УДК 517.958: 536.2

S. N. KHARIN[1], M. M.SARSENGELDIN[2]

ANALYTICAL SOLUTION OF HEAT CONDUCTION EQUATION IN A DOMAIN WITH MOVING BOUNDARY NOT TANGENT TO AXES OF COORDINATES

[1]Kazakhstan-British Technical University (050000 Almaty, 59 Tole bi street);
[2]Suleiman Demirel University (050043, Almaty, 19 Toraigyrov street)

The boundary-value problem for the heat equation is reduced to a system of integral equations, which singularity is conditioned by the extinction of the domain at the initial time. Analytical solution of this system is found using Laplace transform in the form of a convergence series. Its analysis enables to establish asymptotic behavior of the solution at small time depending on boundary conditions.

Introduction. Development of analytical methods of solution of free boundary problems are very important for analysis of dynamics of phenomena of heat and mass transfer with phase transformation, hydrodynamic flows and many other problems. The well-known analytical method is based on the representation of a solution in the form of heat potential with following reduction of the given problem to integral equation [1]. However if the domain with moving boundary degenerates into a point at the initial time, the integral equations become singularity and can not be solved by Picard's method. Asymptotic properties of such equations have been investigated in [2]. Auto-model case when the boundary $\alpha(t)$ is moving according to the law $\alpha(t)=c\sqrt{t}$ is considered in [3] where analytical solution is found. The case of a uniform moving boundary appears in many applications, in particular in the theory of welding.

Problem statement. Definition. The class functions M_β is defined by formula: $f(t)\in M_\beta(h)$ if $f(t)$ is continuous on the interval $(0,t)$ and $\lim\limits_{t\to 0}\frac{f(t)}{t^\beta}=h=const$, where β is any real number.

The main problem can be formulated as following. It is required to find the solution of the heat equation

$$\frac{\partial u}{\partial t}=a^2\frac{\partial^2 u}{\partial x^2} \qquad (1)$$

in the domain $D(t>0, 0<x<\alpha(t))$, degenerating at the initial time: $\alpha(0)=0$ and satisfying the initial condition

$$u(0,0)=0 \qquad (2)$$

and the boundary conditions

$$u(0,t)=\varphi(t), \qquad (3)$$

$$u(\alpha(t),t)=\psi(t). \qquad (4)$$

The functions $\varphi(t)$ and $\psi(t)$ supposed to be continuous, and $\alpha(t)$ is positive, strictly increasing, differentiable function.

The solution of the problem (1) – (4) can be represented in the form of heat potentials:

$$u(x,t)=\frac{1}{2a\sqrt{\pi}}\int_0^t\frac{x}{(t-\tau)^{3/2}}\exp\left[-\frac{x^2}{4a^2(t-\tau)}\right]\nu(\tau)d\tau+$$

$$+\frac{1}{2a\sqrt{\pi}}\int_0^t\frac{x-\alpha(\tau)}{(t-\tau)^{3/2}}\exp\left[-\frac{(x-\alpha(\tau))^2}{4a^2(t-\tau)}\right]\mu(\tau)d\tau \qquad (5)$$

that satisfies the equations (1) and (2) for any functions $\nu(\tau)$ and $\mu(\tau)$. Satisfying the boundary conditions (3) – (4) we get the system of integral equations with respect to $\nu(\tau)$ and $\mu(\tau)$. Eliminating $\nu(\tau)$ gives the integral equation [2]:

$$\mu(t)-\int_0^t K(t,\tau)\mu(\tau)d\tau=f(t), \qquad (6)$$

where

$$K(t,\tau)=\frac{1}{2a\sqrt{\pi}}\left\{\frac{\alpha(t)+\alpha(\tau)}{(t-\tau)^{3/2}}\exp\left[-\frac{(\alpha(t)+\alpha(\tau))^2}{4a^2(t-\tau)}\right]+\right.$$

$$\left.+\frac{\alpha(t)-\alpha(\tau)}{(t-\tau)^{3/2}}\exp\left[-\frac{(\alpha(t)-\alpha(\tau))^2}{4a^2(t-\tau)}\right]\right\}; \quad (7)$$

$$f(t)=\frac{1}{2a\sqrt{\pi}}\times$$

$$\times\int_0^t\frac{\alpha(t)}{(t-\tau)^{3/2}}\exp\left[-\frac{\alpha(t)^2}{4a^2(t-\tau)}\right]\varphi(\tau)d\tau-\psi(t). \quad (8)$$

The kernel $K(t,\tau)$ has singularity at $t=0$, thus the equation (6) is not solvable by Picard's method.

Asymptotic properties of the solution $\mu(t)$ are given by the theorem 1 [2]:

Theorem 1. If $\alpha(t)\in M_\gamma$ and $f(t)\in M_\delta$, then $\mu(t)\in M_{\delta-\gamma+1/2}$ for $\gamma>1/2$ and $\mu(t)\in M_{\delta+2\gamma-1}$ for $0<\gamma<1/2$.

Let us prove now the lemma.

Lemma. If $\overline{\omega}(p)$ is the Laplace transform of the function $\omega(t)$,i.e. $\overline{\omega}(p)=\int_0^\infty e^{-pt}\omega(t)dt$ or in short notation $\overline{\omega}(p)\to\omega(t)$, $\overline{\omega}(p^2)=\overline{\Omega}(p)\to\Omega(t)$ and if $\omega(t)\in M_\beta$, then $\Omega(t)\in M_{\beta+1/2}$, $\Omega'(t)\in M_\beta$.

Proof. According to the Efros theorem [4], if $\overline{f}(p)\to f(t)$, then

$$\overline{G}(p)\overline{f}[q(p)]\to\int_0^\infty f(\tau)g(t,\tau)d\tau,$$

where $g(t,\tau)\leftarrow\overline{G}(p)e^{-\tau q(p)}$

Putting $\overline{G}(p)=1$, $q(p)=\sqrt{p}$ we find that

$$e^{-\tau\sqrt{p}}\to\frac{\tau}{2\sqrt{\pi t^3}}e^{-\frac{\tau^2}{4t}}=g(t,\tau) \text{ and}$$

$$\overline{\Omega}\left(\sqrt{p}\right)\to\omega(t)=$$

$$=\int_0^\infty\frac{\tau}{2\sqrt{\pi t^3}}e^{-\frac{\tau^2}{4t}}\Omega(\tau)d\tau=\frac{2}{\sqrt{\pi t}}\int_0^\infty ze^{-z^2}\Omega(2\sqrt{t}z)dz$$

or

$$\sqrt{t}\omega(t)=\frac{2}{\sqrt{\pi}}\int_0^\infty ze^{-z^2}\Omega(2\sqrt{t}z)dz.$$

Since $\Omega(t)$ is an original satisfying the inequality $\Omega(z)\le Me^{-\delta z}$, $\delta>0$ we can conclude from (12) that $\Omega(t)\in M_{\beta+1/2}$. Differentiation of the last expression shows that $\Omega'(t)\in M_\beta$ .The lemma is proved.

Solution of characteristic equation. If the boundary $\alpha(t)$ does not touch coordinate axis, i.e. $\alpha(t)\in M_1$ and it can be represented in the form

$$\alpha(t)=ct+\alpha_1(t), \quad (9)$$

where $c=const$ and $\alpha_1(t)\in M_\lambda$, $\lambda>1$, then we can use the method of regularization for the equation (6). According to this method we should solve first the characteristic equation for $\alpha(t)=ct$ which can be written in the form

$$\mu(t)-\frac{k}{\sqrt{\pi}}\int_0^t\frac{t+\tau}{(t-\tau)^{3/2}}\exp\left[-\frac{k^2(t+\tau)^2}{(t-\tau)}\right]\mu(\tau)d\tau-$$

$$-\frac{k}{\sqrt{\pi}}\int_0^t\frac{\exp[-k^2(t-\tau)}{(t-\tau)^{1/2}}\mu(\tau)d\tau=f(t), \quad (10)$$

where $k=\dfrac{c}{2a}$.

Using Laplace transform and calculating the first integral in (10) we obtain

$$\int_0^\infty e^{-pt}\int_0^t\frac{t+\tau}{(t-\tau)^{3/2}}\exp\left[-\frac{k^2(t+\tau)^2}{(t-\tau)}\right]\mu(\tau)d\tau=$$

$$=\int_0^\infty\mu(\tau)d\tau\int_\tau^\infty\frac{t+\tau}{(t-\tau)^{3/2}}\exp\left[-pt-\frac{k^2(t+\tau)^2}{(t-\tau)}\right]dt=$$

$$=\sqrt{\pi}\left(\frac{1}{\sqrt{p+k^2}}+\frac{1}{k}\right)\int_0^\infty e^{-\left(\sqrt{p+k^2}+2k\right)^2\tau}\cdot e^{k^2\tau}\mu(\tau)d\tau.$$

Introducing notation

$$e^{k^2t}\mu(t)=\omega(t),\quad e^{k^2t}f(t)=f_1(t) \quad (11)$$

we get using shifting theorem

$$\bar{\mu}(p)=\bar{\omega}(p+k^2),\quad \bar{f}(p)=\bar{f}_1(p+k^2),$$

Then the image of the first integral in (10) is

$$\int_0^\infty e^{-pt}\int_0^t \frac{t+\tau}{(t-\tau)^{3/2}}\exp\left[-\frac{k^2(t+\tau)^2}{(t-\tau)}\right]\mu(\tau)d\tau=$$

$$=\sqrt{\pi}\left(\frac{1}{\sqrt{p+k^2}}+\frac{1}{k}\right)\bar{\omega}\left[\left(\sqrt{p+k^2}+2k\right)^2\right],$$

while for the image of the second integral we have the expression:

$$\int_0^\infty e^{-pt}\int_0^t \frac{\exp[-k^2(t-\tau)}{(t-\tau)^{1/2}}\mu(\tau)d\tau dt=$$

$$=\frac{\sqrt{\pi}}{\sqrt{p+k^2}}\bar{\omega}(p+k^2).$$

Thus the Laplace transform of the equation (10) is

$$\int_0^\infty e^{-pt}\int_0^t \frac{\exp[-k^2(t-\tau)}{(t-\tau)^{1/2}}\mu(\tau)d\tau dt=$$

$$=\frac{\sqrt{\pi}}{\sqrt{p+k^2}}\bar{\omega}(p+k^2).$$

Replace $\sqrt{p+k^2}$ in this functional equation by p we get

$$\bar{\omega}(p^2)-\left(\frac{k}{p}+1\right)\bar{\omega}\left[(p+2k)^2\right]-\frac{k}{p}\bar{\omega}(p^2)=\bar{f}_1(p^2).$$

Using notation

$$\bar{\omega}(p^2)=\bar{\Omega}(p),\quad \bar{f}_1(p^2)=\bar{F}(p) \qquad (12)$$

we get the functional equation for $\bar{\Omega}(p)$

$$p[\bar{\Omega}(p)-\bar{\Omega}(p+2k)]-k[\bar{\Omega}(p)+\bar{\Omega}(p+2k)]$$

and corresponding equation for originals

$$\frac{d}{dt}[\Omega(t)-e^{-2kt}\Omega(t)]-$$

$$-k[\Omega(t)+e^{-2kt}\Omega(t)]=F'(t)+F(0). \qquad (13)$$

Conjugation of the given conditions (2), (3), (4) at $t=0$ enables us to conclude that $\varphi(0)=\psi(0)=0$. Then from the formula (8) we can suppose that $f(t)\in M_\delta$, $\delta>0$ and $\mu(t)\in M_{\delta-1/2}$ due to the theorem 1. Therefore $\omega(t)\in M_{\delta-1/2}$ and $\Omega(0)=0$, $F(0)=0$ due to the lemma.

The equation (13) can be rewritten as the differential equation

$$(1-e^{-2kt})[\Omega'(t)-k\Omega(t)]=F'(t) \qquad (14)$$

with the initial condition

$$\Omega(0)=0.$$

The main problem is to find its solution not in term of $F(t)$ but in term of $f(t)$.

To do it we expand the exponent into a series and represent (14) in the form

$$\Omega'(t)-k\Omega(t)=\sum_{n=0}^{\infty}e^{-2nkt}F'(t).$$

Applying again Laplace transform we get

$$\bar{\Omega}(p)=\sum_{n=0}^{\infty}\frac{p+2nk}{p-k}\bar{F}(p+2nk)$$

thus

$$\bar{\omega}(p)=\bar{\Omega}(\sqrt{p})=\sum_{n=0}^{\infty}\frac{\sqrt{p}+2nk}{\sqrt{p}-k}\bar{F}(\sqrt{p}+2nk)=$$

$$=\sum_{n=0}^{\infty}\frac{\sqrt{p}+2nk}{\sqrt{p}-k}\bar{f}_1[(\sqrt{p}+2nk)^2].$$

To find the original we apply again the Efros theorem putting in (11)

$$\bar{G}(p)=\frac{\sqrt{p}+2nk}{\sqrt{p}-k},\qquad \bar{q}(p)=(\sqrt{p}+2nk)^2.$$

Then

$$\bar{G}(p)e^{-\tau q(p)}=\frac{\sqrt{p}+2nk}{\sqrt{p}-k}e^{-\tau(\sqrt{p}+2nk)^2}=$$

$$=e^{-4n^2k^2\tau}[1+\frac{(2n+1)k}{\sqrt{p}-k}]e^{-4nk\tau\sqrt{p}}\cdot e^{-\tau p}.$$

From the tables of Laplace transform we can find

$$\psi(4nk\tau,t)+(2n+1)k\times$$

$$\times\left[\chi(4nk\tau,t)+ke^{-4nk^2\tau+k^2t}erfc\left(\frac{4nk\tau}{2\sqrt{t}}-k\sqrt{t}\right)\right],$$

where

$$\psi(\tau,t)=\frac{\tau}{2\sqrt{\pi t^3}}e^{-\frac{\tau^2}{4t}},\qquad \chi(\tau,t)=\frac{1}{\sqrt{\pi t}}e^{-\frac{\tau^2}{4t}}.$$

Using Shifting theorem we get

$$g(t,\tau)=e^{-4n^2k^2\tau}W(4nk\tau,t-\tau), \qquad (15)$$

where

$$W(\tau,t)=\begin{cases}\dfrac{\tau}{2\sqrt{\pi t^3}}e^{-\frac{\tau^2}{4t}}+(2n+1)k\times \\ \times\left[\dfrac{1}{\sqrt{\pi t}}e^{-\frac{\tau^2}{4t}}+ke^{k^2t-k\tau}erfc\left(\dfrac{\tau}{2\sqrt{t}}-k\sqrt{t}\right)\right], \\ \text{if} \quad \tau<t \\ 0. \quad \text{if} \quad \tau>t\end{cases} \qquad (16)$$

Thus

$$\omega(t)=\int_0^t\sum_{n=0}^{\infty}e^{-4n^2k^2\tau}W(4nk\tau,t-\tau)f_1(\tau)d\tau$$

and from (11) we obtain the final formula for the solution of the characteristic equation (10)

$$\mu(t)=\int_0^t V(\tau,t)f(\tau)d\tau, \qquad (17)$$

where

$$V(\tau,t)=e^{-k^2t}\sum_{n=0}^{\infty}e^{-(4n^2-1)k^2\tau}W(4nk\tau,t-\tau). \qquad (18)$$

Expressions (16) – (18) show that the function $V(\tau,t)$ Has singularities at $t=0$, $\tau=0$, $\tau=t$. Thus convergence of the series (18) should be proved. The last expression can be written in the form

$$V(\tau,t)=e^{-k^2(t-\tau)}(I_1+I_2+I_3),$$

where the terms in the right side have following estimations:

$$I_1=\sum_{n=0}^{\infty}\frac{2nkt}{\sqrt{\pi}(t-\tau)^{3/2}}e^{-\frac{4n^2k^2t\tau}{t-\tau}}=$$

$$=\frac{2kt}{\sqrt{\pi}(t-\tau)^{3/2}}\sum_{n=0}^{\infty}ne^{-\alpha^2n^2}<\frac{2kt}{\sqrt{\pi}(t-\tau)^{3/2}}\cdot\frac{1}{\alpha^2},$$

$$\alpha^2=\frac{4k^2t\tau}{t-\tau};$$

$$I_2=\sum_{n=0}^{\infty}\frac{k}{\sqrt{\pi(t-\tau)}}e^{-\frac{4n^2k^2t\tau}{t-\tau}}=$$

$$=\frac{k}{\sqrt{\pi(t-\tau)}}\sum_{n=0}^{\infty}e^{-\alpha^2n^2}<\frac{k}{\sqrt{\pi(t-\tau)}}(1+\frac{1}{\alpha});$$

$$I_3=\sum_{n=0}^{\infty}(2n+1)k^2e^{k^2t-k^2\tau(2n+1)^2}\times$$

$$\times erfc\left(\frac{2nk\tau}{\sqrt{t-\tau}}-k\sqrt{t-\tau}\right)<2k^2e^{k^2(t-\tau)}+\frac{e^{k^2t}}{\tau}.$$

Using these inequalities we can estimate $V(\tau,t)$:

$$V(\tau,t)<\frac{e^{-k^2(t-\tau)}}{2k\sqrt{\pi}\cdot\tau\cdot\sqrt{t-\tau}}+$$

$$+\frac{ke^{-k^2(t-\tau)}}{\sqrt{\pi}\cdot\sqrt{t-\tau}}+\frac{e^{-k^2(t-\tau)}}{2\sqrt{\pi}\cdot\sqrt{t\tau}}+2k^2+\frac{e^{k^2\tau}}{\tau}. \qquad (19)$$

Since $|f(t)|\le ht^{\delta}$ we can estimate $\mu(t)$ using (17) – (19)

$$|\mu(t)|\le\int_0^t V(t,\tau)|f(\tau)|d\tau\le h\int_0^t V(t,\tau)\cdot\tau^{\delta}d\tau\le$$

$$\le hN_1\int_0^t\left[\frac{\tau^{\delta-1}}{\sqrt{t-\tau}}+\frac{\tau^{\delta}}{\sqrt{t-\tau}}+\frac{\tau^{\delta-1/2}}{\sqrt{t}}+\tau^{\delta}+\tau^{\delta-1}\right]d\tau=$$

$$=hN_1t^{\delta-1/2}\times$$

$$\times\int_0^1\left[\frac{x^{\delta-1}}{\sqrt{1-x}}+\frac{tx^{\delta}}{\sqrt{1-x}}+\sqrt{t}x^{\delta-1/2}+t^{3/2}x^{\delta}+\sqrt{t}x^{\delta-1}\right]dx\le$$

$$\le h\cdot N_{\delta}\cdot t^{\delta-1/2}, \qquad (20)$$

where the constant N_{δ} can decrease only if δ increases.

The last inequality means that

$$\mu(t)=\int_0^t V(\tau,t)f(\tau)d\tau\in M_{\delta-1/2}.$$

Taking into account that $\alpha(t)\in M_1$, we can see the confirmation of the theorem 1.

It is important for applications to have a simple approximate expression for calculation of $\mu(t)$ for small values of t. To get such approximation we consider again the equation (14) and use the expansion $1-e^{-2kt}=2kt+g(t)$ for small values of t, where $g(t)\in M_2$. Then the equation (14) can be written in the form

$$\Omega'(t)-k\Omega(t)=\frac{F'(t)}{2kt}+R(t), \qquad (21)$$

where

$$R(t)=\left[\Omega'(t)-k\Omega(t)\right]\cdot S(t), \qquad S(t)\in M_1 . \quad (22)$$

Using Laplace transform we get

$$p\bar{\Omega}(p)-k\bar{\Omega}(p)=\frac{1}{2k}\int_p^\infty q\bar{F}(q)dq+\bar{R}(p)$$

thus

$$\bar{\Omega}(p)=\frac{1}{2k}\cdot\frac{1}{p-k}\int_p^\infty q\bar{F}(q)dq+\frac{1}{p-k}\bar{R}(p)$$

and

$$\bar{\omega}(p)=\frac{1}{2k}\cdot\frac{1}{\sqrt{p}-k}\int_{\sqrt{p}}^\infty q\bar{F}(q)dq+\frac{1}{\sqrt{p}-k}\bar{R}(\sqrt{p})=$$

$$=\frac{1}{4k}\cdot\frac{1}{\sqrt{p}-k}\int_p^\infty \bar{f}_1(s)ds+\frac{1}{\sqrt{p}-k}\bar{R}(\sqrt{p}),$$

where

$$\bar{f}_1(s)\to f_1(t)=e^{k^2t}f(t).$$

Using formula [4]

$$\frac{1}{\sqrt{p}-k}\to\frac{e^{k^2t}}{\sqrt{t}}ierfc(-k\sqrt{t})$$

we can write

$$\omega(t)= \qquad (23)$$

$$=\frac{1}{4k}\int_0^t\frac{e^{k^2(t-\tau)}}{\sqrt{t-\tau}}ierfc(-k\sqrt{t-\tau})\cdot\frac{f_1(\tau)}{\tau}d\tau+R_1(t),$$

where

$$R_1(t)=\int_0^t\frac{e^{k^2(t-\tau)}}{\sqrt{t-\tau}}ierfc(-k\sqrt{t-\tau})\cdot R_0(\tau)d\tau , \quad (24)$$

$$R_0(t)\leftarrow\bar{R}(\sqrt{p}). \qquad (25)$$

Let us estimate $R_1(t)$. Since $f_1(t)=f(t)e^{k^2t}\in M_\delta$ and $\alpha(t)=ct\in M_1$, we can derive from the theorem 1 that $\omega(t)\in M_{\delta-1/2}$. From the lemma and formula (22) we obtain that $R(t)\in M_{\delta+1/2}$. Finally, using lemma again to the expression (25) we find that $R_0(t)\in M_\delta$ and from (24) we conclude that

$$R_1(t)\in M_{\delta+1/2}. \qquad (26)$$

We can see that in spite of the fact that the term $R_1(t)$ in the formula (23) contains the function $\omega(t)$, it is an infinitesimal of more higher order in comparison with the first term for small values of t, so it can be neglected. Since $\omega(t)=e^{k^2t}\mu(t)$, $f_1(t)=e^{k^2t}f(t)$, we get the final expression for the solution at small values of t:

$$\mu(t)=\frac{1}{4k}\int_0^t\frac{ierfc(-k\sqrt{t-\tau})}{\tau\sqrt{t-\tau}}\cdot f(\tau)d\tau . \quad (27)$$

Regularization. Let us consider now the general integral equation (6) where the moving boundary is given by the expression (9). It can be represented in the form

$$\mu(t)-\int_0^t K_1(t,\tau)\mu(\tau)d\tau=G(t), \qquad (28)$$

where

$$G(t)=\int_0^t\left[K(t,\tau)-K_1(t,\tau)\right]\mu(\tau)d\tau+f(t)$$

and $K_1(t,\tau)$ is the kernel of characteristic equation, i.e. it is the kernel $K(t,\tau)$ at $\alpha(t)=ct$.

Using formula (17) for the solution of (28) we get

$$\mu(t)=\int_0^t V(\tau,t)G(\tau)d\tau$$

or

$$\mu(t)-\int_0^t V(\tau,t)\int_0^\tau\left[K(\tau,\tau_1)-K_1(\tau,\tau_1)\right]\mu(\tau_1)d\tau_1 d\tau=$$

$$=\int_0^t V(\tau,t)f(\tau)d\tau. \qquad (29)$$

If $\alpha_1(t)\le Pt^{1+\lambda}$, $\lambda>0$, then for any $s(t)\in M_\beta$, $\beta>-1$, *i.e.* $|s(t)|\le qt^\beta$ it can be shown that

$$\int_0^t V(\tau,t)\left|\int_\tau^t\left[K(\tau_1,\tau)-K_1(\tau_1,\tau)\right]s(\tau)\right|d\tau\le$$

$$\le L_\beta qt^{\beta+\lambda-1/2},$$

where L_β is a constant that should decrease when β increases. It enables us to prove that for any $f(t) \in M_\delta$ the integral equation (28) can be solved by Picard's method for the interval

$$t \le t_0 < L_{\delta-1/2}^{-(\lambda-1/2)^{-1}}. \qquad (30)$$

To expand this solution for $t > t_0$ we write the equation (6) in the form

$$\mu(t) - \int_{t_0}^{t} K(t,\tau)\mu(\tau)d\tau =$$

$$= f(t) + \int_{0}^{t_0} K(t,\tau)\mu(\tau)d\tau. \qquad (31)$$

Taking into account that $\alpha(t) > \varepsilon > 0$ for $t \ge t_0$ we can conclude that the kernel $K(t,\tau)$ in the left side is continuous. Since the function $\mu(t)$ for $t < t_0$ is already found above, and the right side of (31) is defined,

We can solve the equation (31), i.e. (6) for any t.

ЛИТЕРАТУРА

1. *Тихонов А.Н., Самарский А.А.* Уравнения математической физики. М.: Гостехтеориздат, 1951.

2. *Харин С.Н.* // Тепловые процессы в электрических контактах и связанные с ними сингулярные интегральные уравнения: Автореф. дис. Алма-Ата, 1968.

3. *Харин С.Н.* // О тепловых задачах с подвижной границей // Изв. АН КазССР. Сер. физ.-мат. наук. 1965. № 3.

4. *Диткин В.А., Прудников А.П.* Интегральные преобразования и операционное исчисление. М.: Физматгиз, 1961.

REFERENCES

1. Tikhonov A.N., Samarskii A.A. *Gostekhteorizdat.* Moskva, **1951** (in Russ.).

2. Kharin S.N. *Avtoreferat dissertatsii.* Alma-Ata, **1968** (in Russ.).

3. Kharin S.N. *Izvestiya AN Kaz SSR, seriya fiziko-matematicheskaya,* 3, **1965** (in Russ.).

4. Ditkin V.A., Prudnikov A.P. *Fizmatgiz,* Moskva, **1961** (in Russ.).

Резюме

Жылыөткізгіштіктің тендеуі үшін шеткі есебі сингулярлығы уақыттың бастапқы сәті саласының тууына сәйкестелген интегралдық тендеулер жүйесіне алып келеді. Осы жүйенің аналитикалық шешімін қатарлардың үйлесімділігі түріндегі Лаплас түрленулерін пайдалану арқылы табуға болады. Бұл шешім уақыттық маңызы шамалы шектеулік жағдайына тәуелді асимтотикалық бағытты шешуді табуға мүмкіндік береді.

Резюме

Краевая задача для уравнения теплопроводности сведена к системе интегральных уравнений, сингулярность которых обусловлена вырождением области в начальный момент времени. Аналитическое решение этой системы находится с использованием преобразования Лапласа в виде сходящихся рядов. Это позволяет установить асимптотическое поведение решения при малых значений времени в зависимости от граничных условий.