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ELECTROMAGNETIC DECAY OF THE TETRAQUARK STATE X(3872) IN A RELATIVISTIC CONSTITUENT QUARK MODEL WITH INFRARED CONFINEMENT

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We further explore the consequences of treating the $X(3872)$ meson as a tetraquark bound state by analyzing its one-photon decay $X \to \gamma + J/\psi$ in the framework of our approach developed in previous papers which incorporates quark confinement in an effective way. To introduce electromagnetism we gauge a nonlocal effective Lagrangian describing the interaction of the $X(3872)$ meson with its four constituent quarks by using the P-exponential path-independent formalism. We calculate the matrix element of the transition $X \to \gamma + J/\psi$ and prove its gauge invariance. We evaluate the $X \to \gamma + J/\psi$ decay width and the longitudinal transverse composition of the J/ψ in this decay. For a reasonable value of the size parameter of the $X(3872)$ meson we and find consistency with the available experimental data.

1 Introduction. This paper is a direct continuation of our previous work [1] where we have analyzed the strong decays of the charmonium-like state $X(3872)$ in the framework of our relativistic constituent quark model which includes infrared confinement in an effective way [2]. In our approach the $X(3872)$ meson is interpreted as a tetraquark state with the quantum numbers $J^{PC} = 1^{++}$ as in [3]. In this paper we analyze the one-photon decay $X \to \gamma + J/\psi$ in the same tetraquark picture. The electromagnetic interaction is incorporated into our relativistic nonlocal effective Lagrangian in a gauge invariant way using the P-exponential path-independent formalism. Then, we calculate the matrix element and width of the $X \to \gamma + J/\psi$ decay.

We begin by collecting the experimental data relevant for our purposes. The $X(3872)$ state discovered by the Belle Collaboration in the decay $B^{\pm} \to K^{\pm}X(3872)$, $X(3872) \to J/\psi\pi^+\pi^-$ is now well established [4]. The $X(3872)$ decays into $\pi^+\pi^- J/\psi$ and has a mass of $m_X = 3872.0 \pm 0.6(stat) \pm 0.5(syst)$ very close to the $M^{D^0} + M^{D^{*0}} = 3871.81 \pm 0.25$ mass threshold [5]. Its width was found to be less than 2.3 MeV at 90% confidence level. The state was confirmed in B - decay by the BARAR experiment [6] and in $p\overline{p}$ production by the Tevatron experiments CDF [7] and DØ [8]. The Belle collaboration has reported [9] evidence for the decay modes $X \to \gamma + J/\psi$:

$$B(B \to XK)B(X \to \gamma + J/\psi) = (1.8 \pm 0.6(stat) \pm 0.1(syst)) \times 10^{-6},$$

$$\frac{\Gamma(X \to \gamma + J/\psi)}{\Gamma(X \to \pi^+\pi^- J/\psi)} = 0.14 \pm 0.05. \qquad (1)$$

These observations imply strong isospin violation because the three-pion decay proceeds via an intermediate ω - meson with isospin 0 where as the two-pion decay proceeds via the intermediate ρ - meson with isospin 1. It is evident that the two-pion decay via the intermediate ρ - meson is very difficult to explain by using an interpretation of the $X(3872)$ as a simple $c\bar{c}$ charmonium state with isospin 0.

In an analysis of $B^+ \to J/\psi\gamma K^+$ decays, the BABAR collaboration [10] found evidence for the radiative decay $X \to \gamma + J/\psi$ with a statistical significance of 3.4σ. They reported the following values for the product of 3.4σ branching fractions

$$B(B^+ \to XK^+)B(X \to \gamma + J/\psi) = (3.3 \pm 1.0(stat) \pm 0.3(syst)) \times 10^{-6}. \qquad (2)$$

All available experimental data up to 2007 were analyzed in [11]. The authors found that [11]:

$$\begin{aligned} &B(B^+ \to XK^+) = 1.30^{+0.20}_{-0.34} \times 10^{-4}, \\ &\frac{\Gamma(X \to \gamma + J/\psi)}{\Gamma(X \to \pi^+\pi^- J/\psi)} = 0.22 \pm 0.06. \end{aligned} \qquad (3)$$

The BABAR collaboration found evidence for the decays $X \to \gamma + J/\psi$ and $X \to \gamma + \psi(2S)$ in their data samp of the decays $B \to c\bar{c}\gamma K$ decays. The measured products of branching fractions are [12]:

$$\begin{aligned} &B(B^\pm \to XK^\pm)B(X \to \gamma + J/\psi) = (2.8 \pm 0.8(stat) \pm 0.1(syst)) \times 10^{-6}, \\ &B(B^\pm \to XK^\pm)B(X \to \gamma + \psi(2S)) = (9.5 \pm 2.7(stat) \pm 0.6(syst)) \times 10^{-6}. \end{aligned} \qquad (4)$$

There have been many theoretical attempts to unravel the structure of the $X(3872)$ and its decays. Many of the theoretical predictions for the decay $X \to \gamma + J/\psi$ published up to now are very model dependent. We mention some of them in turn. All possible $1D$ and $2P$ $c\bar{c}$ assignments for the $X(3872)$ were considered in [13]. The authors obtained $E1$ radiative widths for decays into charmonium $c\bar{c}$ states as well as for some strong decays taking the experimental mass as input. The conclusion was that many of the possible J^{PC} assignments can be eliminated due to the smallness of the observed total width. The suggestion was that radiative transitions could be used to test the remaining J^{PC} assignments. Some tests of the hypothesis that the $X(3872)$ is a weakly bound $D^0\overline{D}^{*0}$ molecule state were suggested in [14]. It was proposed that measuring the $3\pi J/\psi, \gamma J/\psi, \gamma\psi', \overline{K}K^*$ and $\pi\rho$ decay modes of the X will serve as a definitive diagnostic tool to confirm or to rule out the molecule hypothesis. QCD sum rules were used in [15] to calculate the width of the radiative decay of the meson $X(3872)$, which was assumed to be a mixture between charmonium and exotic molecular $[c\bar{q}][q\bar{c}]$ states with $J^{PC} = 1^{++}$.

2 Theoretical framework. The e□ective interaction Lagrangians describing the coupling of the charmonium-like meson such as the $X(3872)$ to four quarks, and the coupling of the charmonium J/ψ state to its two constituent quarks are written in the form, (see Ref. [1]),

$$L_{\text{int}} = g_X X_{q\mu}(x) J^\mu_{Xq}(x) + g_{J/\psi} J/\psi_\mu(x) J^\mu_{J/\psi}(x), \quad (q = u, d). \qquad (5)$$

The nonlocal interpolating quark currents teak

$$J^\mu_{Xq}(x) = \int dx_1 \ldots \int dx_4 \delta\left(x - \sum_{i=1}^4 \omega_i x_i\right) \Phi_X\left(\sum_{i<j} (x_i - x_j)^2\right) \times$$

$$\frac{1}{\sqrt{2}} \varepsilon_{abc}\varepsilon_{dec} \{[q_a(x_4) C\gamma^5 c_b(x_1)][\bar{q}_d(x_3)\gamma^\mu C^5 \bar{c}_e(x_2)] + (\gamma^5 \leftrightarrow \gamma^\mu)\},$$

$$J^\mu_{J/\psi} = \int dy_1 \int dy_2 \delta\left(y - \frac{1}{2}(y_1 + y_2)\right) \Phi_{J/\psi}((y_1 - y_2)^2) \bar{c}_a(y_1) \gamma^\mu c_a(y_2). \qquad (6)$$

where $\omega_1 = \omega_2 = m_c / 2(m_q + m_c) \equiv \omega_c / 2, \quad \omega_3 = \omega_4 = m_q / 2(m_q + m_c) \equiv \omega_q / 2$. The matrix $C = \gamma^0 \gamma^2$ is related to the charge conjugation matrix. We follow [3] and take the tetraquark state to be a linear superposition of the and X_d states acording to

$$X_l \equiv X_{low} = X_u \cos\theta + X_d \sin\theta, \\ X_h \equiv X_{high} = -X_u \sin\theta + X_d \cos\theta, \tag{7}$$

The coupling constant g_X in Eq.(5) is determined by the compositeness condition $Z_H = 0$. The compositeness condition requires that the renormalization constant $Z_H = 0$ of the elementary meson X is set to zero, i.e.

$$Z_H = 1 - \Sigma'_H (m_H^2) = 0, \tag{8}$$

where $\Sigma'_H (m_H^2)$ is the derivative of the mass operator. One has

$$\Sigma'_H (m_H^2) = g_H^2 \Pi'_H (m_H^2) = g_H^2 \left. \frac{d\Pi_H (p^2)}{dp^2} \right|_{p^2 = m_H^2} \tag{9}$$

and where m_H is the meson mass. At this point we take the mesons to be spinless for the sake of simplicity. The generalization to mesons with arbitrary spin is straightforward. The gauge invariant interaction with the electromagnetic field is introduced in two stages. The free Lagrangian of quarks and hadrons is gauged in the standard manner by using minimal substitution:

$$\partial^\mu q \to (\partial^\mu - ie_q A^\mu) q, \quad \partial^\mu \bar{q} \to (\partial^\mu - ie_q A^\mu) \bar{q}, \tag{10}$$

where e_q is the quark's charge ($e_u = \frac{2}{3} e$, $e_d = -\frac{1}{3} e$, etc.). Minimal substitution gives us the first piece of the electromagnetic interaction Lagrangian

$$L_{Int}^{em(1)} = \sum_q e_q A_\mu(x) J_q^\mu(x), \quad J_q^\mu(x) = \bar{q}(x) \gamma^\mu q(x). \tag{11}$$

The second term of the electromagnetic interaction Lagrangian $L_{Int}^{em(2)}$ arises when one expands the gauge exponential in powers of A_μ up to the order of perturbation theory that one is considering. Expanding the Lagrangian up to the first order in A^μ one obtains

$$L_{Int}^{em(2)}(x) = g_H X_{q\mu}(x) \cdot J_{X_q - em}^\mu(x) + g_{J/\psi} J/\psi_\mu(x) \cdot J_{J/\psi_{q-em}}^\mu(x), \quad (q = u, d).$$

$$J_{X_{q-em}}^\mu = \int d\vec{\rho} \Phi_X(\vec{\rho}^2) J_{4q}^\mu(x_1, \ldots, x_4) \{ ie_q [I_x^{x_3} - I_x^{x_4}] + ie_c [I_x^{x_2} - I_x^{x_1}] \}, \tag{12}$$

$$J_{J/\psi - em}^\mu = \int d\rho \Phi_{J/\psi}(\rho^2) J_{2q}^\mu(x_1, x_2) ie_c [I_x^{x_2} - I_x^{x_1}]$$

For example, expanding the Lagrangian Eq. (10) up to the first order in A^μ one obtains

$$J_{X_{q-em}}^\mu(x) = \prod_{i=1}^4 \int d^4 x_i \int d^4 y J_{4q}^\mu(x_1, \ldots, x_4) A_\rho(y) E_X^\rho(x; x_1, \ldots, x_4, y),$$

$$E_X^\rho(x; x_1, \ldots, x_4, y) = \prod_{i=1}^4 \int \frac{d^4 p_i}{(2\pi)} \int \frac{d^4 r}{(2\pi)} e^{-ip_1(x - x_1) + ip_2(x - x_2) + ip_3(x - x_3) - ip_4(x - x_4) - ir(x - y)} \widetilde{E}_X^\rho(p_1, \ldots, p_4, r), \tag{13}$$

$$\widetilde{E}_X^{\rho}(p_1,\dots,p_4,r)=\int_0^1\sum_{j=1}^{3}\left\{e_c\left[-\widetilde{\Phi}_X(-z_{1j})l_{1j}^{\rho}+\widetilde{\Phi}_X(-z_{2j})l_{2j}^{\rho}\right]+e_q\left[-\widetilde{\Phi}_X(-z_{4j})l_{4j}^{\rho}+\widetilde{\Phi}_X(-z_{3j})l_{3j}^{\rho}\right]\right\}$$

$$J^{\mu}_{J/\psi-em}(y)=\int d^4y_1\int d^4y_2\int d^4z J^{\mu}_{2q}(y_1,y_2)A_{\rho}(z)E^{\rho}_{J/\psi}(y;y_1,y_2,z),$$

$$E^{\rho}_{J/\psi}(y;y_1,y_2,z)=\int\frac{d^4p_1}{(2\pi)^4}\int\frac{d^4p_2}{(2\pi)^4}\int\frac{d^4q}{(2\pi)^4}e^{-ip_1(y_1-y)+ip_2(y_2-y)+iq(z-y)}\widetilde{E}^{\rho}_{J/\psi}(p_1,p_2,q), \quad (14)$$

$$\widetilde{E}^{\rho}_{J/\psi}(p_1,p_2,q)=e_c\int_0^1 d\tau\left\{-\widetilde{\Phi}'_{J/\psi}(-z_-)l_-^{\rho}-\widetilde{\Phi}'_{J/\psi}(-z_+)l_+^{\rho}\right\}$$

For calculational convenience we will choose a simple Gaussian form for the vertex function $\widetilde{\Phi}_X(-\Omega^2)$. The minus sign in the argument of the Gaussian function is chosen to emphasize that we are working in Minkowski space. One has

$$\widetilde{\Phi}_X(-\Omega^2)=\exp(\Omega^2/\Lambda_X^2), \quad (15)$$

where the parameter Λ_X characterizes the size of the X - meson. Since Ω^2 turns into $-\Omega^2$ in Euclidean space the form (15) has the appropriate fall-o□ behavior in the Euclidean region. We emphasize that any choice for Φ_X is appropriate as long as it falls o□ su□ciently fast in the ultraviolet region of Euclidean space to render the corresponding Feynman diagrams ultraviolet finite. As mentioned before we shall choose a Gaussian form for Φ_X in our numerical calculation for calculational convenience. The matrix element of the decay $X\to\gamma+J/\psi$ can be calculated from the Feynman diagrams shown.

$$M\left(X_q(p)\to J/\psi(q_1)+\gamma(q_2)\right)=i(2\pi)^4\delta(p-q_1-q_2)\varepsilon^X_{\mu}\varepsilon^{J/\psi}_{\nu}\varepsilon^{\gamma}_{\rho}T^{\mu\nu\rho}(q_1,q_2), \quad (16)$$

3 **Numerical analysis.** Using the calculated matrix elements for the decay $X\to J/\psi+\rho(\omega)$ one can evaluate the decay widths $X\to\gamma+J/\psi$. We employ the narrow width approximation for this purpose. The adjustable parameters of our model are the constituent quark masses m_q, the scale parameter l characterizing the infrared confinement and the size parameters Λ_X. They were determined by using a least square fit to a number of physical observables, see [2]. We correct an error of Ref. [1] in the normalization condition of the X meson, which led to $a\le 30\%$ underestimate of the strong decay widths. Both decay widths become smaller as the size parameter increases. Note that the radiative decay width for $X_h=-X_u\sin\theta+X_d\cos\theta$ is almost an order of magnitude smaller than that for $X_l=-X_u\cos\theta+X_d\sin\theta$. If one takes $\Lambda\in(3,4)GeV$ with the central value $\Lambda_X\in 3{,}5GeV$ then our prediction for the ratio of widths reads

$$\frac{\Gamma(X_l\to J/\psi+\gamma)}{\Gamma(X_l\to J/\psi+2\pi)}\Big|_{theor}=0.15\pm 0.03. \quad (17)$$

which fits very well the experimental data from the BELLE collaboration [9] written down in their Eq. 18

$$\frac{\Gamma(X\to J/\psi+\gamma)}{\Gamma(X\to J/\psi+2\pi)}=\begin{cases}0.14\pm 0.05 & BELLE\ (9)\\ 0.22\pm 0.06 & BARAR\ (11)\end{cases} \quad (18)$$

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РЕЛЯТИВИСТІК КОНСТИТУЕНТТІК КВАРКТІК ҮЛГІДЕ X(3872) ТӨРТКВАРКТІК КҮЙДІҢ ИНФРАҚЫЗЫЛ КОНФАЙНМЕНТПЕН ЭЛЕКТРОМАГНИТТІК ЫДЫРАУЫ

Біздің алдыңғы жұмыстарымыздағыдай $X(3872)$ мезонды төртварктi байланысқан күй ретінде зерттей отырып, біздің тәсілдің аумағында оны бірфотонды ыдырау $X \to \gamma + J/\psi$ ретінде талдаймыз, ол кварктерді ұстап қалудың тиімді тәсілі. Электромагнитизмді енгізу үшін, төртварктан тұратын мезонның әсерлесуін көрсету үшін біз локалды емес тиімді Лагранжиан функциясын өлшейміз. Біз $X \to \gamma + J/\psi$ ыдырауының матрицалық элементін есептеп, калибрлік инварианттылығын дәлелдейміз. $X \to \gamma + J/\psi$ ыдыраудың енін және осы ыдыраудың құрамындағы J/ψ мезонды бағалаймыз. $X(3872)$ мезонның біркелкі параметрін эксперименттегі мәндерден аламыз.

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ЭЛЕКТРОМАГНИТНЫЙ РАСПАД ЧЕТЫРЕХКВАРКОГО СОСТОЯНИЯ X(3872) В РЕЛЯТИВИСТКОЙ КОНСТИТУЕНТНОЙ КВАРКОВОЙ МОДЕЛИ С ИНФРАКРАСНЫМ КОНФАЙНМЕНТОМ

Мы изучаем, рассматривая последствия $X(3872)$ мезона, как четырехкваркое связанное состояние, анализируя его как однофотонный распад $X \to \gamma + J/\psi$ в рамках нашего подхода, разработанного в предыдущих работах, который включает удержание кварков в эффективным способом. Чтобы ввести электромагнетизм, мы измеряем нелокальную эффективную функцию Лагранжиана, описывающую взаимодействие мезона с его четырьмя составляющими кварками. Мы вычисляем матричный элемент $X \to \gamma + J/\psi$ перехода и доказываем его калибровочный инвариантность. Мы оцениваем ширину $X \to \gamma + J/\psi$ распада и продольный поперечный состав J/ψ в этом распаде. Для разумного значения размерного параметра мезона $X(3872)$ мы и берем данные, имеющиеся с эксперимента.