

UDC 681.3

S. N. BORANBAYEV

# MATHEMATICAL MODELS OF PLANNING AND DISTRIBUTIONS OF RESOURCES OF THE NETWORK WITH VARIABLE PARAMETERS

(Представлена академиком НАН РК М. О. Отельбаевым)

The article is devoted to a problem connected to development of mathematical models of planning and distribution of resources of a network with variable parameters. The method for the decision of considered mathematical models is developed.

Let's consider program system in which objects at each level are on the one hand managing directors in relation to operated objects of the given level and objects of a control system of the bottom levels, and at the same time, are operated in relation to objects of a control system of highest levels. In such program system management of computing resources represents process which includes such components as reception of the necessary information about a condition of the subordinated objects of management, processing and the analysis of the initial information, decision-making, finishing of the decision and the necessary information to objects of management.

At construction of such program systems it is the necessary to solve a problem of planning and distribution of available hardware, program and other resources. Hardware resources understand as processors, various kinds of memory, the device of input-output, channels, etc., and as program resources – operational systems, compilers, packages of applied programs, etc.

The important requirement to developed model is the necessity of the description of cyclic process of functioning of the control system characterized by functional parameters. Thus these parameters pay off in view of dynamics of available computing resources of program system. Parameters of the processes occurring in program system are variables.

For the set of the modeled situations arising in program system, corresponding cycles of management are developed. Cycles of management are defined from a condition of qualitative performance of the fullest volume of operation which are represented by the list of operation with the instruction for each of them of sets previous and sets following it operation.

For the decision of a problem of planning of resources in program systems it is possible to use mathematical models of network structure, representing all set of the operation, carried out in system, in the form of the focused network.

Let the network is set by a complex of operation, and technological communications between these operations.

It is required to execute all complex of operation of a network for directive time  $T$ . We shall use terminology [1–3] and to lean on the results received in works [4–8].

The mathematical model of a problem has a following appearance.

$$\frac{dx_i(t)}{dt} = u_i(t), i \in I = \{1, 2, \dots, n\}$$

$$x_i(0) = 0, i \in I = \{1, 2, \dots, n\}$$

$$x_i(T) = 1, i \in I = \{1, 2, \dots, n\}$$

$$\sum_{i=1} d_{ji}(t) u_i(t) \leq V_j(t), j \in J = \{1, 2, \dots, m\}$$

$$h_i(t) \leq u_i(t) \leq H_i(t), i \in I = \{1, 2, \dots, n\}$$

$$u_i(t) = 0 \text{ if } \exists v \in I_i^+, x_v(t) < 1$$

$$u_i(t) > 0 \text{ if } 0 < x_i(t) < 1$$

Where,  $x_i(t)$  – value of a point of operation  $i$ ;  $u_i(t)$  – value of intensity of this operation; initial value of a point of operation  $i$  it is accepted equal to zero; operation  $i$  is considered executed if its point has reached value of unit;  $d_{ji}(t)$  – intensity of an expenditure of a resource of a kind  $j$  at performance of operation  $i$  with individual intensity;



$V_j(t)$  – intensity of receipt of resources of a kind  $j$ ;  
 $J = \{1, 2, \dots, m\}$  – set of kinds of resources;  
 $I = \{1, 2, \dots, n\}$  – set of operation;  $h_i(t)$  – the bottom value of intensity of operation  $i$ ;  $H_i(t)$  – the top value of intensity of operation  $i$ ;  $I_i^+$  – set of operation directly previous operation  $i$ ;  $I_i^-$  – set of operation directly following operation  $i$ .

Let the ordered sequence  $R$  is set technological, according to which operation of a network start to use resources. We consider, that operations do not suppose interruption. The problem consists in performance of a complex of operation of a network for directive time  $T$ , and also definitions of times of the beginning, the termination and intensities performance of all of operation.

Let's consider algorithm of the decision of a considered problem.

1)  $t = 0$ .

2) In  $S$  ordered, according to their position in  $R$ , all operations of a network here is located, technologically ready for performance at the present time.

3) It is checked  $S$ . If  $S$  is empty, we pass to item 10.

4) From  $S$  the next operation gets out. All active operations  $i \in G$  return borrowed or at present to time resources.

5) Intensity of operation is calculated:

$$u_i(t) = z_i(t) \text{ if } z_i(t) \geq h_i(t)$$

$$u_i(t) = 0 \text{ if } z_i(t) < h_i(t)$$

$$z_i(t) = \min \left\{ H_i(t), \min_{j \in J_i} \left\{ V_j(t) / d_{ij}(t) \right\} \right\}$$

If the condition  $u_i(t) \geq h_i(t)$  is satisfied, it means that operation  $i$  can be carried out. Corresponding operation in file  $S$  is marked with a special label. The label specifies the belonging to the of operation of set  $G$ . We pass to item 8.

If performance of operation cannot be continued, it means that there was a critical site and it should be eliminated.

6) If  $\tau$  – the moment of occurrence of a critical site. Corresponding operation is marked with a special attribute.

Are calculated:

$$t^0 = \min_{i \in S} \{t_i^0\},$$

$$x^0 = x(t^0).$$

7) Since the moment  $t^0$  the decision for a complex of operation of a network down to the moment  $t^f$  is under construction:

$$t^f = \max_{i \in S} \{t_i^f\}.$$

Thus on performance of the marked operation  $i$  additional restriction is imposed:

$$t_i^0 \geq \tau.$$

If on an interval  $[t^0, t^f]$  the new critical sites are not revealed, they are calculated:

$$t^k = \min_{i \in S} \{t_i^k\},$$

$$x^0 := x(t^k),$$

$$t^0 := t^k.$$

Also we pass to item 8. If on an interval  $[t^0, t^f]$  there were critical sites we pass to item 6.

8) It is calculated:

$$V_j(t) := V_j(t) - d_{ij} u_i(t),$$

$$t_i^f = (1 - x_i(t)) / u_i(t) + t.$$

If all operations from  $S$  are already chosen, we pass to item 9, otherwise on item 4.

9) They are calculated:

$$t' = \min_{i \in G} \{t_i^f\}$$

$$x_i(t') = x_i(t) + u_i(t)(t' - t), \text{ if } i \in G$$

$$x_i(t') = x_i(t), \text{ if } i \notin G$$

$$V_j(t') := V_j(t) + \sum_{i \in W} d_{ij} u_i(t), \quad j \in J$$

$$r_i(t') := r_i(t) - 1 \text{ if } i \in I_v^-, \quad v \in W$$

All operations  $i \in G \setminus W$  return all resource borrowed by them. The moment of time  $t'$  is appointed a present situation of decision-making. We pass to item 2.

10) If received time of performance of a complex of network operation is no more than directive time then the decision of a problem is received. Otherwise another technological the ordered sequence is constructed and iterations proceed. It is possible to interrupt iterative process after achievement of comprehensible accuracy of the decision, either on the timer, or after performance of the certain quantity of iteration.



# LITERATURE

1. *Coffman E.* Theory of schedules and computers. M.: Science, 1984. 334 p.
2. *Bernstein F.*, etc. Operational system. M.: World, 1977. 336 c.
3. *Hoare C.* Communicating Sequential Processes // CACM. 1978. V. 21, N8. P. 666-677.
4. *Boranbayev S.N.* Algorithm of the decision of a problem of planning and distribution of resources of a network // Reports of the National Academy of Sciences. 2003. N2. P. 15-21.
5. *Boranbayev S.N.* Property of a method of planning and distribution of resources of a network // News of the National Academy of Sciences. A series physical and mathematical. 2003. N1. P. 68-71.
6. *Boranbayev S.N.* Method of planning and distributions of resources of a network // Bulletin of the National Academy of Sciences. 2003. N2. P. 146-150.
7. *Boranbayev S.N.* Optimality condition in optimization network tasks // 989<sup>th</sup> American Mathematical Society Meeting. University of Colorado. Boulder, Colorado, USA. 2003. P. 33.

8. *Boranbayev S.N.* Mathematical Model for the Development and Performance of Sustainable Economic Programs // International Journal of Ecology and Development. 2007. V. 6, N W07.

## Резюме

Параметрлері айнымалы торларға құнарларды жоспарлау және үлестірудегі математикалық модельдерді жасау барысындағы проблемалық мәселелер шешілген. Математикалық модельдерді шешетін әдіс жасалған.

## Резюме

Статья посвящена проблеме, связанной с разработкой математических моделей планирования и распределения ресурсов сети с переменными параметрами. Разработан метод для решения рассматриваемых математических моделей.

*L. N. Gumilyov Eurasian  
National University*

Поступила 20.06.07г.