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DISTRIBUTED INERTIA SPACE MOTION ROD MECHANISM

Annotation

In the mechanics of mechanisms the inertia of linkage elements is usually taken into account through reduction to the center of mass as the resultant force vector and the resultant moment of the couple of forces, and the center of mass is included in the finite element model of the mechanism structure as the nodal point Ref. [1]. It is possible to deduce more accurate finite-element models analytically accounting for distributed inertia of motion.

Keywords: inertia, finite element method, rod mechanism

Ключевые слова: инерция, метод конечных элементов, рычажный механизм

Кілт сөздер: инерция, шеткі элементтер әдісі, иінтіректі механизм.

Model mechanisms

Let’s first consider the plane-parallel motion of the element “*k*” in the fixed system (Fig. 1). At the point O let’s introduce the moving coordinate system $OX^{'}Y^{'}Z^{'}$ with the position determined by the angle θ_k. At the point P_k connecting element *k* with the element (*k-1*) let’s introduce two local coordinate systems: $P_k X_k Y_k Z_k$ moving together with the associated joint, with the axes remaining parallel to the corresponding axes of the coordinate system $OXYZ$ and the coordinate system $P_k X_k^{'} Y_k^{'} Z_k^{'}$ rigidly bound to the element “*k*”.

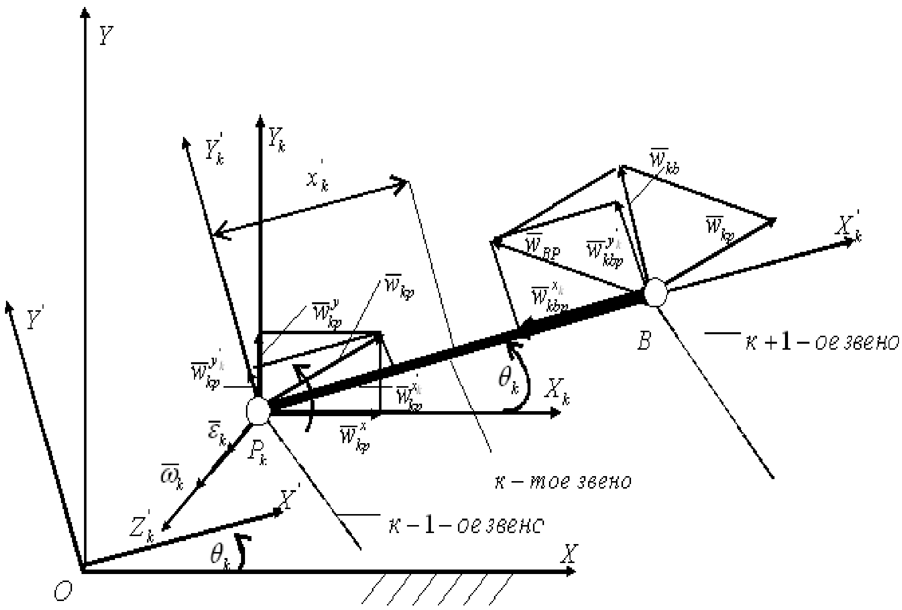

Fig.1. The link mechanism element

Let's assume that the element *k* of the mechanism with the uniform cross-section moves in plane-parallel relative to OXY. And let's assume that in an arbitrary instantaneous position of the mechanism the angle θ_k , the acceleration components $\overline{w}_{kp}^{x}$ and $\overline{w}_{kp}^{y}$ of the point P_k on the element *k* relative to the fixed coordinate system OXY, and $\overline{\omega}_k$, $\overline{\varepsilon}_k$ (angular velocity and angular acceleration of the element *k* with directions, as shown in Figure 1, correspondingly) are known. Then

$$\begin{Bmatrix} w_{kp}^{x'} \\ w_{kp}^{y'} \end{Bmatrix} = \begin{bmatrix} \cos\theta_k & \sin\theta_k \\ -\sin\theta_k & \cos\theta_k \end{bmatrix} \begin{Bmatrix} w_{kp}^{x} \\ w_{kp}^{y} \end{Bmatrix}$$

the acceleration of the point *B* of the element *k* is the sum of the acceleration of the point P_k and the acceleration of the point *B* in its rotation together with the element around this pole. The vector $\overline{w}_{kbp}^{y_k'}$ is perpendicular to the axis of the element, $\overline{w}_{kbp}^{y_k'}$ is always directed from point *B* to the pole P_k. Components of the acceleration of point *B* relative to the coordinate system $OX'Y'$ in matrix form are as follows:

$$\begin{Bmatrix} w_b^{x_k'} \\ w_b^{y_k'} \end{Bmatrix} = \begin{Bmatrix} w_{kp}^{x_k'} \\ w_{kp}^{y_k'} \end{Bmatrix} + \begin{Bmatrix} w_{kbp}^{x_k'} \\ w_{kbp}^{y_k'} \end{Bmatrix} = \begin{bmatrix} \cos\theta_k & \sin\theta_k \\ -\sin\theta_k & \cos\theta_k \end{bmatrix} \begin{Bmatrix} w_{kp}^{x} \\ w_{kp}^{y} \end{Bmatrix} + \begin{Bmatrix} -l_k\omega_k^2 \\ l_k\varepsilon_k \end{Bmatrix}$$

In case the cross-sections along the element are uniform, then the gravitation force of the element may be considered as the load uniformly distributed along the length of the element with the intensity level of $q_{cb}^{k}=\gamma_k A_k$, where γ_k is the specific gravity of the material, and A_k is the cross-sectional area of the element *k*. Due to the need in further calculations of the element let's split this load in two directions:

$$\begin{Bmatrix} q_{cb}^{x_k^{'}} \\ q_{cb}^{y_k^{'}} \end{Bmatrix} = \begin{bmatrix} \cos\theta_k & \sin\theta_k \\ -\sin\theta_k & \cos\theta_k \end{bmatrix} \begin{Bmatrix} 0 \\ -q_{cb}^{k} \end{Bmatrix}$$

The intensity of the uniformly distributed load acting perpendicular to the axis of the element arising from the acceleration $\overline{w}_{kp}^{y_k^{'}}$ is equal to $q_n^{y_k^{'}}=-\gamma_k A_k \overline{w}_{kp}^{y_k^{'}}/g$ and is directed opposite to the direction of $\overline{w}_{kp}^{y_k^{'}}$. The angular velocity $\overline{\omega}_k$ of the element *k* rotation about the pole P_k causes the distributed loads of triangular shape with the intensity $q_b^{x_k^{'}}=\gamma_k A_k \omega_k^2 x_k^{'}/g$ acting along the axis of the element, and always directed from the axis of rotation along the axis $x_k^{'}$. The angular velocity of the element *k* rotational motion about axis $Z_k^{'}$, causes the distributed loads of triangular shape with the intensity $q_b^{y_k^{'}}=-\gamma_k A_k \varepsilon_k x_k^{'}/g$. These loads always act perpendicular to the axis of the element and are directed opposite to the direction of the vector $\bar{\varepsilon}_k \times \bar{l}_k$.

By summing the loads acting along the axis $Y_k^{'}$ it is found that the intensity of the total load is varying along the length of the element in a linear fashion: $q_y(x_k^{'})=a_{kq}+b_{kq}x_k^{'}$; $a_{kq}=-\gamma_k A_k \cos\theta_k-\gamma_k A_k w_{kp}^{y_k^{'}}/g$, $b_{kq}=-\gamma_k A_k \varepsilon_k/g$. Similarly, summing the loads acting along the axis of the element ($X_k^{'}$) it can be seen that their intensity also varies in a linear fashion and is represented by the following equation: $q_x(x_k^{'})=a_{kn}+b_{kn}x_k^{'}$; $a_{kn}=-\gamma_k A_k \sin\theta_k-\gamma_k A_k w_{kp}^{x_k^{'}}/g$, $b_{kn}=\frac{\gamma_k A_k}{g}\omega_k^2$. Thus, the intensity of inertia and gravity forces of the element has quite definite analytical expression. By using them and the finite bar elements method, based on the straight uniform bar Ref. [1-2] the finite-element models, analytically accounting for distributed inertia of the plane-parallel motion, and more accurate for the study of the link mechanisms kinetostatics (dynamic force analysis) are deduced. The finite bar elements method equilibrium equations of the bar element are as follows (Fig. 2):

$$\frac{dN}{dx}+q_x=\frac{dN}{dx}+a_n+b_n x=0,\quad \frac{dQ_y}{dx}+q_y=\frac{dQ_y}{dx}+a_q+b_q x=0;\quad \frac{dM_z}{dx}-Q_y=0;$$

$$\frac{dQ_z}{dx}=0; \quad \frac{dM_y}{dx}-Q_z=0 \quad \frac{dM}{dx}=0 \tag{1}$$

Equations relating deformations and elastic displacements for rods:

$$\varepsilon_x=\frac{du}{dx}, \quad \chi=\frac{d\varphi}{dx}, \quad \chi_y=-\frac{d^2 w_z}{dx^2}, \quad \chi_z=-\frac{d^2 w_y}{dx^2} \tag{2}$$

Equations relating deformations and forces for rods:

$$N=EF\varepsilon_x; \quad M=GJ\chi; \quad M_y=EJ_y\chi_y; \quad M_z=EJ_z\chi_z \tag{3}$$

The «k», «'» are not shown for the convenience of writing. Taking into account positive direction of the external and internal geometric and force factors (Fig.2,3):

$$\begin{gathered}\varphi_{y0}=-\varphi_2^{ij}; \quad \varphi_{yl}=-\varphi_2^{ji}; N_0=-R_{1N\xi}^{ij}; \quad Q_{y0}=-R_{2N\xi}^{ij}; \quad Q_{z0}=-R_{3N\xi}^{ij}; \\ M_0=-R_{1M\xi}^{ij}; \quad M_{y0}=-R_{2M\xi}^{ij}; \quad M_{z0}=-R_{3M\xi}^{ij};\end{gathered} \tag{4}$$

and other factors have the same direction; and, omitting the intermediate rearrangements of (1) - (4), the finite bar elements method basic equilibrium relations for the straight uniform bar element are deduced (Fig. 3):

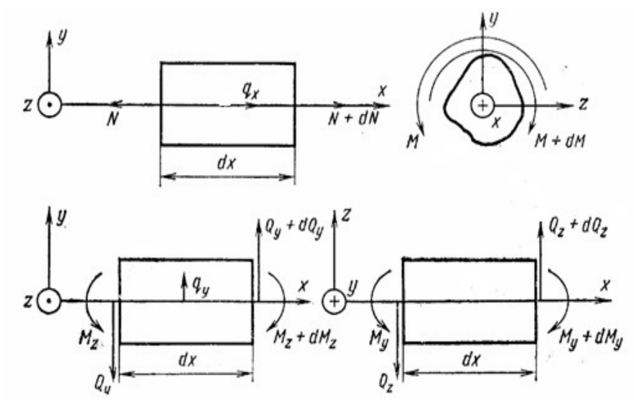

Fig.2. Sign Convention for Force Impacts

$$\begin{Bmatrix}\vec{R}_\xi^{ij} \\ \vec{R}_\xi^{ji}\end{Bmatrix}=\begin{bmatrix}B_{11}^{ij} & B_{12}^{ij} \\ B_{21}^{ij} & B_{22}^{ij}\end{bmatrix}\begin{Bmatrix}\vec{U}_\xi^{ij} \\ \vec{U}_\xi^{ji}\end{Bmatrix}+\begin{Bmatrix}\vec{Q}^{ij} \\ \vec{Q}^{ji}\end{Bmatrix} \tag{5}$$

coupling generalized $\vec{R}_{\xi}^{ij} = \left[R_{1N\xi}^{ij} R_{2N\xi}^{ij} R_{3N\xi}^{ij} R_{1M\xi}^{ij} R_{2M\xi}^{ij} R_{3M\xi}^{ij}\right]^T$, acting in the bar nodal points with the nodal generalized $\vec{R}_{\xi}^{ji} = \left[R_{1N\xi}^{ji} R_{2N\xi}^{ji} R_{3N\xi}^{ji} R_{1M\xi}^{ji} R_{2M\xi}^{ji} R_{3M\xi}^{ji}\right]^T$ displacements $\vec{U}_{\xi}^{ij} = \left[u_1^{ij} u_2^{ij} u_3^{ij} \varphi_1^{ij} \varphi_2^{ij} \varphi_3^{ij}\right]^T$, $\vec{U}_{\xi}^{ji} = \left[u_1^{ji} u_2^{ji} u_3^{ji} \varphi_1^{ji} \varphi_2^{ji} \varphi_3^{ji}\right]^T$. Square submatrices $\left[B_{rq}^{ij}\right] (r = 1,2;\quad q = 1,2)$ stiffness matrices $\left[B^{ij}\right]$ of the bar element do not change Ref. [2, 3], and subvectors $\vec{Q}^{ij}$ and $\vec{Q}^{ji}$ are as shown in (6). Thus, elements (6) of subvectors $\vec{Q}^{ij}$ and $\vec{Q}^{ji}$ analytically accounting not only for the weight, but the distributed inertia of plane-parallel motion of link mechanism element as well are deduced [3]:

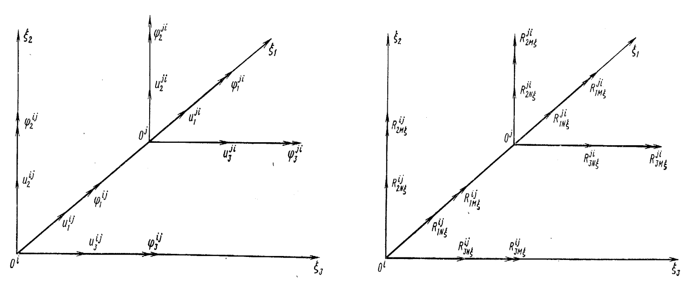

Fig.3. Bar finite element stresses and other displacement components

$$\vec{Q}^{ij} = \begin{Bmatrix} -\dfrac{a_n l}{2} - \dfrac{b_n l^2}{2} \\ -\dfrac{a_q l}{2} - \dfrac{3 b_q l^2}{20} \\ 0 \\ 0 \\ 0 \\ -\dfrac{a_q l^2}{12} - \dfrac{b_q l^3}{30} \end{Bmatrix}, \quad \vec{Q}^{ji} = \begin{Bmatrix} -\dfrac{a_n l}{2} - \dfrac{b_n l^2}{2} \\ -\dfrac{a_q l}{2} - \dfrac{7 b_q l^2}{20} \\ 0 \\ 0 \\ 0 \\ \dfrac{a_q l^2}{12} + \dfrac{b_q l^3}{30} \end{Bmatrix} \tag{6}$$

3. Spatial movement

Let's consider the spatial motion of the k element of the mechanism relative to the fixed coordinate system OXYZ. By Chasles theorem every displacement of the free body from one position to another of can be deduced by linear displacement with the arbitrarily chosen pole and rotation about an arbitrary axis through the pole. Then acceleration of any point M of the free solid body in the projections onto the moving axes $P_k X_k^{'} Y_k^{'} Z_k^{'}$ has the form:

$$\begin{cases} w_{x_k} = w_{P_{xk}} + \frac{d\omega_{y_k}}{dt} z_k - \frac{d\omega_{z_k}}{dt} y_k + \omega_{x_k}\left(\omega_{x_k} x_k + \omega_{y_k} y_k + \omega_{z_k} z_k\right) - \omega_k^2 x_k \\ w_{y_k} = w_{P_{yk}} + \frac{d\omega_{z_k}}{dt} x_k - \frac{d\omega_{x_k}}{dt} z_k + \omega_{y_k}\left(\omega_{x_k} x_k + \omega_{y_k} y_k + \omega_{z_k} z_k\right) - \omega_k^2 y_k \\ w_{z_k} = w_{P_{zk}} + \frac{d\omega_{x_k}}{dt} y_k - \frac{d\omega_{y_k}}{dt} x_k + \omega_{z_k}\left(\omega_{x_k} x_k + \omega_{y_k} y_k + \omega_{z_k} z_k\right) - \omega_k^2 z_k \end{cases}$$

Where $w_{P_{xk}}, w_{P_{yk}}, w_{P_{zk}}$ are the projections of the acceleration w_{Pk} of the pole P_k in coordinates $P_k X_k^{'} Y_k^{'} Z_k^{'}$ (Fig. 1). In this case the centers of gravity of the element cross-sections are located along the axis of $P_k X_k^{'}$, therefore, the coordinates of M are $x_k = x_k^{'}$, $y_k = z_k = 0$; then

$$\begin{cases} w_{x_k} = w_{P_{xk}} + \omega_{x_k}^2 x_k - \omega_k^2 x_k \\ w_{y_k} = w_{P_{yk}} + \frac{d\omega_{z_k}}{dt} x_k + \omega_{y_k}\omega_{x_k} x_k \\ w_{z_k} = w_{P_{zk}} - \frac{d\omega_{y_k}}{dt} x_k + \omega_{x_k} x_k \omega_{z_k} \end{cases}$$

Accelerations w_{xk} along the axis $P_k X_k$ cause distributed inertial forces, and the intensity of the total load varies along the length of the element in a linear fashion and is deduced using the following expression: $n_k(x_k) = a_{kn} + b_{kn} x_k$, where $a_{kn} = -\gamma_k A_k \sin\theta_k - \gamma_k A_k w_{P_{xk}}/g$, $b_{kn} = -\gamma_k A_k \omega_{xk}^2/g + \gamma_k A_k \omega_k^2/g$ by algebraic summing of all loads forcing on the element in the plane $P_k X_k Y_k$, it is found that the intensity of the total load varies along the length of the element in a linear fashion and is deduced using the following expression: $q_k^y(x_k) = a_{kq}^y + b_{kq}^y x_k$, $a_{kq}^y = -\gamma_k A_k \cos\theta_k - \gamma_k A_k w_{P_{yk}}/g$, θ_k - tilting angle of the element k in the plane OXY by algebraic summing of all loads acting on the element in the plane $P_k X_k Y_k$, it is found that the intensity of the total load varies along the length of the element in a linear fashion and is deduced using the following expression:

$$q_k^z(x_k) = a_{kq}^z + b_{kq}^z x_k, \quad a_{kq}^z = -\gamma_k A_k w_{P_{zk}}/g, \quad b_{kq}^z = \frac{\gamma_k A_k}{g}\frac{d\omega_{yk}}{dt} x_k - \frac{\gamma_k A_k}{g}\omega_{zk}\omega_{xk} x_k$$

The finite bar elements method equilibrium equations of the bar element (Fig. 2) in the spatial case are as follows:

$$\frac{dN}{dx} + q_x = \frac{dN}{dx} + a_n + b_n x = 0, \quad \frac{dM}{dx} + m = 0;$$

$$\frac{dQ_y}{dx}+q_y=\frac{dQ_y}{dx}+a_q+b_q x=0;\quad \frac{dM_z}{dx}-Q_y=0;\quad \frac{dQ_z}{dx}+q_z=0;\quad \frac{dM_y}{dx}-Q_z=0 \tag{7}$$

When you set the length of «*l*», the initial displacement and rotation angles (4), Solving (2,3,7) for the elastic displacement, we obtain for the spatial case [3]:

$$u=u_o+(u_l-u_o)\frac{x}{l}+\frac{a_n x}{2EF}(x-l)-\frac{b_n x}{6EF}(x^2-l^2),\ \varphi=\varphi_o+(\varphi_l-\varphi_o)\frac{x}{l}+\frac{ml^2}{2GJ}\frac{x}{l}(1-\frac{x}{l});$$

$$w_y=(1-3\frac{x^2}{l^2}+2\frac{x^3}{l^3})w_{yo}+x(1-\frac{x}{l})^2\varphi_{zo}+\frac{x^2}{l^2}(3-2\frac{x}{l})w_{yl}-x(1-\frac{x}{l})\frac{x}{l}\varphi_{zl}+$$
$$+\frac{a_q^y}{EJ_z}\cdot\frac{x^2}{24}(l-x)^2+\frac{b_q^y}{EJ_z}\cdot\frac{x^2}{120}\cdot\left(2l^3-3l^2x+x^3\right)\quad .$$

$$w_z=(1+\frac{2x^3}{l^3}-\frac{3x^2}{l^2})w_{Z_o}+\frac{x^2}{l^2}(3-2\frac{x}{l})w_{zl}+x\cdot\left(1-\frac{x}{l}\right)^2\varphi_{y_o}+\left(\frac{x}{l}-1\right)\cdot\frac{x^2}{l}\cdot\varphi_{yl}$$
$$+\frac{a_q^z}{EJ_z}\cdot\frac{x^2}{24}(l-x)^2+\frac{b_q^z}{EJ_z}\cdot\frac{x^2}{120}\cdot\left(2l^3-3l^2x+x^3\right)$$

For internal of force factors spatial case we get [3]:

$$N_o=EF\left[\frac{u_l-u_o}{l}+\frac{a_n l}{2EF}+\frac{b_n l^2}{2EF}\right],\ N_l=EF\left[\frac{u_l-u_o}{l}-\frac{a_n l}{2EF}-\frac{b_n l^2}{2EF}\right]$$

$$Q_{yo}=-\frac{6EJ_Z}{l^3}(2w_{yo}+l\varphi_{zo}-2w_{yl}+l\varphi_{zl})+\frac{a_q^y l}{2EJ_y}+\frac{3b_q^y l^2}{20EJ_y}$$

$$Q_{yl}=-\frac{6EJ_Z}{l^3}(2w_{yo}+l\varphi_{zo}-2w_{yl}+l\varphi_{zl})-\frac{a_q^y l}{2EJ_y}-\frac{7b_q^y l^2}{20EJ_y}$$

$$Q_{zo}=-\frac{6EJ_Z}{l^3}(2w_{zo}+l\varphi_{yo}-2w_{zl}+l\varphi_{yl})+\frac{a_q^z l}{2EJ_y}+\frac{3b_q^z l^2}{20EJ_y}$$

$$Q_{zl}=-\frac{6EJ_Z}{l^3}(2w_{zo}+l\varphi_{yo}-2w_{zl}+l\varphi_{yl})-\frac{a_q^z l}{2EJ_y}-\frac{7b_q^z l^2}{20EJ_y}$$

$$M_{z0}=\frac{EJ_z}{l^2}(6w_{yo}+4l\varphi_{zo}-6w_{yl}+2l\varphi_{zl})-\frac{a_q^y l^2}{12EJ_z}-\frac{b_q^y l^3}{30EJ_z}$$

$$M_{zl}=-\frac{EJ_z}{l^2}(6w_{yo}+2l\varphi_{zo}-6w_{yl}+4l\varphi_{zl})-\frac{a_q^y l^2}{12EJ_z}-\frac{b_q^y l^3}{20EJ_z}$$

$$M_{y0} = -\frac{EJ_z}{l^2}(-6w_{zo} + 4l\varphi_{yo} + 6w_{zl} + 2l\varphi_{zl}) - \frac{a_q^z l^2}{12EJ_z} - \frac{b_q^z l^3}{30EJ_z}$$

$$M_{yl} = -\frac{EJ_z}{l^2}(-6w_{zo} + 4l\varphi_{yo} + 6w_{zl} + 2l\varphi_{zl}) - \frac{a_q^z l^2}{12EJ_z} - \frac{b_q^z l^3}{20EJ_z}$$

$$M_o = \frac{GJ}{l}(\varphi_l - \varphi_o) + \frac{ml}{2} \qquad M_l = \frac{GJ}{l}(\varphi_l - \varphi_o) - \frac{ml}{2}$$

Then, considering (5), the subvectors $\vec{Q}^{ij}$ and $\vec{Q}^{ji}$ of the finite bar elements method equilibrium equations for straight uniform bar element analytically accounting not only for the weight, but the distributed inertia of spatial motion (8) of the element k, the following expression is deduced [3]:

$$\vec{Q}^{ij} = \begin{Bmatrix} -\frac{a_{kn}l}{2} - \frac{b_{kn}l^2}{2} \\ -\frac{a_{kq}^y l}{2} - \frac{3b_{kq}^y l^2}{20} \\ -\frac{a_{kq}^z l}{2} - \frac{3b_{kq}^z l^2}{20} \\ \frac{ml}{2} \\ \frac{a_{kq}^z l^2}{12} + \frac{b_{kq}^z l^3}{30} \\ -\frac{a_{kq}^y l^2}{12} - \frac{b_{kq}^y l^3}{30} \end{Bmatrix}, \quad \vec{Q}^{ji} = \begin{Bmatrix} -\frac{a_{kn}l}{2} - \frac{b_{kn}l^2}{2} \\ -\frac{a_{kq}^y l}{2} - \frac{7b_{kq}^y l^2}{20} \\ -\frac{a_{kq}^z l}{2} - \frac{7b_{kq}^z l^2}{20} \\ -\frac{ml}{2} \\ -\frac{a_{kq}^z l^2}{12} - \frac{b_{kq}^z l^3}{30} \\ \frac{a_{kq}^y l^2}{12} + \frac{b_{kq}^y l^3}{30} \end{Bmatrix} \qquad (8)$$

4. Conclusion

Thus, the elements (6) of the subvectors $\vec{Q}^{ij}$ and $\vec{Q}^{ji}$ analytically accounting not only for the weight, but the distributed inertia of the bar mechanism element spatial motion are deduced.

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Резюме

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СЫРЫҚТЫ МЕХАНИЗМНІҢ КЕҢІСТІКТЕГІ ҚОЗҒАЛЫСЫНЫҢ

ТАРАТЫЛҒАН ИНЕРЦИЯСЫ

Басты күш векторы және инерция күшінің моменттері ретінде, әдетте буындар қозғалысының инерция күші салмақ орталығына түсіріледі, ал салмақ орталығы шеткі элементтер әдісі түйін түрінде орналасады. Мұнда талданған қозғалыс инерциясын таратылуы ескеріліп, сырықтық механизм құрылымының нақты шеткі элементтер әдісі алынған. Барлық инерциялық жүктеменің алгебралық қорытындысы перпендикуляр бағытпен және буын өсі бойымен әсер ету арқылы олардың өршеленуі буын ұзындығын

сызықты өзгертеді. Шынжырлы механизм құрылымының соңғы элементтер үлгісі буындарының серпімділігін ескеріп кинетостатика және динамика зерттеу үшін, түзусызықты біртекті ось үшін, осы және шеткі элементтер әдісін пайдалану арқылы нақты дәлелдер алынды. Шеткі элементтер әдісінде белгілі матрицалық қатынасында:

$$\begin{Bmatrix} \vec{R}_{\xi}^{ij} \\ \vec{R}_{\xi}^{ji} \end{Bmatrix} = \begin{bmatrix} B_{11}^{ij} & B_{12}^{ij} \\ B_{21}^{ij} & B_{22}^{ij} \end{bmatrix} \begin{Bmatrix} \vec{U}_{\xi}^{ij} \\ \vec{U}_{\xi}^{ji} \end{Bmatrix} + \begin{Bmatrix} \vec{Q}^{ij} \\ \vec{Q}^{ji} \end{Bmatrix},$$

$\vec{R}_{\xi}^{ij} = \left[R_{1N\xi}^{ij} R_{2N\xi}^{ij} R_{3N\xi}^{ij} R_{1M\xi}^{ij} R_{2M\xi}^{ij} R_{3M\xi}^{ij} \right]^T$, $\vec{R}_{\xi}^{ji} = \left[R_{1N\xi}^{ji} R_{2N\xi}^{ji} R_{3N\xi}^{ji} R_{1M\xi}^{ji} R_{2M\xi}^{ji} R_{3M\xi}^{ji} \right]^T$ жалпыланған реактивті күштерді және буынды серпімді ауысулармен $\vec{U}_{\xi}^{ij} = \left[u_1^{ij} u_2^{ij} u_3^{ij} \varphi_1^{ij} \varphi_2^{ij} \varphi_3^{ij} \right]^T$, $\vec{U}_{\xi}^{ji} = \left[u_1^{ji} u_2^{ji} u_3^{ji} \varphi_1^{ji} \varphi_2^{ji} \varphi_3^{ji} \right]^T$ байланыстырғанмен, қатаңдық матрицасы элементтері $\left[B_{rq}^{ij} \right], (r = 1,2; \quad q = 1,2)$ өзгермейді, ал векторлар $\vec{Q}^{ij}$, $\vec{Q}^{ji}$ жаңа түрге өзгереді:

$$\vec{Q}^{ij} = \begin{Bmatrix} -\dfrac{a_{kn} l}{2} - \dfrac{b_{kn} l^2}{2} \\ -\dfrac{a_{kq}^{y} l}{2} - \dfrac{3 b_{kq}^{y} l^2}{20} \\ -\dfrac{a_{kq}^{z} l}{2} - \dfrac{3 b_{kq}^{z} l^2}{20} \\ \dfrac{ml}{2} \\ \dfrac{a_{kq}^{z} l^2}{12} + \dfrac{b_{kq}^{z} l^3}{30} \\ -\dfrac{a_{kq}^{y} l^2}{12} - \dfrac{b_{kq}^{y} l^3}{30} \end{Bmatrix}, \quad \vec{Q}^{ji} = \begin{Bmatrix} -\dfrac{a_{kn} l}{2} - \dfrac{b_{kn} l^2}{2} \\ -\dfrac{a_{kq}^{y} l}{2} - \dfrac{7 b_{kq}^{y} l^2}{20} \\ -\dfrac{a_{kq}^{z} l}{2} - \dfrac{7 b_{kq}^{z} l^2}{20} \\ -\dfrac{ml}{2} \\ -\dfrac{a_{kq}^{z} l^2}{12} - \dfrac{b_{kq}^{z} l^3}{30} \\ \dfrac{a_{kq}^{y} l^2}{12} + \dfrac{b_{kq}^{y} l^3}{30} \end{Bmatrix}$$

Кілт сөздер: инерция, шеткі элементтер әдісі, иінтіректі механизм.

Резюме

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РАСПРЕДЕЛЕННАЯ ИНЕРЦИЯ ПРОСТРАНСТВЕННОГО ДВИЖЕНИЯ СТЕРЖНЕВОГО МЕХАНИЗМА

Силы инерции движения звеньев обычно приводятся к центру масс в виде главных векторов сил и моментов сил инерции, а центр масс входит конечно-элементную модель в виде узла. Здесь получены более точные конечно-элементные модели конструкций стержневых механизмов, аналитически учитывающие распределенную инерцию движения. Алгебраически суммируя все инерционные нагрузки, действующие по направлению перпендикулярно и вдоль оси звена, видно, что их интенсивность меняется по длине звена линейно. Используя это и метод конечных элементов (МКЭ) для прямолинейного однородного стержня, получены более точные для исследования кинетостатики и динамики конструкций рычажных механизмов конечно-элементные модели с учетом упругости звеньев. В известном матричном соотношении МКЭ:

$$\left\{\begin{matrix}\vec{R}_{\xi}^{ij}\\ \vec{R}_{\xi}^{ji}\end{matrix}\right\}=\begin{bmatrix}B_{11}^{ij} & B_{12}^{ij}\\ B_{21}^{ij} & B_{22}^{ij}\end{bmatrix}\left\{\begin{matrix}\vec{U}_{\xi}^{ij}\\ \vec{U}_{\xi}^{ji}\end{matrix}\right\}+\left\{\begin{matrix}\vec{Q}^{ij}\\ \vec{Q}^{ji}\end{matrix}\right\},$$

связывающее обобщенные реактивные силы $\vec{R}_{\xi}^{ij}=\left[R_{1N\xi}^{ij}R_{2N\xi}^{ij}R_{3N\xi}^{ij}R_{1M\xi}^{ij}R_{2M\xi}^{ij}R_{3M\xi}^{ij}\right]^T$ и $\vec{R}_{\xi}^{ji}=\left[R_{1N\xi}^{ji}R_{2N\xi}^{ji}R_{3N\xi}^{ji}R_{1M\xi}^{ji}R_{2M\xi}^{ji}R_{3M\xi}^{ji}\right]^T$ с узловыми перемещениями $\vec{U}_{\xi}^{ij}=\left[u_1^{ij}u_2^{ij}u_3^{ij}\varphi_1^{ij}\varphi_2^{ij}\varphi_3^{ij}\right]^T$ и $\vec{U}_{\xi}^{ji}=\left[u_1^{ji}u_2^{ji}u_3^{ji}\varphi_1^{ji}\varphi_2^{ji}\varphi_3^{ji}\right]^T$ подматрицы $\left[B_{rq}^{ij}\right],(r=1,2;\quad q=1,2)$ матрицы жесткости $\left[B^{ij}\right]$ не изменяются, а подвекторы $\vec{Q}^{ij}$ и $\vec{Q}^{ji}$ получают новый вид:

$$\vec{Q}^{ij} = \begin{Bmatrix} -\dfrac{a_{kn}l}{2} - \dfrac{b_{kn}l^2}{2} \\ -\dfrac{a_{kq}^{y}l}{2} - \dfrac{3b_{kq}^{y}l^2}{20} \\ -\dfrac{a_{kq}^{z}l}{2} - \dfrac{3b_{kq}^{z}l^2}{20} \\ \dfrac{ml}{2} \\ \dfrac{a_{kq}^{z}l^2}{12} + \dfrac{b_{kq}^{z}l^3}{30} \\ -\dfrac{a_{kq}^{y}l^2}{12} - \dfrac{b_{kq}^{y}l^3}{30} \end{Bmatrix}, \quad \vec{Q}^{ji} = \begin{Bmatrix} -\dfrac{a_{kn}l}{2} - \dfrac{b_{kn}l^2}{2} \\ -\dfrac{a_{kq}^{y}l}{2} - \dfrac{7b_{kq}^{y}l^2}{20} \\ -\dfrac{a_{kq}^{z}l}{2} - \dfrac{7b_{kq}^{z}l^2}{20} \\ -\dfrac{ml}{2} \\ -\dfrac{a_{kq}^{z}l^2}{12} - \dfrac{b_{kq}^{z}l^3}{30} \\ \dfrac{a_{kq}^{y}l^2}{12} + \dfrac{b_{kq}^{y}l^3}{30} \end{Bmatrix}$$

Ключевые слова: инерция, метод конечных элементов, рычажный механизм.

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