АСТРОФИЗИКА

UDC 524.8

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EVOLUTION OF PERTURBATIONS IN THE UNIVERSE DESCRIBED BY THE NONSTATIONARY AND NONLINEAR EQUATION OF STATE

(Presented by academician NAS RK T.B.Omarov)

The growth of the perturbation density in a baryon substance due to a nonstationary and nonlinear characteres of the equation of state of nonbaryon matter in the Universe is searched. This state could equate at the Big Bang process, after which the amplitude was radically decreased.

The problems of development of fluctuations of baryon matter are studied in a number of papers (see, e.g., [1]), however, the physical reason for the growth is poorly understood. The fact is that the conventional approach to the problem is based on investigation into gravitational instability of baryon matter in the Universe. However, the observational data of the last decade proved a significant quantitative predominance of nonbaryon substance over baryon matter in the Universe [2].

Therefore, a natural question arises if nonbaryon substance itself (for example, dark matter) can cause formation of space structures in the Universe. Various aspects of the problem were discussed in [3-4]. Among them, note analysis of antigravitational – in particular, vacuum – instability of cosmological baryon substance. Moreover, it was shown in [5] that vacuum itself can produce objects like dwarf galaxies.

The goal of this work is to study the development of perturbations of baryon matter due to nonbaryon substance, in particular, to analyze the growth in the baryon-matter perturbations during evolution of the Universe with the nonlinear and nonstationary one type equation of state. The necessity of searching the cosmological consequences of a nonstationary equations of state for nonbaryonic substrate was argued, in particular, in review [6].

1. Evolution of the scaling factor

The Einstein equations describing evolution of the scaling factor are written as [1]

$$\ddot{a} = -\frac{4\pi}{3}G(\rho_{nb} + 3p_{nb})a, \tag{1}$$

$$H^2 + \frac{k}{a^2} = \left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi}{3}G\rho_{nb}. \tag{2}$$

The law of conservation of energy follows from Eqs. (1) – (2)

$$\dot{\rho}_{nb}a^3 + 3(\rho_{nb} + p_{nb})a^2\dot{a} = 0. \tag{3}$$

In this system, ρ_{nb} is the density of nonbaryon matter, ρ_{nb} is the pressure, and G is the gravitation constant. Here $H = \frac{\dot{a}}{a}$ is the Hubble constant which, as was noted above, depends on time. In addition, in Eq. (2), k is the space curvature equal to 1 for the closed, 0 for the flat, and –1 for the open models of the Universe, respectively. In order to find how the Universe evolves in time, one should specify the equation of state of nonbaryon matter, which relates the energy density and pressure. For adiabatic processes, the equation is written as $\rho_{nb} = \omega \cdot \rho_{nb}$, where ω is the parameter of state (a constant parameter in the Friedman model). For the known forms of nonbaryon matter, for example, for quintessence, vacuum, phantom energy, it takes on the values $-1<\omega<1/3$, -1, and $\omega<-1$, respectively [2].

According to the statement of the problem, we use the Wetterich parametrization [3,7].

$$\omega(a) = \frac{\omega_0}{1 + b\ln\frac{a_0}{a}}, \tag{4}$$

which describes a wide class of nonstationary equations of state. Here $\omega_0 = -1$, and $b = 0.25$, a_0 is a fixed scaling factor, and a is its current value.

Solving Eqs. (3), (4), we derive

$$\rho_{nb} = \rho_0 x^{-3}(-4 + \ln x)^{-12}, \tag{5}$$

where $x = \frac{a}{a_0}$. Substituting the necessary parameters into Eq. (2), we have the following inhomogeneous differential equation of the first order:

$$\frac{dx}{dt} = Cx^{-1/2}(-4+\ln x)^{-6}, \qquad (6)$$

with the constant $C = \sqrt{\frac{8\pi}{3} G\rho_0}$.

To solve the above equation, we consider the case where $\frac{a}{a_0} \langle\langle 1$. This condition is fulfilled both in the very early Universe and at later stages of its evolution under the corresponding choice of the fixed scaling factor a_0.

Equation (6) is then simplified and is written as:

$$\frac{dx}{dt} = C \cdot 10^{-3} \cdot x^{-1/2}. \qquad (7)$$

Solving Eqs. (7), we derive

$$t = C^{-1} \cdot 10^3 x^{3/2}. \qquad (8)$$

Transforming Eq. (8), we have

$$x = C^{2/3} \cdot 10^{-2} t^{2/3}. \qquad (9)$$

Differentiating Eq. (9) and using the definition of the Hubble constant, we can easily find its expression as

$$H = \frac{\dot{x}}{x} = t^{-1}. \qquad (10)$$

Using an explicit form of the variable x, we derive the following dependence of the scaling factor on time:

$$a = C^{2/3} \cdot 10^{-2} a_0 t^{2/3}. \qquad (11)$$

2. Growth of perturbation density in the universe

Let us write a general nonrelativistic equation describing the development of perturbation density of baryon matter [1, 4] as

$$\ddot{\delta} + 2H\dot{\delta} + (v_s^2 k^2 - 4\pi G\rho_b)\delta = 0, \qquad (12)$$

where v_s is the velocity of sound in baryon matter, k is the wave vector, and ρ_b is the density of baryon matter. For its further analysis, two notes should be made.

Two terms are present in the parentheses in Eq. (12). The first term describes the internal energy of baryon matter, and the second – its external (gravitational) energy. In this case, the relation between these energies changes in the course of evolution of the Universe.

Further, strictly speaking, it is not only the equation of state of nonbaryon matter that changes. So does that of baryon matter in the course of evolution of the Universe. Hence, the relation for baryon-matter density is also not constant but depends on time.

In the very early Universe, matter is of relativistic nature, therefore, $v_s \sim 1$; at later stages, matter becomes nonrelativistic, hence $v_s \to 0$. In addition, we take into account the fact that the wave vector is decreased as $k^2 \propto a^{-2} = \frac{10^4}{C^{4/3} \cdot a_0^2} t^{-4/3}$ according to Eq. (11) and condition $a_0 = t_0$.

The law of conservation (3) with allowance for the presence of not only nonbaryon but also baryon matter is generalized as

$$(\dot{\rho}_{nb} + \dot{\rho}_b)a = -3[(\rho_{nb} + \rho_b) + (p_{nb} + p_b)]\dot{a}. \qquad (13)$$

However, since baryon matter and nonbaryon substance do not interact with each other, the variables entering the equation are independent. Thus, for baryon matter, we derive the following evolutionary equation:

$$\dot{\rho}_b = -\mathrm{K} \cdot H\rho_b, \qquad (14)$$

where K is the coefficient depending on the equation of state of baryon matter (for relativistic gas K = 4 , for dust K = 3). The solution to the equation derived with allowance for Eq. (10) is written as $\rho_b = \hat{\rho}_0\, t^{-\mathrm{K}}$, where $\hat{\rho}_0 = const$.

Substituting these values into Eq. (12), we have a differential equation of the second order with the variable coefficients

$$\ddot{\delta} + P(t)\dot{\delta} + Q(t)\delta = 0, \qquad (15)$$

which are

$P(t) = 2t^{-1}$, $Q(t) = \frac{10^4}{C^{4/3} \cdot t_0^2} t^{-4/3} - 4\pi G\hat{\rho}_0 t^{-K}$, respectively.

Let us study the evolution of baryon matter within the time interval $t_1 = 10^{-36} < t < t_2 = 10^{-6}$ s. Justification of such statement is due to the fact that one of the most important problems in modern cosmology is investigation into the formation of baryon-matter perturbations at the earliest stages of the Universe [8]. Therefore, let us consider the behavior of the function $t_1 \sim 10^{-36} c$ characteristic of the very early Universe, assuming $K = 4$ and using the comparison theorem for finding the maximum allowed value of z.

The choice of the lower limit is due to the fact that the stage of production of baryon matter starts at the instant of time $t_1 = 10^{-36}$ s. The upper limit

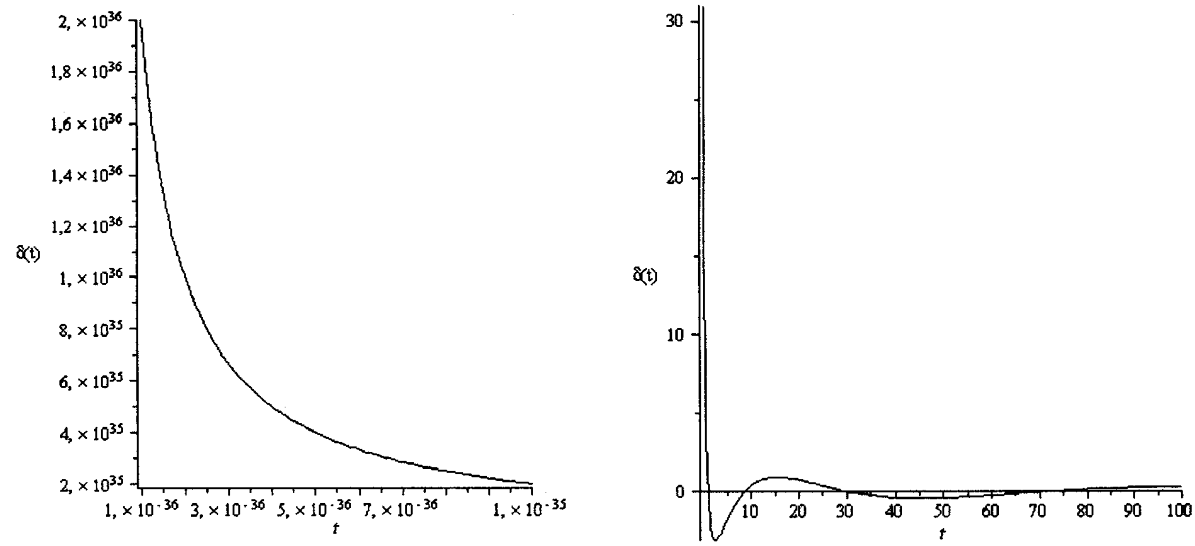

Figure 1.

$t_2 = t_0 \sim 10^{-6}$ s is specified based on the estimation of termination of the hadrons era [1].

2.1 Let the condition $\frac{10^4}{C^{4/3} \cdot t_0^{\,2}} t^{-\frac{4}{3}} >> 4\pi G\hat{\rho}_0 t^{-4}$ meaning predominance of the kinetic energy of baryon matter over the potential energy be valid. In this case, neglecting the second term in the $Q(t)$ of Eq. (15) we find the solve that

$$\delta(t) = C_1 \frac{\sqrt{9it^{2/3}\frac{10^4}{C^{4/3} \cdot t_0^{\,2}} + 27t\sqrt{\left(\frac{10^4}{C^{4/3} \cdot t_0^{\,2}}\right)^3 + i + t^{1/3}\sqrt{\frac{10^4}{C^{4/3} \cdot t_0^{\,2}}}}}}{t\sqrt{i - 3t^{1/3}\sqrt{\frac{10^4}{C^{4/3} \cdot t_0^{\,2}}}}} e^{3t^{1/3}\sqrt{\frac{10^4}{C^{4/3} \cdot t_0^{\,2}}}} +$$

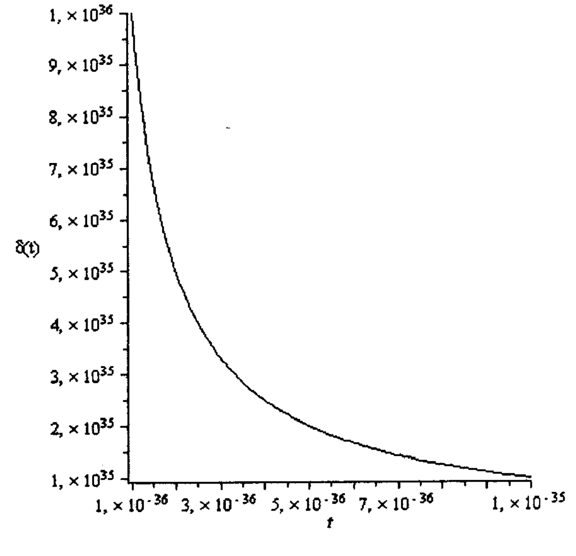

Figure 2.

$$+C_2 \frac{\sqrt{9it^{2/3}\frac{10^4}{C^{4/3} \cdot t_0^{\,2}} - 27t\sqrt{\left(\frac{10^4}{C^{4/3} \cdot t_0^{\,2}}\right)^3 + i - 3t^{1/3}\sqrt{\frac{10^4}{C^{4/3} \cdot t_0^{\,2}}}}}}{t\sqrt{i + 3t^{1/3}\sqrt{\frac{10^4}{C^{4/3} \cdot t_0^{\,2}}}}} e^{-3t^{1/3}\sqrt{\frac{10^4}{C^{4/3} \cdot t_0^{\,2}}}} \quad (16)$$

2.2 Let the inverse condition $\frac{10^4}{C^{4/3} \cdot t_0^{\,2}} t^{-\frac{4}{3}} << 4\pi G\hat{\rho}_0 t^{-4}$, which occurs in the case of predominance of the potential energy over the kinetic one in baryon matter, be satisfied. The solve follows that

$$\delta(t) = C_1 \sinh\frac{\sqrt{4\pi G\hat{\rho}_0}}{t} + C_2 \cosh\frac{\sqrt{4\pi G\hat{\rho}_0}}{t}. \quad (17)$$

2.3 Finally, let us consider the case where the relationship $\frac{10^4}{C^{4/3} \cdot t_0^{\,2}} t^{-\frac{4}{3}} \approx 4\pi G\hat{\rho}_0 t^{-4}$ is valid. From a physical view point, the relationship describes a stable state of perturbation in baryon matter. Thus, solving Eq. (15) becomes

$$\delta(t) = C_1 + C_2 t^{-1}. \quad (18)$$

In so doing, the density changes according to the law $\delta(t) = C_1 + C_2 t^{-1}$, meaning a intense decreasing as compared to the power ($\delta(t) \sim t$) growth rate of baryonmatter perturbations within the conventional Friedmann cosmology with the stationary equation of state for nonbaryon matter [1].

Conclusion

Analyzing these graphs we see that at the matter borning epoch the baryonic matter has the superperturbed state. This state could equate at the Big Bang process, after which the amplitude was radically decreased.

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Резюме

Исследован рост плотности возмущений в барионном субстрате, обусловленный нестационарным и нелинейным характером уравнения состояния небарионной материи во Вселенной. Показано, что при таком подходе они уменьшаются от сверхвозмущенного состояния, т. е. от Большого Взрыва.

Резюме

Стационарлы емес және сызықты емес күй теңдеуімен сипатталатын, барионды емес материяның ерте Ғаламдағы барионды материяның ұйтқуларының дамуына әсерін зерттелді. Бұл ұйтқулар өте үлкен дәрежеден Үлкен Жарылыстан бастап түсетіндігін көрсетті.

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г. Алматы *Поступила 05.10.09 г.*