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RHEOLOGICAL LAWS OF VISCOUS FLUID DYNAMICS

Annotation. Physically substantiated dependence of stress tensor components of velocity gradient arises from the laws of friction, theoretical substantiation is associated with displacement tensor. Adequate simulation of viscous fluid depends on conformity of the flow shear and normal strains to the value of the velocity components in each point.

Keywords: liquid viscosity, pressure, laminar, turbulence.

Тірек сөздер: сұйықтық, тұтқырлық, кернеу, ламинарлық, турбуленттік.

Ключевые слова: жидкость, вязкость, напряжение, ламинарное, турбулентность.

1. Rheological communication arising from the laws of friction. In the course of physics and theoretical mechanics friction law for small velocities adopted in the form of $\vec{F}_{mp} = -k\nu\vec{e}_\nu = -k\vec{\nu}$, to high velocities is generalized in the form of a quadratic dependence on the velocity modulus $\vec{F}_{mp} = -k_2\nu^2\vec{e}_\nu$ [1]. Shear strains $\pi_{yx} = \mu \frac{\partial u}{\partial y}$ (Newton's law of friction) follows from the law for small velocities $\vec{F}_{mp} = -k\vec{\nu}$. It makes sense distribution of this fact to degree formula of friction $\vec{F}_{mp} = -k_m\nu^m\vec{e}_\nu$, $m = 1, 2, 3, \dots$ and setting of quasi-parabolic properties of obtained in this case dynamics equations of viscous fluid.

Let the drag force of particle is proportional to the degree $m = 1, 2, 3, \dots$ speed in a given direction. Suppose that the friction force on the layer y_1 is $\vec{F}_1 = -k_m u_1^m \vec{i}$, respectively, $\vec{F}_2 = -k_m u_2^m \vec{i}$ on layer y_2 . Next, consider the increment $\delta \vec{F} = \vec{F}_2 - \vec{F}_1 = -k_m u_2^m \vec{i} + k_m u_1^m \vec{i} = -k_m \delta u^m \vec{i}$, $\delta \vec{F} \uparrow \downarrow \vec{i}$, $\delta \vec{F} = \vec{f} \delta y$, \vec{f} - linear force density $\delta \vec{F}$, $\vec{f} \uparrow \uparrow \vec{\pi}_{yx}$, therefore, $\vec{f} = k' \vec{\pi}_{yx}$, $\delta \vec{F} = k' \vec{\pi}_{yx} \delta y$, $\vec{\pi}_{yx} \uparrow \downarrow \vec{i}$, $\vec{\pi}_{yx} = -\pi_{yx} \vec{i}$, $k' \vec{\pi}_{yx} \delta y = -k_m \delta u^m \vec{i}$. Resulting equalities $-k' \pi_{yx} \delta y = -k_m \delta u^m$, $\pi_{yx} = \mu_m \frac{\delta u^m}{\delta y}$, $\mu_m = \frac{k_m}{k'}$, in the limit give shear stress

$$\pi_{yx} = \lim_{\delta y \rightarrow 0} \mu_m \frac{\delta u^m}{\delta y} = \mu_m \frac{\partial u^m}{\partial y}, \text{ similar conclusions:}$$

$$\pi_{xy} = \mu_m \frac{\partial v^m}{\partial x}, \pi_{zy} = \mu_m \frac{\partial v^m}{\partial z}, \pi_{yz} = \mu_m \frac{\partial w^m}{\partial y}, \pi_{xz} = \mu_m \frac{\partial w^m}{\partial x}, \pi_{zx} = \mu_m \frac{\partial u^m}{\partial z},$$

of which at degrees $m = 1$ obtain formulas of Newton's law of friction.

Similar arguments sets formula of viscous component $\vec{\pi}_{xx}^o$ of the normal stress $\vec{\pi}_{xx} = -p \vec{i} + \vec{\pi}_{xx}^o$. Let the friction forces equal at point x_1 $\vec{F}_1 = -k_m u_1^m \vec{i}$ and $\vec{F}_2 = -k_m u_2^m \vec{i}$ at point $x_2 = x_1 + \delta x$. Increments are compiled: $\delta \vec{F} = \vec{F}_2 - \vec{F}_1 = -k_m u_2^m \vec{i} + k_m u_1^m \vec{i} = -k_m \delta u^m \vec{i}$. Through a linear density of $\delta \vec{F} = \vec{\varphi} \delta x$, $\vec{\varphi} = k'' \vec{\pi}_{xx}^o$ we have $\delta \vec{F} = k'' \vec{\pi}_{xx}^o \delta x$, $k'' \vec{\pi}_{xx}^o \delta x = -k_m \delta u^m \vec{i}$. By definition, $\vec{\pi}_{xx}^o \uparrow \downarrow \vec{i}$. This expression is scalar multiplied by the unit vector \vec{i} : $(k'' \vec{\pi}_{xx}^o, \vec{i}) \delta x = -k_m (\delta u^m \vec{i}, \vec{i})$. The result is

$$(\vec{\pi}_{xx}^o, \vec{i}) \delta x = |\vec{\pi}_{xx}^o| \cdot |\vec{i}| \cos 180^\circ = -\pi_{xx}^o, -k_m (\delta u^m \vec{i}, \vec{i}) = -k_m \delta u^m |\vec{i}| \cdot |\vec{i}| \cos 0^\circ = -k_m \delta u^m$$

Equalities $-k'' \pi_{xx}^o \delta x = -k_m \delta u^m$, $\pi_{xx}^o = \mu_m \frac{\delta u^m}{\delta x}$, $\mu_m = \frac{k_m}{k''}$, in the limit give formulas of the normal stresses components:

$$\pi_{xx}^o = \lim_{\delta x \rightarrow 0} \mu_m \frac{\delta u^m}{\delta x} = \mu_m \frac{\partial u^m}{\partial x}, \pi_{yy}^o = \mu_m \frac{\partial v^m}{\partial y}, \pi_{zz}^o = \mu_m \frac{\partial w^m}{\partial z}, \text{ QED.}$$

Obviously, full normal stresses are the sum of this components and hydrostatic pressure:

$$\pi_{xx} = -p + \pi_{xx}^o = -p + \mu_m \frac{\partial u^m}{\partial x}, \pi_{yy} = -p + \pi_{yy}^o = -p + \mu_m \frac{\partial v^m}{\partial y}, \pi_{zz} = -p + \pi_{zz}^o = -p + \mu_m \frac{\partial w^m}{\partial z}$$

This justification of normal stresses makes unnecessary hypothesis of the pressure [2].

At degrees $m = 1$ formula obtained normal stresses imposed by Navier in 1822.

The elements of the matrix of displacement \vec{S} are the 1st derivative of the Taylor's series:

$$v_i^m(\vec{r} + \delta \vec{r}, t) = v_i^m(\vec{r}, t) + \frac{\partial v_i^m}{\partial x_j} \delta x_j, \quad i = 1, 2, 3, \quad \vec{S} = \left\{ \frac{\partial v_i^m}{\partial x_j} \right\}_{i,j=1,2,3}$$

Obvious proportionality of derived the stress tensor components to the matrix of displacement components

$$\pi_{ji} = -p \delta_{ij} + \mu_m \frac{\partial v_i^m}{\partial x_j}, \quad i, j = 1, 2, 3, \quad m = 1; 3; 5; 7; 9; \dots$$

which is the theoretical foundation of the power law of friction.

2. On the ineffectiveness of the Newton's law of friction in the modeling of turbulent flows. As shown in the preceding paragraph, the Newton's law of friction $\pi_{yx(H)} = \mu \frac{\partial u}{\partial y}$ is a consequence of friction

for small velocities $\vec{F}_{mp} = -k\vec{v}$, therefore equation, $\rho[\frac{\partial \vec{v}}{\partial t} + (\vec{v}, \nabla)\vec{v}] + \nabla p = \mu \Delta \vec{v} + \rho \vec{F}$, based on the Newton's stress tensor $\pi_{ij} = -p\delta_{ij} + \mu \bar{S}$, is a model of low-rate laminar flows, and can not be a model for high-rate flows. As is well known (see [3]), the Stokes' law of friction $\pi_{yx(c)} = \mu(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x})$ and the Stokes' stress tensor $\pi_c = -[p + (2/3\mu - \mu')div\vec{v}]E + \mu\dot{S}$, according to Landau, is a hypothesis, as they have no physical basis, because there is no matching friction force $\vec{F}_{mp} = -k\vec{v}$, and strain rate tensor \dot{S} is part of the displacement tensor $\bar{S} = \dot{S} + \hat{S}$ for degree $m = 1$, which is another proof of the fallacy of the Stokes' hypothesis, therefore, the Navier-Stokes' equations.

3. Adequate approach to modeling viscous flows. It is quite obvious and does not require proof that in the viscous fluid flows, especially in turbulent or intermittent, the speeds are changed in magnitude and direction, therefore, frictional forces will be variable at the points of the stream

$$\vec{F}_{mp} = -k_m v^m \vec{e}_v, v = |\vec{v}|, \vec{v} = v\vec{e}_v, m > 0, m = 1, 3, 5, 7, 9, \dots$$

For example, the longitudinal velocity is many times larger than the transverse velocity component in longitudinal streaming flow of the plate: $|u| \gg |w|, |u| \gg |v|$, thus in the direction of axis \vec{i} must run one of the friction

$$\vec{F}_{mp} = -ku\vec{i}, \vec{F}_{mp} = -k_3 u^3 \vec{i}, \vec{F}_{mp} = -k_5 u^5 \vec{i}, \vec{F}_{mp} = -k_7 u^7 \vec{i}, \vec{F}_{mp} = -k_9 u^9 \vec{i}$$

etc., and in the transverse directions with the laws of other power-law

$$\vec{F}_{mp} = -kv\vec{j}, \vec{F}_{mp} = -k_3 v^3 \vec{j}, \vec{F}_{mp} = -k_5 v^5 \vec{j}, \dots, \vec{F}_{mp} = -kw\vec{k}, \vec{F}_{mp} = -k_3 w^3 \vec{k}, \vec{F}_{mp} = -k_5 w^5 \vec{k}, \dots,$$

etc., which should be taken into account in the formulas tangential and normal stresses

$$\pi_{ji} = -p\delta_{ij} + \mu_{m_i} \frac{\partial v_i^{m_i}}{\partial x_j}, i, j = 1, 2, 3, m_i = 1, 3, 5, 7, 9, \dots$$

(δ_{ij} Kronecker delta) and in the dynamics equations.

4. Universal model of the dynamics of liquid and gas. Thus, from the laws of friction with different degrees refractive

$$\vec{F}_{mp} = -k_{m_v} v^{m_v} \vec{e}_v, v = |\vec{v}|, \vec{v} = v\vec{e}_v, m_v > 0$$

follow the stresses with the corresponding degrees

$$\pi_{ji} = -p\delta_{ij} + \mu_{m_i} \frac{\partial v_i^{m_i}}{\partial x_j}, i, j = 1, 2, 3, m_i = 1, 3, 5, 7, 9, \dots$$

and equations

$$\rho(\frac{\partial v_i}{\partial t} + \sum_{j=1}^3 v_j \frac{\partial v_i}{\partial x_j}) + \frac{\partial p}{\partial x_i} = \rho F_i + \sum_{j=1}^3 \frac{\partial}{\partial x_j} (\mu_{m_i} \frac{\partial v_i^{m_i}}{\partial x_j}), i = 1, 2, 3, \quad \frac{\partial \rho}{\partial t} + \sum_{i=1}^3 \frac{\partial \rho v_i}{\partial x_i} = 0$$

$$\rho c_v (\frac{\partial T}{\partial t} + \sum_{j=1}^3 v_j \frac{\partial T}{\partial x_j}) = \sum_{j=1}^3 \frac{\partial}{\partial x_j} (\lambda \frac{\partial T}{\partial x_j}) - p div \vec{w} + \sum_{j=1}^3 \sum_{i=1}^3 \mu_{m_i} \frac{\partial v_i^{m_i}}{\partial x_j} \frac{\partial v_i}{\partial x_j},$$

Degree must be odd positive integers to belong quasi-parabolic type equations. Of odd powers of the property lost and the initial-boundary value problems for these equations are incorrect.

These complexes are formed in dimensionless variables:

$$\frac{1}{Re_{m_i}} = \frac{\mu_{m_i} U^{m_i-2}}{\rho L} = \frac{\mu}{\rho UL} \frac{\mu_{m_i} U^{m_i-1}}{\mu} = \frac{1}{Re} Ks_{m_i},$$

$$Ks_{m_i} = \frac{\mu_{m_i} U^{m_i-1}}{\mu} = \alpha \left(\frac{U}{m_i} \right)^{m_i-1} \cdot U^{m_i-1}$$

Euler equation of ideal gas and fluid obtained from universal equations with degrees $m_i = 0 \forall i, Ks_i = 0$. In ideal gas and fluid viscosity is zero (no molecular transport) confirms the validity of the following formula:

$$\mu_{m_i} = \alpha \left(\frac{1}{m_i} \right)^{m_i-1} \mu, \quad \mu_{m_i=0} = \alpha \left(\frac{1}{m_i} \right)^{m_i-1} \mu = \alpha \left(\frac{1}{0} \right)^{0-1} \mu = \alpha \left(\frac{0}{1} \right) \mu = 0, \mu_0 = 0.$$

Equation with the stress tensor can be obtained from Newton's equations with exponents $m_i = 1 \forall i$ and $\mu_1 = \mu$ viscosity coefficient.

Equality $\alpha = 1 \left(\frac{ce \kappa}{M} \right)^{m_i-1}$ follows from the formula $\mu_{m_i} = \alpha \left(\frac{1}{m_i} \right)^{m_i-1} \mu$ with

$$m_i = 1: \mu_1 = \alpha \left(\frac{1}{1} \right)^{1-1} \mu = \alpha \mu, \quad m_i = 1: \mu_1 = \alpha \left(\frac{1}{1} \right)^{1-1} \mu = \alpha \mu, \text{ since } \mu_1 = \mu.$$

5. Comparison with the experimental averaged velocity profile of turbulent flow in a circular tube and areas of applicability of power rheological law. Axial velocity of the one-dimensional steady flow of a viscous fluid in a circular tube is obtained from the equation in cylindrical coordinates

$$\frac{dp}{dz} = \frac{\mu_m}{r} \frac{d}{dr} \left(r \frac{dV_z^m}{dr} \right), V_r = 0, V_\varphi = 0, \frac{dp}{dz} = \text{const} < 0,$$

The solution has the form:

$$V_z^m = -\frac{1}{4\mu_m} \frac{dp}{dz} (a^2 - r^2), V_r = 0, V_\varphi = 0, \frac{V_z}{\max V_z} = \left(1 - \frac{r^2}{a^2} \right)^{\frac{1}{m}}.$$

Figures 1–5 are paintings comparison with the averaged velocity of the turbulent flow (indicated by asterisks), given in the textbook [2] p.670. Draws attention to Figure 1, where the velocity profiles are plotted strength laminar flow in a pipe, the corresponding friction law of Newton and solving the Navier-Stokes equations $m = 1, \mu_1 = \mu$. Averaged turbulent profile matches to Reynolds number $Re = 3\,240\,000$. Figures 2 and 3 shows the comparison of the theoretical values for $m = 7, m = 9$ with the experimental turbulent profile. There is an almost perfect match.

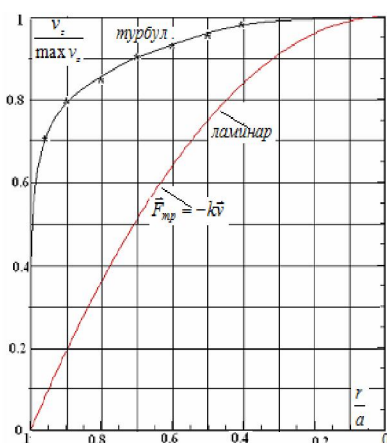


Figure 1

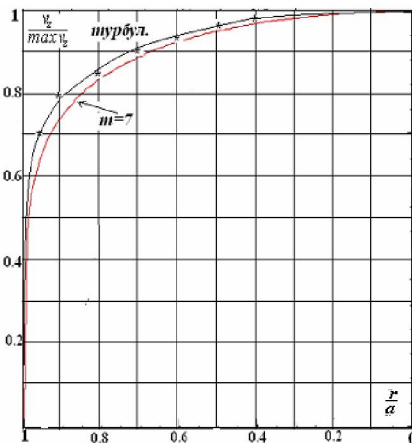


Figure 2

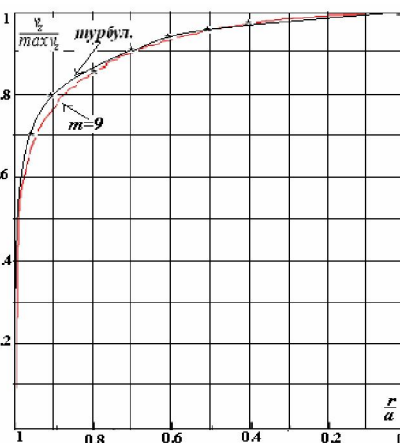


Figure 3

Thus, the most coincident with the experimental results are obtained with the increase of the degrees $m = 7, m = 9$. Obviously, it makes sense simulate certain turbulent flows by varying the dimensionless parameters of these rheological laws.

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Резюме

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ТҰТҚЫРЛЫ СҰЙЫҚТЫҚ ҚОЗҒАЛЫСЫНЫҢ РЕОЛОГИЯЛЫҚ ЗАҢДАРЫ

Кернеулер тензорының компоненттері үйкеліс заңдарынан тәуелділіктері дәлелденген, соған сәйкес осындай байланыстарды теоретикалық негіздеуі ығысу тензорымен орнатылған. Тұтқырлы сұйықтықтың ағыстарын нақты үлгілеу ағыстың әрбір нүктесінде жанама және тік кернеулердің осы нүктедегі жылдамдықтың шамасымен байланыстығына тәуелді, яғни дифференциалдық теңдеулер құбылысты болады.

Тірек сөздер: сұйықтық, тұтқырлық, кернеу, ламинарлық, турбуленттік.

Резюме

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РЕОЛОГИЧЕСКИЕ ЗАКОНЫ ДИНАМИКИ ВЯЗКОЙ ЖИДКОСТИ

Физически обоснована зависимость тензора напряжений градиента скорости, вытекает из законов трения, теоретическое обоснование связано с объемом тензора. Адекватное моделирование вязкой жидкости зависит от соответствия потока сдвига и штаммы нормальной стоимости компонент скорости в каждой точке.

Ключевые слова: жидкость, вязкость, напряжения, ламинарное, турбулентность.

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