$$
$$W_{xx} = \psi_{yy},\ V_x = \psi_y,\ \psi = 2(\ln\varphi)_{xx}.$$
The *A6 equation* is
$$\psi_t + \psi_{xxy} + 3[\psi V]_x = 0,$$
$$V_x = \psi_y,\ \psi = 2(\ln\varphi)_{xx}.$$
The *A7 equation* is
$$\psi_t + \psi_{yyy} + 3[VW]_x = 0,$$
$$W_{xx} = \psi_{yy},\ V_x = \psi_y,\ \psi = 2(\ln\varphi)_{xx}.$$
The *A8 equation* is
$$\psi_t + W_{xyy} + 3[W^2]_x = 0,$$
$$W_{xx} = \psi_{yy},\ \psi = 2(\ln\varphi)_{xx}.$$
The *A9 equation* is
$$\psi_t + \psi_{xyy} + Q_{yy} = 0,$$
$$Q_x = 2U^2 + \psi P,\ P_{yy} = \psi_{xx},\ U_y = \psi_x,\ \psi = 2(\ln\varphi)_{yy}.$$
The *A10 equation* is
$$\psi_t + \psi_{xxy} + 3F_{yy} = 0,$$
$$F_x = PU,\ P_{yy} = \psi_{xx},\ U_y = \psi_x,\ \psi = 2(\ln\varphi)_{yy}.$$
The *A11 equation* is
$$\psi_t + \psi_{yyy} + 3M_{yy} = 0,$$
$$M_x = \psi U,\ U_y = \psi_x,\ \psi = 2(\ln\varphi)_{yy}.$$
The *A12 equation* is
$$\psi_t + V_{yyy} + 3K_{yy} = 0,$$
$$K_x = \psi^2,\ V_x = \psi_y,\ \psi = 2(\ln\varphi)_{yy}.$$
The *A13 equation* is
$$\psi_t + \psi_{xxx} + 3[U^2]_y = 0,$$
$$U_y = \psi_x,\ \psi = 2(\ln\varphi)_{xy}.$$
The *A14 equation* is
$$\psi_t + \psi_{xxx} + 3B_{yy} = 0,$$
$$B_x = P^2,\ P_{yy} = \psi_{xx},\ \psi = 2(\ln\varphi)_{yy}.$$
Here $\psi = \psi(x,y,t)$, $\varphi = \varphi(x,y,t)$ are adequately smooth complex-valued functions,
$$\ln\varphi = |\varphi| + i\arg\varphi,\ -\pi < \arg\varphi \le \pi.$$

Theorem 2. The A1-A14 equations are a complex solidly two-dimensional generalization of the Korteweg-de Vries equation
$$u_t + u_{xxx} + 6uu_x = 0, \tag{9}$$
where $u = 2(\ln f)_{xx}$, $u = u(x,t)$, $f = f(x,t)$ are adequately smooth real functions.

Proof. We demonstrate that a linear transformation (3) driving the A1-A14 equations into the Korteweg-de Vries equation (9). We find partial derivatives of the functions $\psi = \psi(x,y,t)$, $U = U(x,y,t)$, $V = V(x,y,t)$, $W = W(x,y,t)$, $Q = Q(x,y,t)$, $P = P(x,y,t)$, $F = F(x,y,t)$, $M = M(x,y,t)$, $K = K(x,y,t)$, $B = B(x,y,t)$ and take $\psi = v(x',t')$, $U = U^*(x',t')$, $V = V^*(x',t')$, $W = W^*(x',t')$, $Q = Q^*(x',t')$, $P = P^*(x',t')$, $F = F^*(x',t')$, $M = M^*(x',t')$, $K = K^*(x',t')$, $B = B^*(x',t')$, where v, U^*, V^*, W^*, Q^*, P^*, F^*, M^*, K^*, B^* are adequately smooth real functions. Then $v_{y'} = U^*_{y'} = V^*_{y'} = W^*_{y'} = Q^*_{y'} =$

$P^*_{y'} = F^*_{y'} = M^*_{y'} = K^*_{y'} = B^*_{y'} = 0$. We put they into the A1-A14 equations. We obtain the Korteweg-de Vries equation (9) in case of $v = u$ and

$$\frac{a_{12}^2 a_{22}}{|A|^3} = -\frac{a_{12}}{|A|} = 1,\ -\frac{a_{12}a_{22}^2}{|A|^3} = \frac{a_{22}}{|A|} = 1,\ -\frac{a_{12}^3}{|A|^3} = \frac{a_{12}^2}{|A|a_{22}} = 1,\ \frac{a_{12}^4}{|A|^3 a_{22}} = -\frac{a_{12}^3}{|A|a_{22}^2} = 1,$$

$$\frac{a_{12}^2 a_{22}}{|A|^3} = \frac{a_{12}^2}{|A|a_{22}} = 1,\ -\frac{a_{12}a_{22}^2}{|A|^3} = -\frac{a_{12}}{|A|} = 1,\ -\frac{a_{12}^3}{|A|^3} = -\frac{a_{12}^3}{|A|a_{22}^2} = -\frac{a_{12}^3}{a_{22}^3} = 1,\ \frac{a_{12}^4}{|A|^3 a_{22}} = \frac{a_{12}^4}{|A|a_{22}^3} = 1,$$

$$\frac{a_{12}^2 a_{22}}{|A|^3} = \frac{a_{22}}{|A|} = 1,\ -\frac{a_{12}a_{22}^2}{|A|^3} = -\frac{a_{22}^2}{|A|a_{12}} = 1,\ -\frac{a_{12}^3}{|A|^3} = -\frac{a_{12}}{|A|} = 1,\ \frac{a_{12}^4}{|A|^3 a_{22}} = \frac{a_{12}^2}{|A|a_{22}} = 1,$$

$$\frac{a_{22}^3}{|A|^3} = -\frac{a_{22}^2}{|A|a_{12}} = 1,\ \frac{a_{22}^3}{|A|^3} = \frac{a_{22}^3}{|A|a_{12}^2} = 1.$$

Hence the A1-A3, A7 and A13 equations coincide with the Korteweg-de Vries equation (9) in case of $a_{11} + a_{21} = 1$, $a_{22} + a_{12} = 1$. The A4-A6, A8-A10, A12 and A14 equations coincide with the equation (9) in case of $a_{11} + a_{21} = 1$, $a_{22} + a_{12} = 1$ or $a_{11} - a_{21} = 1$, $a_{22} - a_{12} = 1$. The A11 equation coincide with the equation (9) in case of $a_{12} = \dfrac{a_{11}a_{22}}{a_{21} - 1}$. The theorem 2 is proved.

Theorem 3. The A1-A14 equations are presented in the form of the H1-H5 bilinear forms. And conversely, the H1-H5 bilinear forms are presented in the form of the A1-A14 equations.

Proof. *Necessity.* We put $\psi = \psi(x, y, t)$ into the A1-A14 equations. We obtain

$$2(\varphi_{xt}\varphi - \varphi_x\varphi_t)\varphi^{-2} + 2(\varphi_{xxyy}\varphi - 2\varphi_{xxy}\varphi_y + \varphi_{xx}\varphi_{yy} - 2\varphi_{xyy}\varphi_x + 2\varphi_{xy}^2)\varphi^{-2} = 0,$$

$$2(\varphi_{xt}\varphi - \varphi_x\varphi_t)\varphi^{-2} + 2(\varphi_{xxxy}\varphi - 2\varphi_{xxx}\varphi_y + 3\varphi_{xx}\varphi_{xy} - 3\varphi_{xxy}\varphi_x)\varphi^{-2} = 0,$$

$$2(\varphi_{xt}\varphi - \varphi_x\varphi_t)\varphi^{-2} + 2(\varphi_{xyyy}\varphi - 2\varphi_{yyy}\varphi_x - 3\varphi_{xyy}\varphi_y + 3\varphi_{yy}\varphi_{xy})\varphi^{-2} = 0,$$

$$2(\varphi_{xt}\varphi - \varphi_x\varphi_t)\varphi^{-2} + 2(\varphi_{yyyy}\varphi - 4\varphi_{yyy}\varphi_y + 3\varphi_{yy}^2)\varphi^{-2} = 0,$$

$$2(\varphi_{xt}\varphi - \varphi_x\varphi_t)\varphi^{-2} + 2(\varphi_{xxxx}\varphi - 4\varphi_{xxx}\varphi_x + 3\varphi_{xx}^2)\varphi^{-2} = 0.$$

Here $\varphi = \varphi(x, y, t)$ is an adequately smooth complex-valued function. Hence the A1, A5, A9 equations give the H1 bilinear form. The A2, A6, A10 equations give the H2 bilinear form. The A3, A7, A11 equations give the H3 bilinear form. The A4, A8, A12 equations give the H4 bilinear form. The A13, A14 equations give the H5 bilinear form.

Adequacy. We consider the bilinear form (1). We will write down the equation (1) in the form

$$2\partial_x\partial_t(\ln\varphi) + 2\partial_x^m\partial_y^n(\ln\varphi) + G = 0, \tag{10}$$

where $G = G(x, y, t)$ is an adequately smooth complex-valued function. We take the derivatives of the equation (10) $k-1$ by x and l by y. We obtain

$$\partial_t\left[2\partial_x^k\partial_y^l(\ln\varphi)\right] + \partial_x^{m-1}\partial_y^n\left[2\partial_x^k\partial_y^l(\ln\varphi)\right] + \partial_x^k\partial_y^l\left[\partial_x^{-1}G\right] = 0, \tag{11}$$

We take $\psi = 2\partial_x^k\partial_y^l(\ln\varphi)$, where $k + l = 2$, $k, l = \overline{0,2}$, $\psi = \psi(x, y, t)$, $\varphi = \varphi(x, y, t)$ are adequately smooth complex-valued functions. Then from (11) we have

$$\partial_t\psi + \partial_x^{m-1}\partial_y^n\psi + \partial_x^k\partial_y^l\Phi = 0, \tag{12}$$

where $\Phi_x = G$, $\Phi = \Phi(x, y, t)$ is an adequately smooth complex-valued function. The equation (12) contains the A1-A14 equations. The theorem 2 is proved.

III. Conclusion. We presented the method of the construction of new solidly two-dimensional soliton A1-A14 equations on the given H1-H5 bilinear forms. We showed that the A1-A14 equations are (2+1)-dimensional generalizations of the Korteweg-de Vries equation and the H1-H5 bilinear forms are (2+1)-dimensional generalizations of the classical bilinear form of Hirota.

Similarly it is possible to construct the AI-AXII equations on the given the HI-HIV bilinear forms.

The ***HI form*** is $\left(D_t D_x + D_x^4 + 4D_x^3 D_y + 3D_x^2 D_y^2\right)(\varphi \cdot \varphi) = 0$.

The ***HII form*** is $\left(D_t D_x + D_x^4 + 4D_x D_y^3 + 3D_x^2 D_y^2\right)(\varphi \cdot \varphi) = 0$.

The ***HIII form*** is $\left(D_t D_x + D_y^4 + 4D_x^3 D_y + 3D_x^2 D_y^2\right)(\varphi \cdot \varphi) = 0$.

The ***HIV form*** is $\left(D_t D_x + D_y^4 + 4D_x D_y^3 + 3D_x^2 D_y^2\right)(\varphi \cdot \varphi) = 0$.

The ***AI equation*** is $\psi_t + \psi_{xxx} + 4\psi_{xxy} + 3\psi_{xyy} + 6\psi\psi_x + 12UU_x + 12[\psi U]_x + 3[\psi V]_x = 0$,

$$U_x = \psi_y,\ V_{xx} = \psi_{yy},\ \psi = 2(\ln\varphi)_{xx}.$$

The ***AII equation*** is $\psi_t + \psi_{xxx} + 4\psi_{xxy} + 3\psi_{xyy} + 6\psi\psi_y + 3VV_y + 12[\psi V]_y + 3[UV]_y = 0$,

$$U_x = \psi_y,\ V_y = \psi_x,\ \psi = 2(\ln\varphi)_{xy}.$$

The ***AIII equation*** is $\psi_t + \psi_{xxx} + 4\psi_{xxy} + 3\psi_{xyy} + 3V_{yy} + 12F_{yy} + 3W_{yy} = 0$,

$$V_x = P^2,\ F_x = PQ,\ W_x = 2Q^2 + \psi P,\ Q_y = \psi_x,\ P_{yy} = \psi_{xx},\ \psi = 2(\ln\varphi)_{yy}.$$

The ***AIV equation*** is

$$\psi_t + \psi_{xxx} + 4\psi_{yyy} + 3\psi_{xyy} + 6\psi\psi_x + 12UU_x + 12[UV]_x + 3[\psi V]_x = 0,$$

$$U_x = \psi_y,\ V_{xx} = \psi_{yy},\ \psi = 2(\ln\varphi)_{xx}.$$

The ***AV equation*** is

$$\psi_t + \psi_{xxx} + 4\psi_{yyy} + 3\psi_{xyy} + 6UU_y + 12[\psi V]_y + 12\psi\psi_y + 3[UV]_y = 0,$$

$$V_x = \psi_y,\ U_y = \psi_x,\ \psi = 2(\ln\varphi)_{xy}.$$

The ***AVI equation*** is $\psi_t + \psi_{xxx} + 4\psi_{yyy} + 3\psi_{xyy} + 3V_{yy} + 12\Phi_{yy} + 3W_{yy} = 0$,

$$V_x = P^2,\ \Phi_x = \psi Q,\ W_x = 2Q^2 + \psi P,\ Q_y = \psi_x,\ P_{yy} = \psi_{xx},\ \psi = 2(\ln\varphi)_{yy}.$$

The ***AVII equation*** is

$$\psi_t + N_{xyy} + 4\psi_{xxy} + 3\psi_{xyy} + 6NN_x + 12VV_x + 12[\psi V]_x + 3[\psi N]_x = 0,$$

$$V_x = \psi_y,\ N_{xx} = \psi_{yy},\ \psi = 2(\ln\varphi)_{xx}.$$

The ***AVIII equation*** is

$$\psi_t + V_{yyy} + 4\psi_{xxy} + 3\psi_{xyy} + 6VV_y + 12\psi\psi_y + 12[\psi U]_y + 3[UV]_y = 0,$$

$$V_x = \psi_y,\ U_y = \psi_x,\ \psi = 2(\ln\varphi)_{xy}.$$

The ***AIX equation*** is $\psi_t + L_{yyy} + 4\psi_{xxy} + 3\psi_{xyy} + 3K_{yy} + 12F_{yy} + 3W_{yy} = 0$,

$$K_x = \psi^2,\ F_x = PQ,\ W_x = 2Q^2 + \psi P,\ L_x = \psi_y,\ Q_y = \psi_x,\ P_{yy} = \psi_{xx},\ \psi = 2(\ln\varphi)_{yy}.$$

The ***AX equation*** is

$$\psi_t + V_{xyy} + 4\psi_{yyy} + 3\psi_{xyy} + 6VV_x + 12UU_x + 12[UV]_x + 3[\psi V]_x = 0,$$

$$U_x = \psi_y,\ V_{xx} = \psi_{yy},\ \psi = 2(\ln\varphi)_{xx}.$$

The ***AXI equation*** is

$$\psi_t + V_{yyy} + 4\psi_{yyy} + 3\psi_{xyy} + 6VV_y + 12\psi\psi_y + 12[\psi V]_y + 3[UV]_y = 0,$$

$$V_x = \psi_y,\ U_y = \psi_x,\ \psi = 2(\ln\varphi)_{xy}.$$

The ***AXII equation*** is $\psi_t + L_{yyy} + 4\psi_{yyy} + 3\psi_{xyy} + 3K_{yy} + 12\Phi_{yy} + 3W_{yy} = 0$,

$$K_x = \psi^2,\ \Phi_x = \psi Q,\ W_x = 2Q^2 + \psi P,\ L_x = \psi_y,\ Q_y = \psi_x,\ P_{yy} = \psi_{xx},\ \psi = 2(\ln\varphi)_{yy}.$$

Here $\psi = \psi(x, y, t)$, $\varphi = \varphi(x, y, t)$ are adequately smooth complex-valued functions,

$$\ln\varphi = |\varphi| + i\arg\varphi,\ -\pi < \arg\varphi \le \pi.$$

Thus the AI, AII, AIII equations give the HI bilinear form. The AIV, AV, AVI equations give the HII bilinear form. The AVII, AVIII, AIX equations give the HIII bilinear form. The AX, AXI, AXII equations give the HIV bilinear form.

Similarly the author constructed the (3+1)-and (4+1)-dimensional KdV equations on the given the multidimensional bilinear forms.

For each equation pair Lax or auxiliary linear system is constructed. It proves the integrability of the equation and allows to solves the equation using a method of a return problem of dispersion.

The bilinear form allows to find soliton solutions of the equation, using Hirota's method.

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КОРТЕВЕГА-ДЕ ФРИЗДІҢ (2+1)-ӨЛШЕМДІ ТАЛДАП ҚОРЫТЫЛҒАН ТЕҢДЕУЛЕРІ

Көпөлшемді сызықты емес солитондық теңдеулер соңғы жылдары қарқынды зерттеулердің нысанына айналды. Олар әмбебап математикалық үлгілер болып табылады, себебі олар әртүрлі физикалық ахуалдарды бейнелейді. Автор берілген қоссызықты H1–H5 және HI–HIV формасына A1–A14 және AI–AIII жаңа кеңістікті екіөлшемді солитонды теңдеулерін құру әдісін ұсынады. A1–A14 және AI–AXII теңдеулері Кортега-де Фриз теңдеулерінің (2+1)-өлшемді талдап қорытылған теңдеулер болса, ал қоссызықты H1–H5 және HI–HIV формалары Хирото классикалық қоссызықты формасының (2+1)-өлшемді қорытылған формасы болып табылады.

A. V. Alekseeva

(2 +1)-DIMENSIONAL GENERALIZATIONS OF THE KORTEWEG-DE VRIES

Multidimensional nonlinear soliton equations are the subject of intense research in recent years. They are universal mathematical models because they describe different physical situations. The author presents a method of constructing new space of two-dimensional soliton equations A1-A14 and AI-AXII and defines a bilinear form H1-H5 and HI-HIV. Equations A1-A14 and AI-AXII are (2 +1)-dimensional generalization of the Korteweg-de Vries equation. Bilinear form H1-H5 and HI-HIV are (2 +1)-dimensional generalization of the classical Hirota bilinear form.