

## THE DYNAMICS OF THE STRUCTURED PARTICLES

The mechanics of the structured particles develops. It was shown that the expansion of the classical mechanics arising at replacement of material points on structured systems leads to its consent with thermodynamics and allows using the classical mechanics for studying of processes of evolution of physical objects in the nature.

The classical mechanics meets with the big difficulties if one analyzes the mechanisms of structures creations, i.e. all those type of dynamics, which are connected with dissipation, and an openness of systems. The most vivid example of such difficulties is the irreversibility problem. Since L. Boltzmann the interest to this problem is not decreasing [1-3].

In the basis of the existing explanations of irreversibility the property of mixing of the Hamilton's systems and averaging of phase space on physically small volume are used. The explanation of the irreversibility is based on postulation of casual fluctuations [2, 3]. Thus the casualty is the irreversibility reason. But it means absence of determinism of the nature. On other side if the world is cognizable, the deterministic mechanism of irreversibility exists. Our research has shown that such mechanism exists in the frameworks of the expanding of the classical mechanics. The expansion consists in replacement of model of system in the form of set of material points on model of system in the form of the set of structured particles [10]. The dynamics of such system can be described with the help of the motion equation of the structured particles consisting of potentially interacting material points

or equilibrium subsystems (ES). Such model possesses the big generality because the wide range of nonequilibrium systems can be presented in the form of ES set. The analysis of the submitted model of the nonequilibrium systems is constructed under the following conditions: 1). Energy of ES should be presented as the sum of internal energy and energy of ES motion; 2). the each element of system should belong to one ES; 3). the ES are in equilibrium during all time. The first condition is necessary for introduction the internal energy into the description of systems dynamics as the key parameter characterizing the energy exchanges between ES. The second condition allows avoiding the difficulties arising due to mixing of particles between different ES. This condition is equivalent to the weak enough ES interaction that does not disturb its equilibrium.

The task of this work is to show how and why the irreversibility are appearing in the system consisting of the structural particles. For this purpose we will analyze distinguish between the models of system from material points which and the system form of the ES. We will also show how it is possible to obtain the motion equation for ES and how Lagrange, Hamilton, Liouville equations for ES follow from it. We will explain interrelation of the

classical mechanics with thermodynamics and how it is possible to define entropy in the classical mechanics.

We will show that the description of dynamics of nonequilibrium system in the classical mechanics is possible if this system will be represented as a set of ES consisting of potentially interacting material points and the energy of ES will be presented as the sum of the motion energy and internal energy. Basing on such model we obtain the equation of ES interaction from the law of energy conservation. These equations will connect the microstreams of energy caused by pair interactions of particles with macrostreams of energy between ES.

## 2. The equation of ES motion

It is known, that the law of conservation of energy and a principle of the least action follow from Newton's equation. But also it is possible to obtain back Newton's equation from the law of conservation of energy. It is important, because the law of conservation of energy is correct always, but Newton's equation is convenient only for potential forces. Therefore the ES motion equation is necessary to obtain from the energy conservation law, because the change of internal energy of the structured particles is connected with the non-potential forces of a friction. These forces are not taking into account by the Newton's equation.

In this case the motion equations for two ES can be obtained in two stages. At first we obtain the motion equation for the system of material points. After that we obtain the motion equations for two interacting ES. Forces between ES can be obtained from their potential energy of interaction.

Let us firstly show how the motion equation for a system of  $N$  material points (further we will call it particles) with weights  $m = 1$  can be obtained from the energy conservation law [10].

Forces between pairs of particles are central and potential. The systems energy  $E$  is equal to the sum of kinetic energy  $T_N = \sum_{i=1}^N m v_i^2 / 2$ , their potential energy in a field of external forces,  $-U^{env}$  and the potential energy of their interaction-  $U_N(r_{ij}) = \sum_{i=1}^{N-1} \sum_{j=i+1}^N U(r_{ij})$ , where  $r_{ij} = r_i - r_j$ ,  $r_i, v_i$  - are the coordinate and velocity of  $i$ -th particle. Thus,  $E = E_N + U^{env} = T_N + U_N + U^{env} = const$ . By transition to corresponding variables we submitted the systems energy as a sum of CM motion energy and internal

energy. After derivation this energy on time, we will obtain:

$$V_N M_N \dot{V}_N + \dot{E}_N^{ins} = -V_N F^{env} - \Phi^{env}, \quad (1)$$

here  $r_i = R_N + \tilde{r}_i$ ,  $M_N = mN$ ,  $F^{env} = \sum_{i=1}^N F_i^{env}(R_N, \tilde{r}_i)$ ,

$$\dot{E}_N^{ins} = \dot{T}_N^{ins}(\tilde{v}_i) + \dot{U}_N^{ins}(\tilde{r}_i) = \sum_{i=1}^N \tilde{v}_i (m \dot{\tilde{v}}_i + F(\tilde{r}_i)_i),$$

$$v_i = V_N + \tilde{v}_i, \quad F_i^{env} = \partial U^{env} / \partial \tilde{r}_i,$$

$\Phi^{env} = \sum_{i=1}^N \tilde{v}_i F_i^{env}(R_N, \tilde{r}_i)$ ,  $\tilde{v}_i, \tilde{r}_i$  - are the coordinate and velocity of  $i$ -th particle in relative to the system CM,  $V_N, R_N$  - are the coordinate and velocity of the system's CM.

The eq. (1) represents balance of energy of system in a field of external forces. The first term in the left hand side determines change of kinetic energy of system. The second term determines the change of internal energy of system. Because  $\sum_{i=1}^N \tilde{v}_i = 0$ , the change of internal energy will be distinct from zero only when the characteristic scale of inhomogeneity of an external field is commensurable with system scale. In this case depending on a configuration of an external field can vary or a kinetic energy of system rotation or energy of the relative motion of elements. In the both cases the force changing an internal energy is non-potential.

As it follows from the Newton equation the particle motion is defined by the work of potential forces, which transform the energy of an external field into the kinetic energy. But the system motion is defined by work of potential and non-potential forces that transform the energy of an external field into kinetic energy of the system motion and into internal energy accordingly. By multiplying the eq. (1) on  $V_N$  and dividing on  $V_N^2$ , we obtained [10]:

$$M_N \dot{V}_N = -F^{env} - \alpha_N V_N, \quad (2)$$

where  $\alpha_N = (\Phi^{env} + \dot{E}_N^{ins}) / V_N^2$  is friction coefficient. Let us call eq. (2) as generalized Newton equation (GNE) for the structured particle. The first term in the right hand side defines system acceleration, and the second term defines change of its internal energy. The GNE is reduced to the Newton equation if one neglects the relative motion of elements, i.e. when the internal energy does not change. In this case the dynamics of system is similar to the reversible dynamics of an elementary particle.

The eqs. (1, 2) can be obtained also by multiplying the Newton's equation on corresponding

velocity and then summarizing all equations. But if we simply summarize Newton's equations the non-potential forces will be lost and the right hand side will be equal to the potential forces. Thus the Newton's equation does not define full dynamics of system since it does not include the non-potential forces. It confirms Leibnitz idea that *vise viva*, i.e. energy is a fundamental parameter but not momentum or force [8].

Let us explain how to obtain the ES interaction equations. For this purpose we take the system consisting of two ES- $L$  and  $K \cdot L$  - is a number of elements in the  $L$ -ES, and  $K$  - in  $K$ -ES, i.e.  $L+K=N$ . Let  $LV_L + KV_K = 0$ , where  $V_L$  and  $V_K$  are velocities of  $L$  and  $K$  ES. By derivations energy of system on time, we will obtain:  $\sum_{i=1}^N v_i \dot{v}_i + \sum_{i=1}^{N-1} \sum_{j=i+1}^N F_{ij} v_{ij} = 0$ , where  $F_{ij} = \partial U / \partial r_{ij}$ . For finding the equation for  $L$ -ES, we gather at the left hand side only the terms defining the change of kinetic and potential energy of interaction of  $L$ -ES elements among themselves. All other terms we displaced into the right hand side and combined the groups of terms in such a way that each group contained of the terms with identical velocities. In accordance with Newton equation, the groups, which contain terms with velocities of the elements from  $K$ -ES, are equal to zero. As a result the right hand side of the equation will contain only the terms which determine the interaction of the elements  $L$ -ES with the elements  $K$ -ES. Thus we will have:

$$\sum_{i_L}^L v_{i_L} \dot{v}_{i_L} + \sum_{i_L=1}^{L-1} \sum_{j_L=i_L+1}^L F_{i_L j_L} v_{i_L j_L} = \sum_{i_L=1}^L \sum_{j_K=1}^K F_{i_L j_K} v_{j_K}$$

where double indexes are entered for a designation of an accessory of a particle to corresponding subsystem. If we will make replacement:  $v_{i_L} = \tilde{v}_{i_L} + V_L$ , where  $\tilde{v}_{i_L}$  - is a velocity of  $i_L$  - particle in relative to CM of  $L$ -ES, then we obtain the eq. for  $L$ -ES. The eq. for  $K$ -ES can be obtained in the same way. As a result we will have [7]:

$$V_L M_L \dot{V}_L + \dot{E}_L^{ins} = -\Phi_L - V_L \Psi \quad (3)$$

$$V_K M_K \dot{V}_K + \dot{E}_K^{ins} = \Phi_K + V_K \Psi \quad (4)$$

where

$$\dot{E}_L^{ins} = \sum_{i_L=1}^{L-1} \sum_{j_L=i_L+1}^L \tilde{v}_{i_L j_L} [m \dot{\tilde{v}}_{i_L j_L} / L + F_{i_L j_L}], \quad \Psi = \sum_{i_L=1}^L F_{i_L}^K,$$

$$M_L = mL, \quad \Phi_L = \sum_{i_L=1}^L \tilde{v}_{i_L} F_{i_L}^K, \quad \Phi_K = \sum_{i_K=1}^K \tilde{v}_{i_K} F_{i_K}^L,$$

$$F_{i_L}^K = \sum_{j_K=1}^K F_{i_L j_K}, \quad F_{j_K}^L = \sum_{i_L=1}^L F_{i_L j_K}, \quad M_K = mK,$$

$$\dot{E}_K^{ins} = \sum_{i_K=1}^{K-1} \sum_{j_K=i_K+1}^K \tilde{v}_{i_K j_K} [m \dot{\tilde{v}}_{i_K j_K} / K + F_{i_K j_K}].$$

The eqs. (3,4) are the ES interaction equations. They describe energy exchange between ES. Independent variables of ES are macroparameters and microparameters. Macroparameters are coordinates and velocities of ES motion. Microparameters are coordinates and velocities of ES elements. These equations bind together two types of the description: on the macrolevel and on the microlevel. The description on the macrolevel determines of ES dynamics as a whole and on the microlevel determines dynamics of ES elements. The potential force,  $\Psi$ , determines the motion of ES as a whole. This force is the sum of the potential forces acting on elements of one ES from another. The non-potential forces which determined by the terms  $\Phi_L$  and  $\Phi_K$ , will transform the motion energy of ES into the internal energy as a result of chaotic motion of elements one ES in the field of the forces of another ES. They are dependent on velocities and cannot be expressed as a gradient from any scalar function. These forces are equivalents to dissipative forces. As well as in case of system in an external field these forces are distinct from zero when the characteristic scale of inhomogeneity of a field of forces of one ES is commensurable with scale of another.

The ES motion equations corresponding to the eqs. (3,4) can be written [10]:

$$M_L \dot{\tilde{V}}_L = -\tilde{\Psi} - \alpha_L \tilde{V}_L \quad (5)$$

$$M_K \dot{\tilde{V}}_K = \tilde{\Psi} + \alpha_K \tilde{V}_K \quad (6)$$

where  $\alpha_L = (\dot{E}_L^{ins} + \Phi_L) / V_L^2$ ;  $\alpha_K = (\Phi_K - \dot{E}_K^{ins}) / V_K^2$ .

The eqs. (5, 6) are GNE for ES. The second terms in the right hand side of the equations determine the force changing internal energy of ES. This force is equivalent to the friction force. The efficiency of transformation of energy of relative motion of ES into internal energy are determined by the factors  $\alpha_L, \alpha_K$ . For to find the solution of the eqs. (5,6) it is necessary to add them by the equations for microparameters which are the Newton's equations for material points of ES. If the relative velocities of ES elements are equal to zero the force of friction is also equal to zero and the GNE will be transformed into the Newton equation for two bodies. For

example it is possible when distances between ES are great enough.

### 3. The generals of Lagrange, Hamilton and Liouville equations for ES

The Hamilton principle for material points is deduced from differential D'Alembert principle with the help of the Newton equation [8]. For this purpose the integral on time from the virtual work  $\delta w^e$  made by effective forces is equated to a zero. Integration on time is carried out provided that external forces possess power function. It means that the canonical principle of Hamilton is fair only for cases when  $\sum F_i \delta R_i = -\delta V$  where  $i$  - is a number of particles, and  $F_i$  - is force acting on this particle. But for ES it is impossible to demand performance of a condition of conservatism of forces because the non-potential forces exist. Non-potential forces change ES internal energy. Therefore Lagrange, Hamilton, Liouville equations for structural particles must be deduced basing on ES motion equations. Liouville equation for ES looks like [5]:

$$df/dt = -f \sum_{L=1}^R \partial F_L / \partial V_L \quad (7)$$

Here  $f$  is a distribution function for a set of ES,  $L = 1, 2, 3 \dots R$ ,  $F_L$  is a non-potential force acting on ES,  $V_L$  is a velocity of  $L$ -ES. These forces can be found with the help of eqs. (5, 6). The right hand side of the eq. (7) is not equal to zero since forces of ES interaction depend on velocities of elements.

The state of system as a set of ES can be determined by the point in the phase space which consists of  $6R-1$  coordinates and momentums of ES, where  $R$  is a number of ES. Let us call this space as S-space to distinguish it from usual phase space for material points. The S-space unlike usual phase space is compressible though total energy of all elements is a constant. The rate of compression of S-space is determined by the rate of transformation of motion energy of the ES into their internal energy. The volume of compression of S-space is determined by energy of the ES motion. The impossibility of return of internal energy of ES in its energy of motion is caused by impossibility of change of momentum ES due to the motion of its material points.

### 4. The equations of interaction of systems and thermodynamics

Let us consider how thermodynamics can follow from the classical mechanics [9]. According to the basic equation of thermodynamics the work of

external forces acting on the system are splitting on two parts. The first part is connected to reversible work. The change of the motion energy of system as whole can be put in conformity for this energy part. The second part of energy will go on heating. It is connected with the internal degrees of freedom of system. The internal energy of ES corresponds to this part.

Let us take the motionless nonequilibrium system consisting of " $R$ " of ES. Each of ES consists of great number of elements  $N_L \gg 1$ , where  $L = 1, 2, 3 \dots R$ ,

$N = \sum_{L=1}^R N_L$ . Let  $dE$  is the change of the energy of a system (do not confuse  $E$  with the internal energy of ES -  $E^{ins}$ ). It is known from the thermodynamics:  $dE = dQ - PdY$ . Here, according to common terminology,  $E$  is the energy of a system;  $Q$  is the thermal energy;  $P$  is the pressure;  $Y$  is the volume. The equation of a systems interaction also includes two types of energy. The one part goes to the change of ES motion. The other part changes the internal energy. The interrelation of classical mechanics and thermodynamics in more details is considered in [10].

Entropy can be entered into the classical mechanics as the rate characterizing increasing of the internal energy ES at the expense of energy of their motion. Then the entropy increasing will be defined so [7, 10]:

$$\Delta S = \sum_{L=1}^R \left\{ N_L \sum_{k=1}^{N_L} \left[ \sum_s F_{ks}^L v_k dt \right] / E_L \right\} \quad (8)$$

Here  $E_L$  is the kinetic energy of  $L$ -ES;  $N_L$  is the number of elements in  $L$ -ES;  $L = 1, 2, 3 \dots R$  is the number of ES;  $s$  - is a number of the external elements which interact with elements  $k$  belonging to the  $L$ -ES;  $F_{ks}^L$  is a force, acted on  $k$ -element;  $v_k$  - is a velocity of the  $k$ -element.

### 5. Conclusion

The obtained results lead to the following conclusions. System evolution in non-homogeneity space is determined by external force. The external force breaks up on potential and non-potential parts. The motion energy of the system changes by the potential component of the force. The system internal energy changes by the work of non-potential part. This work is distinct from zero if the scale of heterogeneity of external forces is less or commensurable with the system scale.

Potential and non-potential forces define evolution of the closed nonequilibrium system in the

homogeneous space, which represented by a set of ES, as well as in the case of a system motion in an external field. But the difference is following. These forces are caused by interaction between ES instead of external forces. The potential part of forces between ES changes their kinetic motion energy. The work of non-potential part of the force transforms the energy of ES motion into the internal energy. Therefore the phase space, which defined in coordinates and the velocities of ES CM, is compressible. We call that space as S-space. S-space compression is defined by Liouville equation for ES. The system equilibrates when the ES motion energy transforms into their internal energy. It defines the irreversibility mechanism.

The existence of the non-potential forces in nonequilibrium systems throws light on the nature of non-integrability of Hamilton systems [2]. Really, the self-agreement between changes of potential and kinetic energy of particles would exist if the forces were only potential. And it would mean systems integrability or possibility of its description by means of Newton's equation. But Newton's equation for material points excludes non-potential forces. Therefore it does not allow considering all streams of energy in nonequilibrium system. Hence it does not allow describing nonequilibrium systems with the help of Hamilton formalism.

For the same reasons there are difficulties of the description of strong interactions by means of an initial formalism of Hamilton. These difficulties can be overcome using the modified equations for case when potential interactions of elements inside ES are stronger than interactions between ES as, for example, in the case of interaction of elementary particles. Then it will be possible to take into account the energy of excitation of internal degrees of freedom of particles and change of their internal energy. It is one of possible reasons of broken symmetry.

As a whole, the obtained equations connect the classical mechanics and thermodynamics. The explanation of the First law of thermodynamics is based on the fact that the work of subsystems' interaction forces changes both the energy of their motion and their internal energy. The explanation of the Second law of thermodynamics is connected with irreversible transformation of the subsystems' relative motion energy into their internal energy. Moreover the impossibility of occurrence of

unstructured particles in the classical mechanics follows from it. It is equivalent to the infinite divisibility of the matter.

Thus we have obtained that the expansion of the classical mechanics arising at replacement of material points on structured systems leads to its consent with thermodynamics and allows using the classical mechanics for studying of processes of evolution of physical objects in the nature.

#### REFERENCES

1. Cohen E.G. Boltzmann and stat. mechanics dynamics: Models and Kinetic Methods for non-equilibrium many body systems. NATO Sci. Series E: Applied Sciences-V. 371, p. 223 (1998).
2. Prigogine I. From the being to becoming. M. 343. (1980), 342 p.
3. Zaslavsky G.M. Chaotic dynamic and the origin of Statistical laws. Physics Today. Aug.. Part 1: 39-46. (1999).
4. Somsikov V.M. Non-recurrence problem in evolution of a hard-disk system// Journal American Institute of physics. V. 643:149-156. (2002).
5. Somsikov V. M. Equilibration of a hard-disks system// Intern. J. Bifurcation And Chaos 14, No 11: 4027-4033. (2004).
6. Landau. L.D., Lifshits Ye. M. Statistical Physics. 1976. Nauka, Moscow.
7. Somsikov V. M. Thermodynamics and classical mechanics. Journal of physics: Conf. series 13 : 7-16. (2005).
8. Lanczos.C. The variation principles of mechanics. University of Toronto press. (1962), 408p.
9. Rumer Yu.B., Ryvkin M.Sh. Thermodynamics Stat. Physics and Kinematics. 1977. Nauka, Moscow.
10. Somsikov V. M The restrictions of classical mechanics in the description of dynamics of nonequilibrium systems and the way to get rid of them. arXiv:0805.1186v1 [physics.class-ph] 8 May 2008.

#### Резюме

Құрылымданған бөлшектердің динамикасы дамуда. Материалдық нүктелердің құрылымданған жүйелерге алмасуы кезінде пайда болатын классикалық механиканың кеңеюі оның термодинамикамен сәйкестігіне әкеп соғатындығы және табиғаттағы физикалық нысандар эволюциясы процестерін зерттеу үшін классикалық механиканы пайдалануға мүмкіндік беретіндігі көрсетілген.

#### Резюме

Развивается динамика структурированных частиц. Показано, что расширение классической механики, возникающее при замене материальных точек на структурированные системы, приводит ее к согласию с термодинамикой и позволяет использовать классическую механику для изучения процессов эволюции физических объектов в природе.

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## КИНЕТИЧЕСКАЯ МОДЕЛЬ РАВНОВЕСНОГО ПОГРАНИЧНОГО СЛОЯ МЕЖДУ ТЕПЛОЙ ПЛАЗМОЙ И МАГНИТНЫМ ПОЛЕМ

Исследование структуры равновесных пограничных слоев между холодной и тепловой плазмой и магнитным полем имеет длинную историю и начинается с работ Чепмена и Ферраро [1]. Авторы естественным образом сталкиваются с эффектом поляризации пограничных слоев, возникающих в замагниченной плазме вследствие разных значений масс и, следовательно, ларморовских радиусов электронной и ионной компонент. Многие решения, например [2], получены в приближении квазинейтральности. Возникающее в таких приближенных решениях электрическое поле требует, конечно, дополнительного исследования и полного решения задачи. Другие решения получены в классе электронейтральных [3,4], что достигается наложением определенных связей на параметры задачи. Некоторые исследователи, например [5], постулируют интегральную нейтральность переходных слоев. И, наконец, в группе работ, например [6-7], делается вывод о том, что плазма переходных слоев в общем случае поляризована.

Ниже будет показано, что электронейтральные решения справедливы только при некоторых определенных соотношениях между параметрами задачи.

При исследовании устойчивости границы плазма – магнитное поле, как правило, пренебрегают предварительным строгим исследованием равновесия этой границы. В качестве равновесного решения в учебнике [8], например, берется априори максвелловская функция распределения с неоднородными значениями плотности и температуры. Рассматривается, в частности, случай, когда характерный размер неоднородности границы намного превышает ларморовские радиусы компонент плазмы. Но этим не может быть исчерпана общая проблема границы. В такой постановке остается за пределами рассмотрения проблема самосогласования электромагнитных полей, а значит степень адекватности предлагаемых моделей реальной границе и её устойчивости. Так, в классе самосогласованных

решений плазма с максвелловским распределением по скоростям не может образовать равновесной границы с удерживающим её магнитным полем.

### Постановка задачи

Пусть в нижнем полупространстве (см. рис. 1) сосредоточена плазма, а в верхнем полупространстве – удерживающее её магнитное поле. На границе этих областей в плоскости  $xoy$  образуется самосогласованный переходный слой.

Задача одномерна, все величины зависят от переменной  $z$ . Исследуемая среда описывается системой из кинетического уравнения

$$\frac{\partial f_a}{\partial t} + \vec{v} \frac{\partial f_a}{\partial \vec{r}} + e_a \{ \vec{E} + [\vec{v} \vec{B}] \} \frac{\partial f_a}{\partial \vec{P}_a} = 0, \quad (1)$$

и уравнений Максвелла с самосогласованным электромагнитным полем (внешние источники отсутствуют)

$$\text{rot} \vec{B} = \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t} + \frac{1}{\varepsilon_0 c^2} \vec{j}, \quad \text{div} \vec{B} = 0, \quad (2)$$

$$\text{rot} \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \quad \text{div} \vec{E} = \frac{\rho}{\varepsilon_0},$$

где

$$\rho = \sum_a e_a \int \mathcal{G}_a d\vec{P},$$

$$\vec{j} = \sum_a e_a \int \vec{v} \mathcal{G}_a d\vec{P}.$$

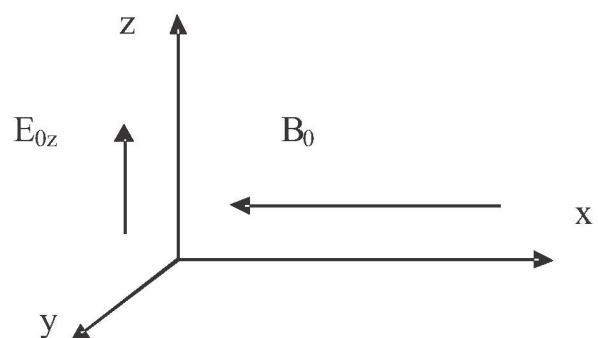


Рис. 1

Задача (1),(2) решается методом теории возмущений

$$\begin{aligned} f_\alpha(\vec{P}, \vec{r}, z, t) &= f_{0\alpha}(\vec{P}) + \delta f_\alpha(\vec{P}, \vec{r}, z, t); \\ \vec{E}(\vec{r}, z, t) &= \vec{E}_0(z) + \delta \vec{E}(\vec{r}, z, t); \\ \vec{B}(\vec{r}, z, t) &= \vec{B}_0(z) + \delta \vec{B}(\vec{r}, z, t). \end{aligned} \quad (3)$$

Плазма считается слабонеравновесной  $\delta f_\alpha(\vec{P}, \vec{r}, z, t) < f_{0\alpha}(\vec{P})$ .

**Вывод уравнений, описывающих электромагнитное поле пограничного слоя**

Равновесная функции распределения конструируется как функция от интегралов движения  $f_{0\alpha}(\vec{P}) = f_{0\alpha}(W, P_y, P_x)$ , где полная энергия и обобщенные импульсы имеют вид:

$$\begin{aligned} W &= \frac{1}{2} m_\alpha (v_x^2 + v_y^2 + v_z^2) + e_\alpha \phi(z), \\ P_y &= m_\alpha v_y + e_\alpha A_y(z), \\ P_x &= m_\alpha v_x. \end{aligned} \quad (4)$$

Здесь  $\phi(z), A_y(z)$  - электрический и магнитный потенциалы ( $\vec{E}_0 = -\text{grad}\phi, \vec{B}_0 = \text{rot}\vec{A}$ ).

Выбор равновесной функции распределения в виде [3]

$$f_{0\alpha}(W, P_y) = \left( \frac{m_\alpha}{2\pi\theta_\alpha} \right)^{\frac{3}{2}} n_{0\alpha} (1 + a_\alpha) \exp \left\{ -\frac{W_\alpha}{\theta_\alpha} - \frac{a_\alpha P_{y\alpha}}{2m_\alpha\theta_\alpha} \right\} \quad (5)$$

где степень анизотропии  $a_\alpha$  определяется формулой

$$a_\alpha = \left( \frac{\theta_\alpha}{\theta_{\alpha y}} - 1 \right), \quad (6)$$

а интегралы движения  $W_\alpha$  и  $P_{y\alpha}$  - формулы (4), позволяет получить уравнения для потенциалов самосогласованного электромагнитного поля:

$$\eta \frac{d^2 \psi}{d\xi^2} = \exp \left( \frac{\psi}{\beta} - \frac{\alpha_e}{1 + \alpha_e} \frac{a^2}{2\beta\mu} \right) - \exp \left( -\psi - \frac{\alpha_i}{1 + \alpha_i} \frac{a^2}{2} \right), \quad (7)$$

$$\begin{aligned} \frac{d^2 a}{d\xi^2} &= \frac{\alpha_e}{1 + \alpha_e} \frac{a}{\mu} \exp \left( \frac{\psi}{\beta} - \frac{\alpha_e}{1 + \alpha_e} \frac{a^2}{2\beta\mu} \right) + \\ &+ \frac{\alpha_i}{1 + \alpha_i} a \exp \left( -\psi - \frac{\alpha_i}{1 + \alpha_i} \frac{a^2}{2} \right). \end{aligned} \quad (8)$$

Здесь введены следующие безразмерные величины:

$$\begin{aligned} \mu &= \frac{m}{M}, \beta = \frac{\theta_{xe}}{\theta_{xi}}, \eta = \frac{\theta_{xi}}{Mc^2}, \psi = \frac{e}{\theta_{xi}} \phi, \\ a &= \frac{e}{c} \frac{1}{\sqrt{\theta_{xi} M}} A_y, \xi^2 = \frac{4\pi e^2 n_0}{Mc^2} x^2. \end{aligned} \quad (9)$$

Функция распределения (5) введена в работе Никольсона [3] при построении равновесного решения для слоя тепловой плазмы, разделяющей области с одинаково направленными магнитными полями. Никольсон рассматривает наиболее простой случай электронейтральных решений. Такие решения можно получить только, если наложить на параметры задачи следующую связь:

$$\frac{1}{\theta_{ex} m_e} \frac{\alpha_e}{1 + \alpha_e} = \frac{1}{\theta_{ix} m_i} \frac{\alpha_i}{1 + \alpha_i}. \quad (10)$$

В этом случае уравнение для электрического потенциала (7) дает тривиальное решение  $\psi(\xi) = 0$ . В качестве граничного условия к оставшемуся уравнению для магнитного потенциала (8) Никольсон выбирает:

$$a(\xi = 0) = 0, \quad (11)$$

оставляя вторую константу  $C = \left( \frac{da}{d\xi}(\xi = 0) \right)^2$ , представляющую собой квадрат магнитного поля, произвольной.

Оказывается, что с помощью функции распределения (5) можно построить решение для равновесной границы между тепловой плазмой и магнитным полем. В этом случае в качестве граничных для уравнений (7), (8) следует поставить следующие условия:

$$\begin{aligned} \psi(-\infty) &= 0, a(-\infty) = 0, E(-\infty) = \\ -\psi'(-\infty) &= 0, B(-\infty) = a'(-\infty) = 0. \end{aligned} \quad (12)$$

Полученная система уравнений Максвелла (7), (8), (12) описывает самосогласованное электромагнитное поле равновесного пограничного слоя между плазмой и магнитным полем. Мы отказываемся от условия электронейтральности (10), ограничивающего область применимости решения. Всюду в области переходного слоя существует отличное от нуля равновесное электрическое поле (8).

Здесь введены следующие безразмерные величины:

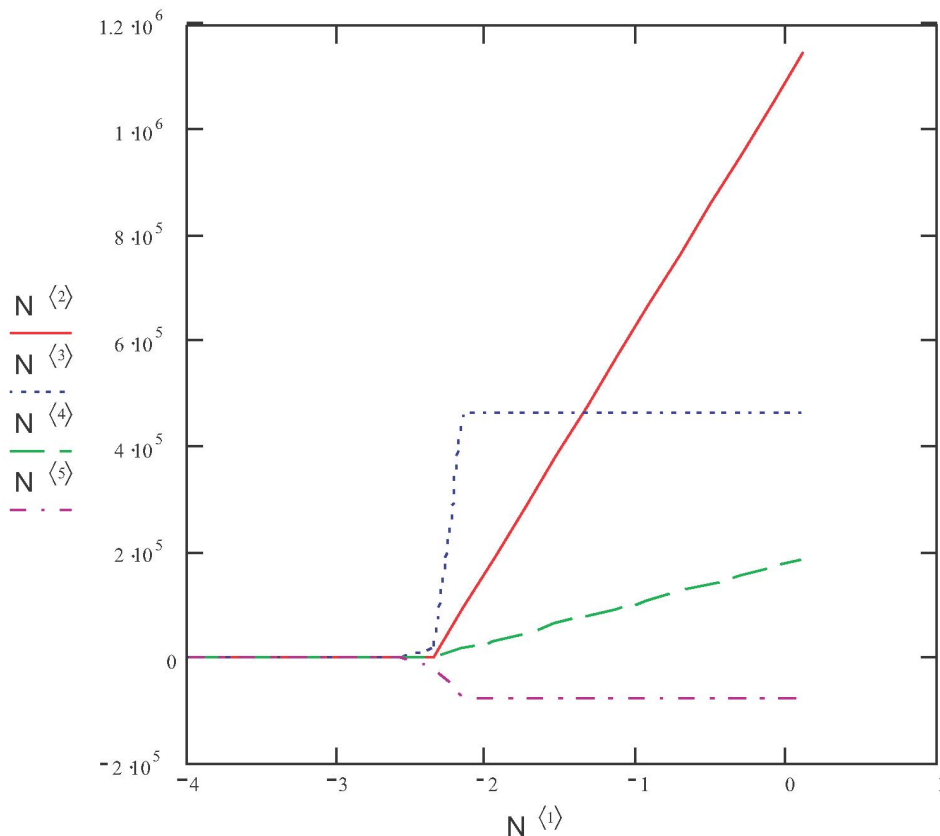


Рис. 2. Профили характеристик пограничного слоя

$$\mu = \frac{m}{M}, \beta = \frac{\theta_{xe}}{\theta_{xi}}, \eta = \frac{\theta_{xi}}{Mc^2}, \psi = \frac{e}{\theta_{xi}} \phi, \quad (9)$$

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$$\frac{1}{\theta_{ez} m_e} \frac{\alpha_e}{1 + \alpha_e} = \frac{1}{\theta_{iz} m_i} \frac{\alpha_i}{1 + \alpha_i}. \quad (10)$$

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Оказывается, что с помощью функции распределения (5) можно построить решение для равновесной границы между тепловой плазмой и магнитным полем. В этом случае в качестве граничных для уравнений (7), (8) следует поставить следующие условия:

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решения. Всюду в области переходного слоя существует отличное от нуля равновесное электрическое поле.

#### Методика численного решения задачи

Численное решение непосредственно задачи (7), (8), (12) даст тривиальный нуль. Необходимо прежде найти асимптотику уравнений (7), (8) в области  $a \rightarrow 0, \psi \rightarrow 0$ . Оставляя малые члены, порядка  $\psi$  и  $a^2$ , можно найти асимптотику системы:

$$\begin{aligned} \frac{d^2 \psi}{d\xi^2} &= \frac{1+\beta}{\beta\eta} \psi + \frac{1}{2\eta} \left( \frac{\alpha_i}{1+\alpha_i} - \frac{1}{\beta\mu} \frac{\alpha_e}{1+\alpha_e} \right) a^2, \\ \frac{d^2 a}{d\xi^2} &= \left( \frac{1}{\mu} \frac{a_e}{1+\alpha_e} + \frac{\alpha_i}{1+\alpha_i} \right) a. \end{aligned} \quad (13)$$

Решение этой системы есть:

$$\begin{aligned} a &= c_2 \exp(\sqrt{b}\xi), B = a' = c_2 \sqrt{b} \exp(\sqrt{b}\xi), \\ \psi &= \frac{c_2 d}{2\sqrt{s}(\sqrt{b} - \sqrt{s})} \exp(\sqrt{b}\xi) + c_3 \exp(\sqrt{s}\xi), \\ E = -\psi' &= -\frac{c_2 d \sqrt{b}}{2\sqrt{s}(\sqrt{b} - \sqrt{s})} \exp(\sqrt{b}\xi) - c_3 \sqrt{s} \exp(\sqrt{s}\xi). \end{aligned} \quad (14)$$

При численном счете надо стартовать с асимптотического решения (14) в некоторой точке  $\xi$ . Здесь введены обозначения:

$$\begin{aligned} s &= \frac{1+\beta}{\beta\eta}, d = \frac{1}{2\eta} \left( \frac{\alpha_i}{1+\alpha_i} - \frac{1}{\beta\mu} \frac{\alpha_e}{1+\alpha_e} \right), \\ b &= \left( \frac{1}{\mu} \frac{a_e}{1+\alpha_e} + \frac{\alpha_i}{1+\alpha_i} \right) \end{aligned} \quad (15)$$

Коэффициенты  $c_2$  и  $c_3$  могут быть произвольными, в частности можно положить

$$c_2 = c_3 = 1.$$

#### Результаты решения

Численное решение задачи (7), (8), (12) с учетом асимптотики (14) получено в пакете MATHCAD и представлено на рис. 2.

Расчеты проведены при значениях параметров  $s = 0.5, \eta = 0.5, \beta = 1$ . Кривая 1 представляет собой профиль магнитного потенциала, кривая 2 – профиль магнитного поля, кривая 3 – профиль электрического потенциала, кривая 4 – профиль электрического поля. Кривые магнитного потенциала и магнитного поля являются типичными для пограничного слоя между плазмой и

магнитным полем. Но при этом оказывается, что в общем случае произвольных параметров плазма погранслоя поляризована, см. профиль электрического поля (кривая 4).

#### Заключение

Электронейтральные решения представляют собой узкий класс решений и существуют только при определенных соотношениях между параметрами задачи, см. формулу (10). Для получения строгого решения задачи о переходном слое между тепловой плазмой и магнитным полем необходимо прибегнуть к численному счету системы (7), (8), (12). Оказывается, что в общем случае плазма стационарного пограничного слоя поляризована, и это электрическое поляризационное поле необходимо учитывать при исследовании неустойчивости погранслоев.

#### ЛИТЕРАТУРА

1. Chapman S., Ferraro V.C.A. A New Theory of Magnetic Storms: Part I – The Initial Phase// Terr. Magn. Atmos. Elect. V. 36. № 1. P. 77-97. 1931.
2. Морозов А.И., Соловьев Л.С. Кинетическое рассмотрение некоторых равновесных плазменных конфигураций// ЖЭТФ. Т. 40. №5. С. 1316-1324. 1961.
3. Nicholson R.B. Solution of the Vlasov Equations for a Plasma in an Externally Uniform Magnetic Field // Phys. Fluids. V. 6. № 11. P. 1581-1586. 1963.
4. Harris E.G. On a Plasma Sheath Separating Regions of Oppositely Directed Magnetic Field // Nuovo Cim. V. XXIII. № 1. P. 115 – 121. 1962.
5. Сузов Ю.С. Структура пограничного слоя между разреженной плазмой и магнитным полем// Ж. прикл. механ. и техн. физ. № 2. С. 15-22. 1965.
6. Sestero A. Charge Separation Effects in the Ferraro-Rosenbluth Cold Plasma Sheath Model// Phys. Fluids. V. 8. № 4. P. 739-744. 1965.
7. Ляхов В.В., Шабанский В.П. Релятивистская модель пограничного слоя между магнитным полем и отражающимся от него потоком холодной плазмы// Деп. в ВИНТИ. №5588-85 Деп. 1985.
8. Александров А.Ф., Богданкевич Л.С., Рухадзе А.А. Основы электродинамики плазмы. М.: Высшая школа. 424с. 1988.

#### Резюме

Власовтың стационарлық тендеуін шешу негізінде жылу плазмасы мен магниттік өріс арасындағы тепе-теңдіктегі шекаралық қабаттың кинетикалық моделі құрылған. Қазір пайдаланылатын электронейтралдық шешімдердің жіңішке кластық шешім болып табылатындығы және олардың есептер параметрлері арасында белгілі бір қатынастар кезінде орын алатындығы көрсетілген. Жалпы жағдайда стационарлық шекаралық қабат плазмасы поляризацияланады екен.

**Summary**

The kinetic model of equilibrium boundary layer between thermal plasma and magnetic field is built on the basis of solution of Vlasov's stationary equation. It is shown that electroneutral solutions used nowadays represent the narrow

class of solutions and exist only at specific relations between parameters the problem. It turns out that generally plasma of a stationary boundary layer is polarized.

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