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ANALYSIS OF KINEMATIC OF THE SPECIAL MECHANISMS OF THE HIGH CLASSES

Annotation

In this paper the exact method of kinematic analysis of the special mechanisms of the high classes which determines the position and velocity explicitly. The proposed approach allows us to develop unique software for the design of these mechanisms. Theoretical results are confirmed by synthesis the transmission mechanism of the IV class with specified geometrical parameters.

Ключевые слова: кинематический анализ, механизм высоких классов, програмное обеспечение, механизм IV класса, геометрический параметр.

Кілт сөздер: кинематикалық талдау, жоғары класс тетігі, бағдарламалық қамтамасыз ету, IV класс тетігі, геометриялық параметр.

Keywords: kinematic analysis of the mechanism of high-class software mechanism IV class of geometrical parameters.

Introduction

Mechanisms of the high classes (MHC) have unique characteristics. They can withstand the huge power loads, raise large masses, control by many parameters and more. Therefore, c

lassification of mechanisms on second and high classes proposed by I.I. Artobolevsky Ref [1] , U.A. Dzholdasbekov Ref [2] requires clarification for the complete solution of the problems of kinematics, kinetostatics and dynamics.

Mechanisms of general purpose directly include arbitrary mechanisms with the Assur`s group of the high classes. The transmission, directive and transfer mechanisms of the high classes are mechanisms of special purpose. This classification allows to develop a general theory of the mechanism of the first groups to identify their functional capacity, and for the second - the theoretical basis of calculation of the kinematic, force and dynamic parameters and the design of mechanisms with given certain characteristics.

The research of the kinematics of arbitrary mechanisms of the high classes is reduced to solving algebraic equations of the sixth and higher degree. Therefore, these equations can not be solved analytically. Scientists have developed the approximate and numerical methods for solving equations of kinematics. The proposed approach allows us to solve the problem of the kinematics of the special mechanisms of the high classes in an explicit analytic form. In the future, these results can be used in the design of lifting devices, the working bodies of earth-moving machines, robots with closed contours, the individual devices of aircraft.

Kinematic analysis of mechanisms of the high classes

For kinematic analysis of arbitrary mechanisms of the high classes use the approach Ref [3].

Statement of the problem. Let MHC consists of n mobile units. The numbers of output units vary from *1* to $n-m$, and of the input with respect to the from the $j=n-m+1$ to *n*. Then vector equation of independent closed countours MHC fixed coordinate system has the form:

$$\sum_i \vec{l}_i + \sum_j \vec{l}_j + \vec{l}_0 = 0 \qquad (1)$$

where i, j are indexes, whose values range over the number respectively initial and input units, belonging to the considered k - th countour, where $\vec{l}_0$ is the vector of the fixed link. The equation (1) can be rewritten as

$$\sum_{i=0}^{n} \vec{l}_i = 0 \text{ or } \sum_{i=0}^{n} l_i \vec{e}_i = 0$$

where l_i is the absolute value and $\vec{e}_i$ is unit vector of the *i*-th vector in the corresponding independent contour.

Then the vector equations of the form (1) for a mechanism with a group of Assyrian group of the IV class of the second order with rotational pairs and two degrees of freedom are represented as (Fig. 1)

$$\left.\begin{array}{l} \vec{l}_1 + \vec{l}_2 + \vec{l}_3 + \vec{l}_5 - \vec{l}_6 - \vec{l}_0 = 0 \\ \vec{l}_1^* + \vec{l}_3^* + \vec{l}_4 + \vec{l}_5 - \vec{l}_6 - \vec{l}_0) \end{array}\right\} \qquad (2)$$

where $\vec{l}_i$ -vectors of input links $i = 1,2,3,4$ $\vec{l}_j$, $\vec{l}_j$ vectors of output links $j = 5,6.$ The asterisk "*" means the membership of the vector to $\vec{l}_i^*$ to the *i*-th basis link.

The equations (1) and (2) can be represented in projections on the axis of the steady coordinate systems by multiplying them by the corresponding unit vectors $\vec{i}_0$ and $\vec{j}_0$.

The problem. Find the positions of the output links for the transmission mechanism of the IV class with the given desired laws of motion of the input links and geometric parameters:

$$\varphi_i = \varphi_i(\varphi_5, \varphi_6, P), \; i = 1,2,3,4 \qquad (3)$$

where $\mathrm{P} = \left\{ l_1, l_1^*, l_2, l_3, l_3^*, l_4, l_5, l_6, \alpha_1, \alpha_3, l_0, \varphi_0 \right\}.$

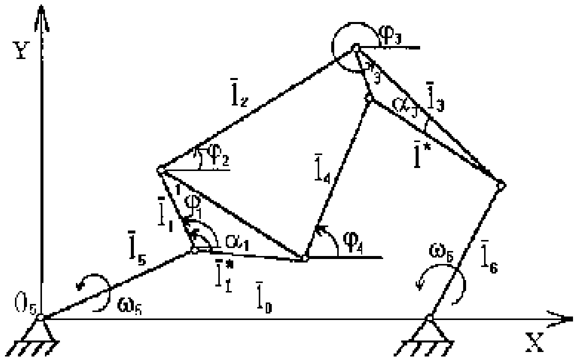

Fig. 1 – Mechanism of IV class

We will follow to the analytic method of the kinematic analysis of arbitrary MHC developed [3].

Solution to the problem. Vector equation (2) MHC represented for k = 2 in the scalar form (in projections) as a system of trigonometric equations

$$\left.\begin{aligned} &l_1\cos\varphi_1 + l_2\cos\varphi_2 + l_3\cos\varphi_3 + l_5\cos\varphi_5 - l_6\cos\varphi_6 - l_0\cos\varphi_0 = 0\\ &l_1\sin\varphi_1 + l_2\sin\varphi_2 + l_3\sin\varphi_3 + l_5\sin\varphi_5 - l_6\sin\varphi_6 - l_0\sin\varphi_0 = 0\\ &l_1^*\cos(\varphi_1-\alpha_1) + l_3^*\cos(\varphi_3-\alpha_3) + l_4\cos\varphi_4 + l_5\cos\varphi_5 - l_6\cos\varphi_6 - l_0\cos\varphi_0 = 0\\ &l_1^*\sin(\varphi_1-\alpha_1) + l_3^*\sin(\varphi_3-\alpha_3) + l_4\sin\varphi_4 + l_5\sin\varphi_5 - l_6\sin\varphi_6 - l_0\sin\varphi_0 = 0 \end{aligned}\right\} \quad (4)$$

with unknown $\varphi_1, \ldots, \varphi_4$ (Fig. 1).

The vector method of eliminating variables reduces the number of unknown variables. Following the vector approach from the first equation of vector equations (3) we find $\vec{l_2}$ and from the second $\vec{l_4}$ that is,

$$\left.\begin{aligned} \vec{l}_2 &= -(\vec{l}_1 + \vec{l}_3) - (\vec{l}_5 - \vec{l}_6 - \vec{l}_0)\\ \vec{l}_4 &= -(\vec{l}_1^{\,*} + \vec{l}_3^{\,*}) - (\vec{l}_5 - \vec{l}_6 - \vec{l}_0) \end{aligned}\right\} \quad (5)$$

Next, using the scalar product, we have

$$\left.\begin{aligned} (\vec{l}_2)^2 &= (\vec{l}_1 + \vec{l}_3)^2 + 2(\vec{l}_1 + \vec{l}_3)(\vec{l}_5 - \vec{l}_6 - \vec{l}_0) + (\vec{l}_5 - \vec{l}_6 - \vec{l}_0)^2\\ (\vec{l}_4)^2 &= (\vec{l}_1^{\,*} + \vec{l}_3^{\,*})^2 + 2(\vec{l}_1^{\,*} + \vec{l}_3^{\,*})(\vec{l}_5 - \vec{l}_6 - \vec{l}_0) + (\vec{l}_5 - \vec{l}_6 - \vec{l}_0)^2 \end{aligned}\right\} \quad (6)$$

After simple transformations of the system (6) to find the vectors $\vec{l_1}$ and $\vec{l_1}^*$ we obtain a special system of scalar equations of the following form:

$$
\begin{aligned}
&2(\vec{l}_3+\vec{l}_5-\vec{l}_6-\vec{l}_0)\vec{l}_1=\vec{l}_2^{\,2}-\vec{l}_1^{\,2}-\vec{l}_3^{\,2}-(\vec{l}_5-\vec{l}_6-\vec{l}_0)^2-2\vec{l}_3(\vec{l}_5-\vec{l}_6-\vec{l}_0)\\
&2(\vec{l}_3^{\,*}+\vec{l}_5-\vec{l}_6-\vec{l}_0)\vec{l}_1^{\,*}=\vec{l}_2^{\,2}-\vec{l}_1^{\,2}-\vec{l}_3^{\,*2}-(\vec{l}_5-\vec{l}_6-\vec{l}_0)^2-2\vec{l}_3^{\,*}(\vec{l}_5-\vec{l}_6-\vec{l}_0)
\end{aligned}
\qquad (7)
$$

It should be noted that $\vec{l}_1^{\,*}$ is associated with $\vec{l}_1$ and $\vec{l}_3^{\,*}$ with $\vec{l}_3$, where l_i is module of the vector $\bar{l}_i$ the scalar product of vectors $\vec{l}_i$ are $\vec{l}_i\cdot\vec{l}_i$ and $\left(\vec{l}_i+\vec{l}_j\right)^2$. In the system (7) the angular coordinates φ_2 and φ_4 of vectors $\vec{l}_2$ and $\vec{l}_4$ are eliminate.

Thus we have obtained a special system of algebraic equations, in which the number of equations matches the number of independent closed vector contours.

$$
\begin{aligned}
&2\left(l_3x_3+l_5x_5-l_6x_6-l_0\right)l_1x_1+2\left(l_3y_3+l_5y_5-l_6y_6\right)l_1y_1=l_2^2-l_1^2-l_3^2-\\
&-\left(l_5x_5+l_6x_6-l_0\right)^2-\left(l_5y_5-l_6y_6\right)^2-2l_3x_3\left(l_5x_5-l_6x_6-l_0\right)-2l_3y_3\left(l_3y_3-l_5y_5\right)
\end{aligned}
$$

(8)

Where

$$x_1=\cos(\varphi_1), y_1=\sin(\varphi_1),$$

$$
\begin{aligned}
x_1^* &= x_1\cos\alpha_1+y_1\sin\alpha_1,\\
y_1^* &= x_1\sin\alpha_1-y_1\cos\alpha_1,
\end{aligned}
$$

$$x_3=\cos(\varphi_3), y_3=\sin(\varphi_3),$$

$$
\begin{aligned}
x_3^* &= x_3\cos\alpha_3+y_3\sin\alpha_3y_3,\\
y_3^* &= x_3\sin\alpha_3-y_3\cos\alpha_3.
\end{aligned}
$$

$$x_4=\cos(\varphi_4), y_4=\sin(\varphi_4),$$

$$x_5=\cos(\varphi_5), y_5=\sin(\varphi_5),$$

$$x_6=\cos(\varphi_6), y_6=\sin(\varphi_6)\cdot$$

Sinchev B. has developed various methods of solving systems of nonlinear equations (8). It is further proposed an accurate method for solving the kinematics of special mechanisms of the high classes.

An accurate method of kinematic analysis of mechanisms of the high classes

We present the concept of a special mechanism under the classification.

Definition. A mechanism intended for an approximate replay of the given relationship between the angular or linear displacement of input and output units is called a transmission mechanism.

This mechanism belongs to mechanisms for special purposes. Thus, the transmission mechanism shall have certain property and this property is described by the following equation:

$$\vec{e}_3 = \vec{f}(\vec{e}_5, \vec{e}_6)\cdot \tag{9}$$

Here $\vec{f}$ is given vector function, $\vec{e}_3$ is the unit vector of output link 3, $\vec{e}_5, \vec{e}_6$ are unit vectors of output links 5 and 6.

We transform (8) to the following convenient form:

$$\left.\begin{array}{l} A_1 x_1 + B_1 y_1 = D \\ A_2 x_1 + B_2 y_1 = D \\ x_1^2 + y_1^2 = 1 \end{array}\right\} \tag{10}$$

where

$$A_1 = 2(l_3 x_3 + l_5 x_5 - l_6 x_6 - l_0) l_1,$$
$$B_1 = 2(l_3 y_3 + l_5 y_5 - l_6 y_6 - l_0) l_1,$$

$$D_1 = l_2^2 - l_1^2 - l_3^2 - (l_5 x_5 - l_6 x_6 - l_0)^2 - (l_5 y_5 - l_6 y_6)^2 -$$
$$- 2 l_3 x_3 (l_5 x_5 - l_6 x_6 - l_0) - 2 l_3 y_3 (l_5 y_5 - l_6 y_6),$$

$$A_2 = 2\left(l_3^* x_3 + l_5 x_5 - l_6 x_6 - l_0\right) l_1^* \cos\alpha_1 - 2\left(l_3^* y_3 + l_5 y_5 - l_6 y_6\right) l_1^* \sin\alpha_1 ,$$
$$B_2 = 2\left(l_3^* x_3 + l_5 x_5 - l_6 x_6 - l_0\right) l_1^* \sin\alpha_1 + 2\left(l_3^* y_3 + l_5 y_5 - l_6 y_6\right) l_1^* \cos\alpha_1 ,$$

$$D_2 = l_4^2 - l_1^{*2} - l_3^{*2} - (l_5 x_5 - l_6 x_6 - l_0)^2 - (l_5 y_5 - l_6 y_6)^2 -$$
$$- 2 l_3^* (l_5 x_5 - l_6 x_6 - l_0) x_3^* - 2 l_3^* (l_5 y_5 - l_6 y_6) y_3^* .$$

The system (10) is equivalent to the relation

$$(A_1 D_2 - A_2 D_1)^2 + (D_1 B_2 - D_2 B_1)^2 + (D_1 B_2 - D_2 B_1)^2 - (A_1 B_2 - A_2 B_1) = 0 \tag{11}$$

which is an algebraic equation of sixth degree with respect to x_3 or y_3 for the mechanism of the IV class of general purpose. The equation (11) is solved only by numerical methods.

Therefore, in the case the mechanism of special purposes, ie, the transmission mechanism should take into account the relation (9). For simplicity, we assume that $l_6 = 0$. Then, the mechanism with two degrees of freedom is the one with one degree of freedom.

Without loss of generality, we can assume that (9) has the explicit form:

$$\varphi_3 = \frac{1}{3}\varphi_5 \qquad (12)$$

First, we substitute into the known coefficients $A_1, B_1, D_1,\ A_2, B_2, D_2$ the angular coordinate (12), then the function x_3 and y_3 will acquire the form:

$$x_3 = \cos(\frac{1}{3}\varphi_5), y_3 = \sin(\frac{1}{3}\varphi_5)\ , \qquad (13)$$

Thus, only the coefficients $A_1, B_1, D_1,\ A_2, B_2, D_2$ only in the explicit form depend on the generalized angular coordinate φ_5 and on lengths of links. Then from (10) we find

$$x_1 = \frac{D_1 B_2 - D_2 B_1}{A_1 B_2 - A_2 B_1}, y_1 = \frac{A_1 D_2 - A_2 D_1}{A_1 B_2 - A_2 B_1}, \qquad (14)$$

Substituting these values (14) into (4) and taking into account (12), we find the trigonometric functions x_2, y_2, x_4, y_4.

This is the essence of the method for determining the positions of links special mechanism of IV class. This precise method of kinematic analysis is easily extended to other mechanisms of the high classes.

Designing the special mechanism of the high class

The results are verified by the example of the transmission mechanism IV class which it provides the connection between the angular coordinates of the input and output links in the form of (12) (Fig. 2).

This special mechanism with rotary pairs has given properties by classification and connection (12). This transmission mechanism IV class has the following geometric dimensions:

$l_0 = OD = 51mm,\ l_1 = AB = 26mm,\ l_1^* = AC = 19mm,\ l_2 = BF = 77mm,\ l_3 = FD = 99mm,$
$l_3^* = ED = 109mm,\ l_4 = EC = 56mm,\ l_5 = OA = 26mm,\ l_6 = 0mm,\ \alpha_1 = 46^0, \alpha_3 = 8^0.$

Generalized coordinate of input link 5 depends on the time. Then the equation of connection (12) has the form: $\varphi_3(t) = \frac{1}{3}\varphi_5(t)$. Other angular coordinates given by formula (14) and the initial system (4), which is already a linear system. Speed of links are found by differentiating the angular coordinates, the kinetic and potential energies - by known formulas. Then the dynamic model of the mechanism is based on an operator Lagrange II rind. Forces of reaction in the joints of the mechanism are known system.

Fig. 2 – Transmission mechanism of IV class

Conclusion

We note that the above classification and developed an accurate method of kinematic analysis for the first time allowed to find the positions of links of mechanism of the high classes in explicit analytic form. This approach can be easily extended to the directive and the transfer mechanisms of high classes. This approach will develop a universal software for design of mechanisms of the high classes for devices and machines. Synthesized transmission mechanism IV class.

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Резюме

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ЖОҒАРЫ КЛАСТАРДЫҢ АРНАЙЫ ТЕТІКТЕРІ

КИНЕМАТИКАСЫНЫҢ ТАЛДАУЫ

Жоғары кластардың арнайы тетіктерінің кинематикалық талдауының нақты әдісі қарастырылған, айқын түрдегі жағдайы мен жылдамдықтары анықталған. Ұсынылып отырған тәсіл осы тетіктерді жобалау үшін бірегей бағдарламалық қамтамасыз етуді әзірлеуге мүмкіндік береді. Теориялық нәтижелері геометриялық параметрмен белгіленген IV класты ауыстыратын механизм синтезімен дәлелденген.

Кілт сөздер: кинематикалық талдау, жоғары класс тетігі, бағдарламалық қамтамасыз ету, IV класс тетігі, геометриялық параметр.

Резюме

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АНАЛИЗ КИНЕМАТИКИ СПЕЦИАЛЬНЫХ МЕХАНИЗМОВ

ВЫСОКИХ КЛАССОВ

В статье рассматривается точный метод кинематического анализа специальных механизмов высоких классов, определяются положения и скорости в явном виде.

Предлагаемый подход позволит разработать уникальное программное обеспечение для проектирования этих механизмов. Теоретические результаты подтверждены синтезом передаточного механизма IV класса с заданными геометрическими параметрами.

Ключевые слова: кинематический анализ, механизм высоких классов, програмное обеспечение, механизм IV класса, геометрический параметр.

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