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A. Seitmuratov[1], M. Ramazanov[2], N. Medeubaev[2], B. Kaliev[1]

[1]The Korkyt Ata Kyzylorda State University, Kyzylorda, Kazakhstan,
[2]The E.A. Buketov Karaganda State University, Karaganda, Kazakhstan.
E-mail: angisin_@mail.ru, ramamur@mail.ru, medeubaev65@mail.ru, kaliev_b@mail.ru

MATHEMATICAL THEORY OF VIBRATION OF ELASTIC OR VISCOELASTIC PLATES, UNDER NON-STATIONARY EXTERNAL INFLUENCES

Abstract. In this work, attempt was made to present the mathematical theory of oscillations of an elastic or viscoelastic plate to study its dynamic behavior under nonstationary external influences. On the basis of this approach, exact equations of longitudinal and transverse oscillations of viscoelastic plates are derived with and without allowance for initial displacements and stresses, approximate equations for the physical nonlinearity of the material. For all problems, expressions are obtained for all displacements and stresses along the thickness of the plate and basic boundary-value formulated problems that lead to longitudinal or transverse vibrations of the plate. On the basis of exact equations some approximate equations that follow with some degree of accuracy are analyzed and approximate boundary value problems formulated for them.

Key words: elastic, viscoelastic, nonstationary, oscillations, nonlinear, deformable, longitudinal, transverse.

Investigation of wave processes in restricted deformable bodies are reduced to complicated mathematical problem, which is generally at the present stage can not be solved either analytical or numerical methods.

Even for deformable media which is described by the simplest models, such as elastic and viscoelastic media, many nonstationary problems have not been investigated and there are no methods to solve these problems in accurate formulation. Therefore, many applied problems in various fields of technology are solved on simplified models that reduce the spatial problems of dynamics to two-dimensional or one-dimensional ones. Such simplified models are plates, rods and shells.

All naturally occurring environments on the nature of the dynamic behavior can be divided into perfectly elastic and differential elastic.

. The first group includes environments of mechanical characteristics that are close to each other. When studying dynamics and wave processes, such media can be considered as homogeneous medium with averaged mechanical characteristics.

Для дифференциально-упругих сред механические характеристики, составляющие двухкомпонентную среду, значительно различаются друг от друга.

For differential-elastic media, the mechanical characteristics which constituting the two-component medium differ significantly from each other.

We shall consider two-component media consisting of two elastic components or from an elastic porous skeleton and liquid filler.

We introduce the concept of the porosity of the medium.

The porosity of the medium will be denoted by a quantity determined by the formula:

$$K_0 = V_p / V_s, \tag{1}$$

where - the volume of the second component relative to the first medium in a volume element as a whole, V_s - the total volume of the elementary sample.

We'll assume, that $K_0 = const$.

The first type of two-component medium consisting of two elastic components is characterized by different conditions of adhesion between them and in the general case of the stress-strain relationship, we write in the form:

$$\sigma_{ij}^{(1)} = \sigma_{ij}^{(1)}\left(\varepsilon_x^{(1)}, \varepsilon_y^{(1)}, \varepsilon_z^{(1)}; \varepsilon_x^{(2)}, \varepsilon_y^{(2)}, \varepsilon_z^{(2)}; \varepsilon_{xy}^{(1)}, \varepsilon_{yz}^{(1)}, \varepsilon_{xz}^{(1)}; \varepsilon_{xy}^{(2)}, \varepsilon_{yz}^{(2)}, \varepsilon_{xz}^{(2)}\right) \tag{2}$$

$$\sigma_{ij}^{(2)} = \sigma_{ij}^{(2)}\left(\varepsilon_x^{(1)}, \varepsilon_y^{(1)}, \varepsilon_z^{(1)}; \varepsilon_x^{(2)}, \varepsilon_y^{(2)}, \varepsilon_z^{(2)}; \varepsilon_{xy}^{(1)}, \varepsilon_{yz}^{(1)}, \varepsilon_{xz}^{(1)}; \varepsilon_{xy}^{(2)}, \varepsilon_{yz}^{(2)}, \varepsilon_{xz}^{(2)}\right),$$

where the index " 1 " refers to the first component, the index " 2 " - to the second.

The second type of two-component medium requires some explanation.

The term "pore" refers to a medium with openly communicating pores.

The connection between the constituent components of the medium will be considered imperfect, i.e. The liquid component can not flow out of the medium.

Let the continuous deformable medium consist of two elastic continua with different mechanical characteristics whose densities will be denoted by $\rho_j\,(j = 1{,}2)$, the displacement vectors of the points.

$$\vec{U}^{(j)}\ (j = 1{,}2).$$

Depending on the deformation stresses are [1]

$$\sigma_{\alpha\alpha}^{(1)} = -\alpha_2 + \lambda_1 e^{(1)} + 2\mu_1\varepsilon_{\alpha\alpha}^{(1)} + \lambda_3 e^{(2)} + 2\mu_3\varepsilon_{\alpha\alpha}^{(2)};$$

$$\frac{1}{2}\left(\sigma_{\alpha\beta}^{(1)} + \sigma_{\beta\alpha}^{(1)}\right) = 2\mu_1\varepsilon_{\alpha\beta}^{(1)} + 2\mu_3\varepsilon_{\alpha\beta}^{(2)}; \tag{3}$$

$$\frac{1}{2}\left(\sigma_{\alpha\beta}^{(1)} - \sigma_{\beta\alpha}^{(1)}\right) = -\lambda_5\left(h_{\beta\alpha} - h_{\alpha\beta}\right),$$

for the first component,

$$\sigma_{\alpha\alpha}^{(2)} = \alpha_2 + \lambda_2 e^{(2)} + 2\mu_2\varepsilon_{\alpha\alpha}^{(2)} + \lambda_4 e^{(1)} + 2\mu_4\varepsilon_{\alpha\alpha}^{(1)};$$

$$\frac{1}{2}\left(\sigma_{\alpha\beta}^{(2)} + \sigma_{\beta\alpha}^{(2)}\right) = 2\mu_2\varepsilon_{\alpha\beta}^{(2)} + 2\mu_3\varepsilon_{\alpha\beta}^{(1)}; \tag{4}$$

$$\frac{1}{2}\left(\sigma_{\alpha\beta}^{(2)} - \sigma_{\beta\alpha}^{(2)}\right) = \lambda_5\left(h_{\beta\alpha} - h_{\alpha\beta}\right),$$

For the second component,
in this case take place depending

$$\mu_4 = \mu_3;$$

$$\alpha_2 = \lambda_3 - \lambda_4 \tag{5}$$

here $\alpha_2, \lambda_2 \mu_2$ elastic constants,

$$\varepsilon_{\alpha\beta}^{(j)} = \frac{1}{2}\left(\frac{\partial U_\alpha^{(j)}}{\partial\beta} + \frac{\partial U_\beta^{(j)}}{\partial\alpha}\right);\ (j = 1{,}2); \tag{6}$$

$$h_{\alpha\beta} = \frac{\partial U_\alpha^{(2)}}{\partial\beta} + \frac{\partial U_\beta^{(1)}}{\partial\alpha};$$