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CURVES OF CONNECTING RODS SIX-MEMBERED MECHANISMS II AND IV CLASSES

(Представлена академиком НАН Г.У.Уалиевым)

1. Introduction

Six-membered mechanism IV class is of great interest to scientists mechanics.

Many scientists have speculated about its advantages for some kinematic and dynamic criteria compared to the same six-membered mechanism of class II (Figure 1). This assumption is based on the fact that having the same number of links it contains a four-bar closed contour variable [1-4].

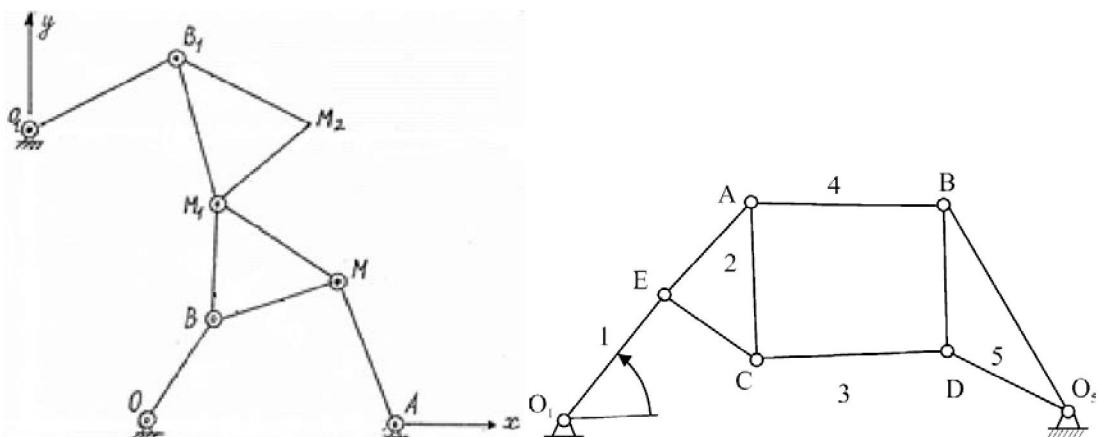


Figure 1 - The six-membered mechanisms Class II and Class IV

One of these criteria is the order of the polynomial curves of connecting rod. The higher the order of the polynomial, the more complicated curve can reproduce mechanism [5-7]. Consequently, the greater the chances of the mechanism. In [8] it is shown that the mechanism of six-membered class II reproduces curves of connecting rod, corresponding to the polynomial of the 18th order. In this paper we investigate the mechanism of six-membered class IV, which differs from a similar mechanism of class II only its structure.

Recurrence relations. Consider dyad OBM (Figure 2), where OB = a, BM = l, BM₁ = k, $\angle MBM_1 = \omega$. Then for the points M and M₁, we have [8]:

$$\left\{ \begin{array}{l} x = x_1 + \frac{(l \cos \omega - k)x_1 + y_1 l \sin \omega}{2k\rho_1^2} (\rho_1^2 + k^2 - a^2) \\ \pm \frac{x_1 l \sin \omega - (l \cos \omega - k)y_1}{2k\rho_1^2} \sqrt{4k^2\rho_1^2 - (\rho_1^2 + k^2 - a^2)^2}, \\ y = y_1 - \frac{x_1 l \sin \omega - (l \cos \omega - k)y_1}{2k\rho_1^2} (\rho_1^2 + k^2 - a^2) \\ \pm \frac{(l \cos \omega - k)x_1 + y_1 l \sin \omega}{2k\rho_1^2} \sqrt{4k^2\rho_1^2 - (\rho_1^2 + k^2 - a^2)^2}, \end{array} \right.$$

$$OR \quad \rho^2 = x^2 + y^2 = \frac{l \cos \omega}{k} \rho_1^2 \pm \frac{l \sin \omega}{k} \sqrt{4k^2 \rho_1^2 - (\rho_1^2 + k^2 - a^2)^2} + \\ + l^2 + \frac{k - l \cos \omega}{k} a^2 - kl \cos \omega \quad (1)$$

Note that the double sign of the square corresponds to two positions M (x, y) at the same position of the point M1 (x1, y1) (Figure 3).

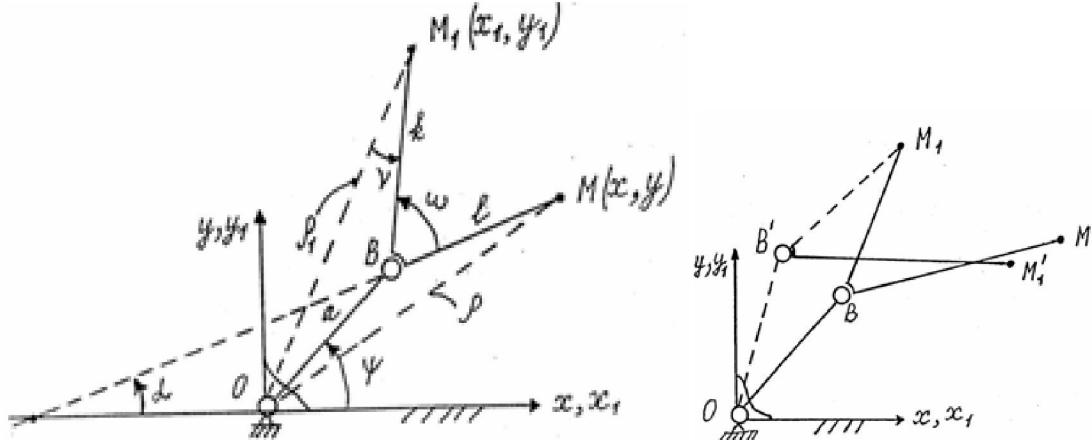


Figure 2 - dyad Figure 3 - Two assembly dyad

If $k = a$, then:

$$\begin{cases} x = x_1 \frac{a + l \cos \omega}{2a} + y_1 \frac{l \sin \omega}{2a} \pm \frac{x_1 l \sin \omega - y_1 (l \cos \omega - a)}{2a \rho_1} \sqrt{4a^2 - \rho_1^2} \\ y = y_1 \frac{a + l \cos \omega}{2a} - x_1 \frac{l \sin \omega}{2a} \pm \frac{x_1 (l \cos \omega - a) + y_1 l \sin \omega}{2a \rho_1} \sqrt{4a^2 - \rho_1^2} \\ \rho^2 = \frac{l \cos \omega}{a} \rho_1^2 \pm \frac{l \sin \omega}{a} \rho_1 \sqrt{4a^2 - \rho_1^2} + l^2 + a^2 - 2al \cos \omega \end{cases}$$

2. Curves of connecting rods six-membered mechanisms of class II.

We show that the addition of the dyad increases the order of the equation of the curve of the connecting rod three times if the initial unit performs rotational motion (Figure 4). Mechanism II class is obtained by successive addition to the initial link AM dyads M-B-O and M1-B1-O1, with the condition $k = a$, that is $M_1 B = BO = a$ and $M_2 B_2 = B_1 O_1 = a_1$. Let us denote: $AM = r$, $OA = d$, $MB = l$, $\angle MBM_1 = \omega$, $M_1 B_1 = l_1$, $\angle M_1 B_1 M_2 = \omega_1$, $OD = g$, $O_1 D = c$, then:

$$\left(V_1 \pm V_2 \frac{\sqrt{4a_1^2 - \rho_1^2}}{\rho_1} \right) \left(V_3 \pm V_4 \frac{\sqrt{4a_1^2 - \rho_1^2}}{\rho_1} \right)^2 = \left(V_5 \mp V_2 \frac{\sqrt{4a_1^2 - \rho_1^2}}{\rho_1} \right) \left(V_6 \mp V_7 \frac{\sqrt{4a_1^2 - \rho_1^2}}{\rho_1} \right)^2,$$

$$\text{where } V_1 = \frac{l_1 \cos \omega_1}{a_1} \rho_1^2 - \frac{g(a_1 + l_1 \cos \omega_1) + cl_1 \sin \omega_1}{a_1} x_1 - \frac{gl_1 \sin \omega_1 - c(a_1 + l_1 \cos \omega_1)}{a_1} y_1 + l_1^2 + a_1^2 - 2a_1 l_1 \cos \omega_1 + g^2 + c^2,$$

$$V_2 = \frac{l_1 \sin \omega_1}{a_1} \rho_1^2 - \frac{c(l_1 \cos \omega_1 - a_1) - gl_1 \sin \omega_1}{a_1} x_1 + \frac{g(l_1 \cos \omega_1 - a_1) + cl_1 \sin \omega_1}{a_1} y_1, \quad V_3 = \frac{ll_1 \cos \omega \cos \omega_1}{a_1} \rho_1^2 - \\ - \frac{A_1(a_1 + l_1 \cos \omega_1) + A_2 l_1 \sin \omega_1}{2a_1} x_1 + \frac{A_2(a_1 + l_1 \cos \omega_1) - A_1 l_1 \sin \omega_1}{2a_1} y_1 + \cos \omega(l_1^2 + a_1^2 - 2a_1 l_1 \cos \omega_1) + A_3,$$

$$V_4 = \frac{ll_1 \cos \omega \cos \omega_1}{a_1} \rho_1^2 - \frac{A_1 l_1 \sin \omega_1 - A_2(l_1 \cos \omega_1 - a_1)}{2a_1} x_1 + \frac{A_1(l_1 \cos \omega_1 - a_1) + A_2 l_1 \sin \omega_1}{2a_1} y_1, \quad V_5 = 4a^2 - V$$

$$V_6 = \frac{A_4(a_1 + l_1 \cos \omega_1) - A_5 l_1 \sin \omega_1}{2a_1} x_1 + \frac{A_4 l_1 \sin \omega_1 - A_5(a_1 + l_1 \cos \omega_1)}{2a_1} y_1 + \frac{l l_1 \sin \omega \cos \omega_1}{a_1} \rho_1^2 + l(l_1^2 + a_1^2 - 2a_1 l_1 \cos \omega_1) \sin \omega - A_6$$

$$V_7 = \frac{A_4 l_1 \sin \omega_1 - A_5(l_1 \cos \omega_1 - a_1)}{2a_1} x_1 + \frac{A_4(l_1 \cos \omega_1 - a_1) + A_5 l_1 \sin \omega_1}{2a_1} y_1 + \frac{l l_1 \sin \omega \sin \omega_1}{a_1} \rho_1^2$$

Freed from the radical and of the denominator, we obtain the equation of the curve the connecting rod, which is described by point M_2 as follows:

$$\rho_1^2 [\rho_1^2 (V_1 V_3^2 - V_5 V_6^2) + (V_1 V_4^2 - V_5 V_7^2 + 2V_2 V_3 V_4 + 2V_2 V_6 V_7) (4a_1^2 - \rho_1^2)] = \\ (4a_1^2 - \rho_1^2) [(2V_5 V_6 V_7 - V_2 V_3^2 - V_2 V_6^2) \rho_1^2 - V_2 (V_4^2 + V_7^2) (4a_1^2 - \rho_1^2)]$$

Since all the operators V are algebraic functions of x_1 and y_1 of the second order, then the resulting expression is an algebraic equation of the eighteenth order [8]. That is, the curves of connecting rods of six-membered mechanisms of class II are algebraic equations not above the eighteenth order.

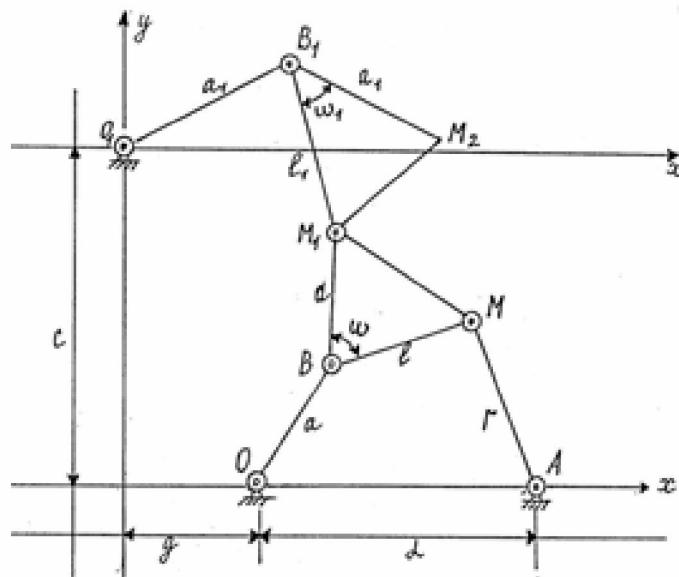


Figure 4 - Class II mechanism of six-membered

3. Bipolar coordinates

Consider a triangle with sides a_1, a_2, a_3 (Figure 5). We will choose a fourth point inside or outside the triangle. Three other sizes b_1, b_2, b_3 (Fig. 5) of quadrilateral were obtained compound of this point to points of the triangle. 1-3 indices are chosen so that the opposing sizes have the same index. The fourth point is determined by the length of two of them, for example, b_1, b_2 . The length of the third is not chosen at random, and follows from the geometric interrelation. Of the six sizes of the quadrilateral - five can be chosen freely within defined borders. Sixth size can be determined from the known five sizes. We denote the angle between a_1 and b_3 sizes as α , and the angle between a_2 and b_3 as β , we obtain the equation: $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$. If we take the squared, then this equation can take the form: $\cos^2 \alpha + \cos^2 \beta + \cos^2(\alpha + \beta) - 2\cos \alpha \cos \beta \cos(\alpha + \beta) - 1 = 0$. By cosine theorem we have: $b_2^2 = a_1^2 + b_3^2 - 2a_1 b_3 \cos \alpha$, $b_1^2 = a_2^2 + b_3^2 - 2a_2 b_3 \cos \beta$, $a_3^2 = a_1^2 + a_2^2 - 2a_1 a_2 \cos(\alpha + \beta)$. If we express \cos, \cos, \cos through the sizes, we have after some transformations [9,10]:

$$a_1^2 b_1^2 (a_1^2 + b_1^2 - a_2^2 - b_2^2 - a_3^2 - b_3^2) + a_2^2 b_2^2 (a_2^2 + b_2^2 - a_3^2 - b_3^2 - a_1^2 - b_1^2) + a_3^2 b_3^2 (a_3^2 + b_3^2 - a_1^2 - b_1^2 - a_2^2 - b_2^2) + a_1^2 a_2^2 a_3^2 + a_1^2 b_2^2 b_3^2 + a_2^2 b_3^2 b_1^2 + a_3^2 b_1^2 b_2^2 = 0.$$

If there are five sizes, it is possible to calculate the sixth. For example, if there are a_1, a_2, a_3, b_1 and b_2 , for calculating - the sixth size b_3 equation can be used as: $b_3^4 B_{32} + b_3^2 B_{31} + B_{30} = 0$. For the coefficients

B_{32}, B_{31}, B_{30} obtained the following expressions: $B_{32} = a_3^2$, $B_{31} = a_3^2 (a_3^2 - a_1^2 - a_2^2 - b_1^2 - b_2^2) - (a_1^2 - a_2^2)(b_1^2 - b_2^2)$, $B_{30} = (a_1^2 b_1^2 - a_2^2 b_2^2)(a_1^2 + b_1^2 - a_2^2 - b_2^2) - a_3^2 (a_1^2 - b_2^2)(b_1^2 - a_2^2)$

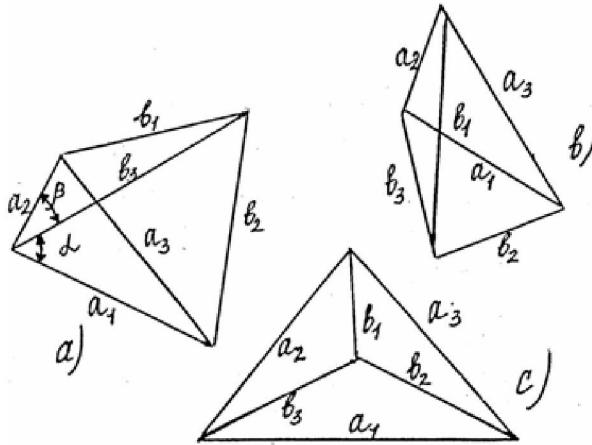


Figure 5 - Bipolar coordinates

Of b_3^2 may be defined the two values. They are applicable only when they are real and positive.

4. Curves links of connecting rods six-membered mechanisms Class IV

Use bipolar coordinates, then the relation between the radius vectors $\rho_{k_i B}$ and $\rho_{k_j B}$ is (Figure 5) [9,10]:

$$\begin{aligned} & \rho_{k_j b}^8 * (\rho_{k_i b}^4 * U_{24} + \rho_{k_i b}^2 * U_{14} + U_{04}) + \rho_{k_j b}^6 * (\rho_{k_i b}^6 * U_{33} + \rho_{k_i b}^4 * U_{23} + \\ & + \rho_{k_i b}^2 * U_{13} + U_{03}) + \rho_{k_j b}^4 (\rho_{k_i b}^8 * U_{42} + \rho_{k_i b}^6 * U_{32} + \rho_{k_i b}^4 * U_{22} + \\ & \rho_{k_i b}^2 * U_{12} + U_{02}) + \rho_{k_j b}^2 * (\rho_{k_i b}^8 * U_{41} + \rho_{k_i b}^6 * U_{31} + \rho_{k_i b}^4 * U_{21} + \rho_{k_i b}^2 * U_{11} + \\ & + U_{01}) + \rho_{k_i b}^8 * U_{40} + \rho_{k_i b}^6 * U_{30} + \rho_{k_i b}^4 * U_{20} + \rho_{k_i b}^2 * U_{10} + U_{00} = 0, \end{aligned} \quad (2)$$

where $\rho_{k_i b}^2 = X_{k_i b}^2 + Y_{k_i b}^2$, $\rho_{k_j b}^2 = X_{k_j b}^2 + Y_{k_j b}^2$ and U_{ij} ($i,j=0,\dots,4$) - dependent on the size of the mechanism. Let us consider the dyad $A_G A_k K_j K_i$. We will assume that $A_G A_k = A_k K_i = a$, $A_k K_j = l$, угол $\angle K_j A_k K_i = \omega$. Then the recurrence relation (1) for the points K_j and K_i in the coordinate system $A_G XY$ will:

$$\begin{cases} X_{k_j a} = X_{k_i a} \frac{a + l \cos \omega}{2a} + Y_{k_i a} \frac{l \sin \omega}{2a} \pm \frac{X_{k_i a} l \sin \omega - Y_{k_i a} (l \cos \omega - a)}{2a} * \sqrt{(4a^2 - \rho_{k_i a}^2) / \rho_{k_i a}^2}, \\ Y_{k_j a} = Y_{k_i a} \frac{a + l \cos \omega}{2a} - X_{k_i a} \frac{l \sin \omega}{2a} \pm \frac{X_{k_i a} (l \cos \omega - a) + Y_{k_i a} l \sin \omega}{2a} * \sqrt{(4a^2 - \rho_{k_i a}^2) / \rho_{k_i a}^2}, \\ \rho_{k_j a}^2 = \frac{l \cos \omega}{a} \rho_{k_i a}^2 \pm \frac{l \sin \omega}{a} \rho_{k_i a}^2 \sqrt{(4a^2 - \rho_{k_i a}^2) / \rho_{k_i a}^2} + l^2 + a^2 - 2al \cos \omega, \end{cases} \quad (3)$$

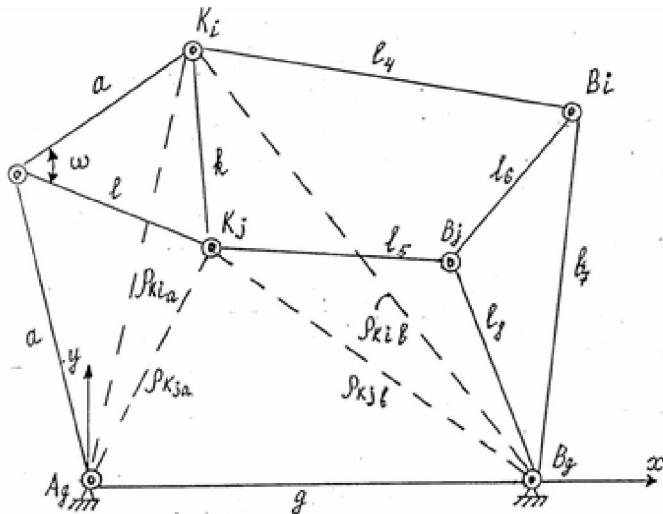


Figure 5 - six-membered mechanism of Class IV

where the signs \pm refer to two point position K_j in relation to the point K_i dyad $A_G A_k K_j K_i$. For definiteness, we take the "+" sign. For quadrilateral $A_G B_G K_i K_j$ the following relations hold:

$$\rho_{k_i b}^2 = (x_{k_i a} - g)^2 + y_{k_i a}^2 = \rho_{k_i a}^2 + g^2 - 2gx_{k_i a}, \quad \rho_{k_j b}^2 = (x_{k_j a} - g)^2 + y_{k_j a}^2 = \rho_{k_j a}^2 + g^2 - 2gx_{k_j a}$$

Substituting this in the expression (2) with (3). Let us find the a polynomial curve that draws mechanism his point of K_i . For this purpose get rid of the denominator and the radical. There was a lot mathematical transformations - "replacement" and "permutation". For these transformations has been used a computer program for analytic calculations «REDUCE» [11]. Then the point K_i of mechanism six-membered class IV reproduces the curve:

$$B_0^6 \cdot R_{i2}^6 \cdot S \cdot T^4 (2U_{24} \cdot U_{42} + U_{33}^2) + B_0^8 \cdot R_{i2}^4 \cdot S \cdot T^4 \cdot U_{24}^2 + 2A_0^7 \cdot R_{i2}^5 \cdot S^5 \cdot T \cdot U_{24} \cdot U_{33} + \\ 28A_0^6 \cdot B_0^2 \cdot R_{i2}^4 \cdot S^4 \cdot T \cdot U_{24}^2 + 42A_0^5 \cdot B_0^2 \cdot R_{i2}^5 \cdot S^4 \cdot T \cdot U_{24}U_{33} + 14A_0 \cdot B_0^6 \cdot R_{i2}^5 \cdot S^2 \cdot T^3 \cdot U_{24}U_{33} + \\ + F(A_0, B_0, R_{i2}, S, T, U_{ij}) = 0, \quad i, j = 0, \dots, 4,$$

where $A_0, B_0, R_{i2}, S, T, U_{ij}$ mean: $B_0 = -4al\rho_{k_i a}^2 \sin \omega + 4aglx_{k_i a}^2 \sin \omega + 4ag(l \cos \omega - a)y_{k_i a}^2$,
 $T = 4a^2 - (x_{k_i a}^2 - g)^2 - y_{k_i a}^2$, $A_0 = -4al\rho_{k_i a}^2 \cos \omega +$
 $+ 4(l \cos \omega - a)agx_{k_i a}^2 - 4agly_{k_i a}^2 \sin \omega + 4a^2(2al \cos \omega - a^2 - l)^2$,

$R_{i2} = \rho_{k_i a}^2$, $S = (x_{k_i a}^2 - g)^2 + y_{k_i a}^2$ and $F(A_0, B_0, R_{i2}, S, T, U_{ij})$ - this is the Sum terms in the polynomial of degree less than 34, it is completely contained in [12]. A_0, B_0, R_{i2}, S, T relative to $x_{k_i a}$ and $y_{k_i a}$ are members of the second degree. Therefore it is possible to estimate the order of the polynomial. We see that it is not more than 36.

5. Conclusion

Thus, we have polynomial curve connecting rod, which produces point K_i six-membered mechanism IV class. It is much higher than the order of the polynomial six-membered mechanism of class II. And that means that the mechanism of class IV superior to its analogue class II for a specific kinematic criteria.

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Резюме

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ШАТУННЫЕ КРИВЫЕ ШЕСТИЗВЕННЫХ МЕХАНИЗМОВ II И IV КЛАССОВ

МВК, благодаря наличию Ассуровых групп с изменяемым замкнутым контуром, обладают большими кинематическими и динамическими возможностями по сравнению с их аналогами II класса. Покажем это для направляющих шестизвенных МВК IV класса с вращательными парами. Одним из кинематических критериев для сравнения различных направляющих механизмов является порядок полинома воспроизводимых ими шатунных кривых, коэффициенты которого зависят от метрических параметров механизма. В работах Левитского Н.И. посвященных изучению свойств шатунных кривых многозвенных направляющих механизмов II класса, были выведены уравнения некоторых видов шатунных кривых и установлена для них зависимость между максимально возможным алгебраическим порядком шатунной кривой и числом звеньев механизма, причем установлено, что шестизвенный механизм II класса с вращательными парами воспроизводит чертящей точкой М, в общем случае, шатунную кривую, описываемую полиномом не выше 18-го порядка. Чем выше порядок полинома шатунной кривой, тем более сложную кривую может воспроизвести направляющий механизм и, следовательно, тем шире его кинематические возможности.

В данной работе для сравнительного исследования (с точки зрения порядка полинома шатунных кривых) выбран один из МВК - направляющий шестизвенный механизм IV класса с вращательными парами как наиболее близкий к направляющему шестизвенному механизму II класса (количество звеньев и количество кинематических вращательных пар у них одинаково) и отличающийся от него лишь своей структурой.

Автором дана оценка порядка полинома точки шатунного звена направляющего шестизвенного механизма IV класса. Он, в общем случае, не более 36 порядка. Таким образом, он оказался значительно выше, чем у аналогичного шестизвенного механизма II класса и значит по выбранному критерию шестизвенный механизм IV класса превосходит свой аналог II класса.

Ключевые слова: шатун, кривая, механизм II класса, механизм IV класса.

Резюме

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II ЖӘНЕ IV КЛАССЫ АЛТЫБУЫНДЫ МЕХАНИЗМДЕРДІҢ ҚИСЫҚТА БҮЛГАҚТАРЫ

Өзгермелі тұйық контурлы Ассур группаларына байланысты II классты баламаларымен салыстырында, ЖКМ ете үлкен кинематикалық және динамикалық мүмкіндіктерге ие болып отыр. Бұл мүмкіндіктерді айналмалы жұптармен IV классты алтыбуынды бағыттаушы ЖКМ-де көрсетуге болады. Әртүрлі бағыттаушы механизмдерге салыстыру үшін кинематикалық өлшемнің бір түрі коэффиценті механизмнің метрлік параметрлеріне тәуелді қисықты бүлгектарға жаңғырту үшін полином тәртібі болып табылады. II

класты көпбуынды бағыттаушы механизмдердің қисықты бұлғақтарының қасиеттерін зерттеуге арналған Н.И.Левитскийдің жұмыстарында қисықты бұлғақтардың кейбір түрлерінің есептерін көрсетіп, қисық бұлғақ және механизм буындарының саны аралығындағы тәуелділікті алгебралық тәртіпте бекіткен. Оның ішінде айналмалы жұппен II класты алтыбуынды механизм М нұктесін сыза өтіп, қисықты бұлғақты 18 тәртіптен жоғарыламай сипаттайты. Қисықты бұлғақ полином тәртібі жоғарылаған сайын, бағыттаушы механизмнің күрделі сзығын алуға болады.

Берілген жұмыста зарттеуді салыстыру үшін, қисықты бұлғақтың полином тәртібі бойынша ЖКМ үшін айналмалы жұппен IV класты алтыбуынды бағыттаушы механизм (кинематикалық айналмалы жұп саны және буындар саны бірдей) құрылымы жөнінен айырмашылығы бар, II класты бағыттаушы механизмге ең жақын екенін көрсетеді.

IV класты алтыбуынды бағыттаушы механизмнің бұлғақты буыны нұктесінің полином тәртібіне баға берілген. Ол жалпы жағдайда 36 тәртіптен аспайды. Сонымен, бұл II класты алтыбуынды механизмнің баламасынан анағұрлым көптігін көрсетеді. IV класты алтыбуынды механизм берілген өлшем бойынша II класты баламадан артып отыр.

Тірек сөздер: бұлғақ, қисық сзығық, II класты механизм, IV класты механизм.

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