TRANSPORT PROPERTIES OF INERTIAL CONFINEMENT FUSION PLASMAS

Abstract. In this paper the transport properties of non-isothermal dense deuterium-tritium plasmas were studied. Based on the effective interaction of potentials between particles, the Coulomb logarithm for a two-temperature nonisothermal dense plasma was obtained. These potentials take into consideration long-range multiparticle screening effects and short-range quantum-mechanical effects in two-temperature plasmas. Transport processes in such plasmas were studied using the Coulomb logarithm. The obtained results were compared with the theoretical works of other authors and with the results of molecular dynamics simulations.

Keywords: dense plasma, Coulomb logarithm, inertial confinement fusion, transport properties.

Introduction. Nowadays, much attention is paid to the high density energy of the substance and matters at high pressures and temperatures. Research in the field of fusion power with inertial confinement (ITS) in the heavy ion beams have a special significance among the works devoted to various aspects of this problem. Basically, these heavy ion accelerators are well known as the main tool in experimental studies of nuclear physics, elementary particle physics and dense plasma physics [1-3]. However, at present time, the lack of new theoretical and experimental data on the transport properties of deuterium-tritium (DT) plasma produced under compression of target with heavy ions beam, requires an adequate qualitative description of the interaction of heavy ions with a dense plasma in a wide range of parameters and provides an additional impetus to the research in this area. Understanding and control of high-pressure behavior of DT fuel are crucial for the success of the experiments with the ignition. Accurate knowledge of transfer coefficients in dense DT plasma is essential for the correct description of the processes occurring in the inertial confinement. This issue has been the subject of many theoretical and experimental studies [4-7]. In some researches [8], the transport properties of dense plasmas, such as diffusion and viscosity were studied on the basis of molecular dynamics simulation (MD) using density functional theory to describe the electron component of the plasma (DFTMD). Kress and others [9] obtained values of viscosity and diffusion of DT plasma, using a density functional theory at finite temperatures on the basis of Kohn-Sham theorem on molecular dynamics and DFTMD.

One of the most promising approaches for the study of transport properties of dense DT plasma is a binary collision approximation, and we can use two approaches. One of them is to calculate the transport coefficients determined on the basis of the particles scattering which interact via potential. The second approach is producing kinetic equation, which collision integral has logarithmically divergent integral on impact parameters, which can be replaced by the Coulomb logarithm.

In this paper, previously proposed model [17, 18, 19] for description of the dense plasma properties on the basis of effective interaction potentials [20, 21] is extended for calculation of the ion transport properties and thermal conductivity for deuterium and deuterium-tritium plasma of inertial confinement. Below we demonstrate a brief description of the model and the results of calculation of plasma transport
properties. In order to show the correctness of the model, its results are compared with the results of quantum molecular dynamics of KMD and TFTMD modelling.

**Coulomb logarithm on the basis of the effective potential.** Transport properties are obtained from on the basis of the Coulomb logarithm using the effective potential for inertial confinement plasma. Coulomb logarithm is determined by the scattering angle in the center-of-mass system with the pair Coulomb collisions [15-17]:

$$
\lambda_{st} = \frac{1}{b_\perp} \int_0^{\theta_{am}} \sin^2 \left( \frac{\theta_c}{2} \right) b \, db ,
$$

(1)

The scattering angle in the center-of-mass system $\theta_c$ is defined as [15]:

$$
\theta_c = \pi - 2b \int_{r_c}^{r_m} \frac{dr}{r^2} \left( 1 - \frac{\Phi_{\alpha\beta}(r)}{E_c} \right)^{1/2} ,
$$

(2)

where $E_c = \frac{1}{2} m_{\alpha\beta} v^2$ - is energy of center-of-mass system, $m_{\alpha\beta} = m_\alpha m_\beta / (m_\alpha + m_\beta)$ - resulted mass of the particles of $\alpha$ and $\beta$ (ion or electron); $b_\perp = Z_\alpha Z_\beta / (m_{\alpha\beta} v^2)$. As a minimum impact parameter there was $b_{min} = \max \{ b_\perp, \lambda_{\alpha\beta} \}$, where $\lambda_{\alpha\beta} = \hbar / \sqrt{2\pi m_{\alpha\beta} k_B T}$ - thermal wavelength de Broglie.

The following dimensionless variables, such as the connection parameter are used:

$$
\Gamma_{ee} = \frac{e^2}{a k_B T_e} , \quad \Gamma_{\mu} = \frac{Z_\alpha^2 e^2}{a k_B T_\mu} \left( \frac{n_\mu}{n_e} \right)^{1/2} , \quad \Gamma_{st} = \frac{Z_\gamma^2 e^2}{a k_B T_{st}} ,
$$

(3)

where $e$ - electron charge, $a = (3/4\pi n_e)^{1/3}$ - the average interparticle distance between the particles, $k_B$ - Boltzmann constant. In the formula (2) $\Phi_{\alpha\beta}(r)$ - the potential of interaction between the particles and the distance of closest approach $r_0$ for a given impact parameter $b$ is determined from the equation:

$$
1 - \frac{\Phi_{\alpha\beta}(r_0)}{E_c} - b^2 = 0 .
$$

(4)

As it is well known, calculation of collective screening effects in the interaction of plasma particles is essential for the correct description of the static and dynamic properties of the plasma. In this paper the dense plasma is considered, which is also important for the quantum effects at small interparticle distances. In addition, we will use electron-ion effective potential, which takes into account both the quantum effects at small distances, and the effect of the screening - at large distances [18-19]:

$$
\Phi_{\alpha\beta}(r) = \frac{Z_\alpha Z_\beta}{r} \frac{1}{\gamma^2 \sqrt{1 - (2k_D/\lambda_{ee} \gamma^2)^2}} \left[ \frac{1/\lambda_{ee}^2 - B^2}{1 - B^2 \lambda_{\alpha\beta}^2} \exp(-Br) - \frac{1/\lambda_{ee}^2 - A^2}{1 - A^2 \lambda_{\alpha\beta}^2} \exp(-Ar) \right] ,
$$

$$
- \frac{Z_\alpha Z_\beta e^2 (1 - \delta_{\alpha\beta})}{r} \exp(-r/\lambda_{\alpha\beta}) ,
$$

(5)

here

$$
A^2 = \frac{\gamma^2}{2} \left( 1 + \sqrt{1 - \left( \frac{2k_D}{\lambda_{ee} \gamma^2} \right)^2} \right) , \quad B^2 = \frac{\gamma^2}{2} \left( 1 - \sqrt{1 - \left( \frac{2k_D}{\lambda_{ee} \gamma^2} \right)^2} \right) ,
$$

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where \(2k_{D} / \left( \kappa_{ee} \gamma^{2} \right) < 1, \) \(k_{D}^{2} = k_{e}^{2} + k_{i}^{2} \) - screening parameter that takes into account the electrons and ions, \(\gamma^{2} = k_{e}^{2} + 1 \kappa_{ee}^{2} \) For non-isothermal plasma we use electron-ion characteristic temperature \(T_{ei} \) [22-23]. In the research [22] it is shown, that for the correct description of the properties of the plasma electron-ion temperature should be expressed in the form: \(T_{ei} = \sqrt{T_{e}T_{i}} \). These effective potentials can be used for non-isothermal and isothermal plasma.

![Coulomb Logarithm](image)

Figure 1 – Coulomb logarithm of the two-temperature DT plasma depending (a) on of the coupling parameters (\(\Gamma\)) and (b) on the parameter of degeneration (\(\Theta\)).

In the study of the transport properties of dense high-temperature plasma, the values of the Coulomb logarithm are very vital. This paper presents the study of the transport properties of dense plasma on the basis of Coulomb logarithm using the effective potential [5].

Figure 1 shows the calculated values of the Coulomb logarithm, depending on the coupling parameter \(\Gamma\) and the degeneracy parameter \(\Theta = k_{B}T / E_{F} \) \(E_{F}\) Fermi energy) at different ratios of the electron and ion temperatures in the dense DT two-temperature plasma. We considered the loosely coupled plasma, in Fig. 1 a) the values of the Coulomb logarithm for \(\Gamma_{ee} < 1\) and \(\Gamma_{in} < 1\) were shown. The value of the Coulomb logarithm decreases with increasing of \(T_{e} / T_{i}\) ratio. At a given electrons temperature \(T_{e}\), the lower values of the Coulomb logarithm at higher values \(T_{e} / T_{i}\) are the result of strong screening of the atomic component of the plasma. Fig. 1 b) shows that the increase in the degeneracy parameter leads to increase of the value of the Coulomb logarithm. At constant density we have the lowest values of the screening length for higher temperatures. This leads to higher values of the Coulomb logarithm.

**Transport properties of inertial confinement dense plasma.** The phenomenon of transfer in a dense plasma is of considerable interest in the various areas of science and technology (plasma physics, inertial confinement, physics of hot dense matter, etc.) [24-25]. In particular, intense studies of fusion power require more reliable information on the transfer of coefficients, i.e., thermal conductivity coefficient, diffusion and viscosity. We will consider the dense DT plasma particles interacting through the effective potential (5).

The coefficient of diffusion, viscosity, and thermal conductivity of the plasma are connected with the effective collision rate using equations:
\[ D = \frac{k_B T}{m_e \nu_{\text{eff}}} \]  
\[ \eta = \frac{5}{4} \sqrt{\frac{m}{\pi}} \frac{(k_B T)^{3/2}}{e^x \lambda} \]  
\[ \kappa = \frac{5 n_e k_B^2 T}{m_e \nu_{\text{eff}}} \]

where \( e \) - electron charge, \( m_e \) - electron mass, \( n \) - plasma particle density, and
\[ \nu_{\text{eff}} = \left( \frac{4}{3} \right) \sqrt{2 \pi e^4 \lambda} \sqrt{\frac{m_e}{4 \pi n_e}} \left( k_B T \right)^{3/2} \]

effective collision frequency is directly proportional to the logarithm of the Coulomb. The diffusion coefficients of \( D \) and viscosity \( \eta \) are given in the following form: \( D^* = D/\omega_p a^2 \) and \( \eta^* = \eta n_i M \omega_p a^2 \), \( \kappa^* = \kappa (m_e \omega_p/a) \), where \( \omega_p = (4 \pi n_i/M)^{1/2} Z e \) - the plasma frequency for the mass of \( M \) ions. In this paper, we use \( M = (2 + 3)/2 = 2.5 \text{amu} \) [26] for considering DT.

Figure 2 – The diffusion coefficient, depending on the coupling parameter \( \Gamma \), \( T_e = T_i \): (a) for different density parameter values \( r_s \), (b) for different ratios of the electron and ion temperatures at \( r_s = 1 \).

Figure 3 – The coefficients (a) of thermal conductivity and (b) viscosity depending on the coupling parameter \( \Gamma \) for different values of the parameter of density
\( r_g, T_e = T_i. \)

Figures 2 a), 2 b) and 3 a), 3 b) show the results of diffusion, thermal conductivity and viscosity coefficient of dense plasma depending on the coupling parameter at \( r_g = 0.5, r_g = 1 \) and \( r_g = 1.5. \) In order to have a more detailed physical description of processes, obtained by numerical methods, the results will be calculated on the basis of the effective interaction potential (5), which takes into account both the quantum and the collective effects. Figures 2 a) and 3 a), 3 b) shows that at lower values of coupling parameter, the coefficients of viscosity and diffusion have higher values. For higher density, the transfer coefficients have lower values.

The coefficients of diffusion, thermal conductivity and viscosity at different temperatures and densities on the basis of the Coulomb logarithm using an effective potential (5) are obtained. Figures 4 a) and 4 b) show the coefficients of diffusion and viscosity of the dense plasma DT calculated on the basis of the Coulomb logarithm depending on the coupling parameter (\( \Gamma \)) at the plasma density, \( \rho = 6.135 \text{ g/cm}^3, \rho = 13.45 \text{ g/cm}^3, \rho = 26.3 \text{ g/cm}^3, \rho = 108 \text{ g/cm}^3, \) respectively. It is evident that coefficients of diffusion and viscosity increase with increasing temperature.

![Graphs](https://example.com/graphs.png)

Figure 4 – Transport coefficients of dense DT plasma depending on the temperature for different densities: (a) diffusion, (b) the viscosity

Figures 5 a) and 5 b) show the results of thermal conductivity of deuterium plasma from temperature for different values of the density \( \rho = 43.105 \text{ g/cm}^3 \) and \( \rho = 199.561 \text{ g/cm}^3. \) Solid red line is the thermal conductivity, obtained on the basis of the effective interaction potential (5), the black triangles are KMD modeling results [27]. Blue dot-dashed line is the total Coulomb logarithm \( \lambda = \ln \Lambda. \) In the paper [27], it was estimated KMD modeling of thermal conductivity of deuterium plasma in a wide range of densities and temperatures. S.X. Hu and others [27] used the following function to describe the results of calculations of KMD modeling of deuterium thermal conductivity on inertial confinement explosions:

\[
\kappa_{\text{KMD}} = \frac{20 (2/\pi)^{\frac{3}{2}} k_b T_i^\lambda}{\sqrt{m_e Z_{\text{eff}}^2 e^q}} \cdot \frac{0.095 (Z_{\text{eff}} + 0.24)}{1 + 0.24 Z_{\text{eff}}} \cdot \frac{1}{\ln \Lambda_{\text{KMD}}}. \tag{10}
\]
Figures 5 a) and 5 b) clearly show, that the more temperature increases the more thermal conductivity increases. We should note that at high values of density, the result obtained on the basis of the effective potential at low temperatures approaches to the result of quantum molecular dynamics method.

Figures 6 a) and 6 b) show the dependence of the diffusion coefficient and the viscosity of the coupling parameter in comparison with the results obtained on the basis of the hyperchained approximation (HCA) [28], the method of molecular dynamics (MD) [28-30], in the framework of the kinetic theory [33-34], as well as the Spitzer-Landau theory [31]. Daligalata-Balruda theory is based on approximation of pair scattering taking into account the correlation effects on the basis of the use of effective interaction potential [28-30, 32]. The effective potential of Daligalata-Balruda is connected with the mean-field potential, which is included in the pair interaction potential. Wallenborn-Baus used the renormalized kinetic theory and the generalized kinetic theory of correlation functions in phase space [34]. Figures 6 a) and 6 b) show that the results obtained on the basis of the effective potential (5) are well correspond with the results of other studies in the weakly bound limit $\Gamma_{vv} < 1$, but vary at $\Gamma_{vv} \geq 1$. The difference in weakly connected case $\Gamma_{vv} \sim 1$ is caused by non-ideal and quantum effects.
We calculated the diffusion and viscosity of DT plasma for density $\rho = 5 \text{ g/cm}^3$ and temperature in the range of $2 - 10 \text{ eV}$ using the Coulomb logarithm on the basis of the effective potential, taking into account the quantum diffraction effect at short distances and the effect of screening at large distances. Figures 7 a) and 7 b) show the comparison of calculation data on diffusion and viscosity in the DT plasma with the theoretical results of other authors [9], calculated on the basis of the density functional theory at finite temperatures using Kohn-Sham theorem in combination with molecular dynamics and functional theory density without the exchange term to describe the electron component of the plasma (DFTMD). The results are well correspond with the results of KMD and DFTMD modeling at higher temperatures, and therefore, we conclude that our method can be used in this way. At below $3 \text{ eV}$ temperatures, comparison with KMD and DFTMD results shows deterioration of correspondence, since at these temperatures, the ideality effect becomes important. Compared with the results of KMD, obtained viscosity data are not as good as for the diffusion, where the temperature dependence differs significantly, while the results obtained for the viscosity on the basis of the effective potential corresponds to the DFTMD modeling results.

![Graphs showing diffusion and viscosity of DT plasma](image)

Figure 7 – Diffusion (a) and viscosity (b) of dense DT plasma depending on the temperature at $\rho = 5.0 \text{ g/cm}^3$.

**Conclusion.** The study of transport properties in dense DT plasma on the basis of two-temperature effective interaction potential, which takes into account the quantum effects of diffraction at small distances and screening at large distances has been conducted. The results obtained using Coulomb logarithm and transfer coefficients for various plasma parameters are correspond with the theoretical and experimental results of other authors, and with the results of MD modeling. According to the results it is clear that the transport properties of dense plasma can be adequately expressed in terms of the Coulomb logarithm based on the effective potentials. Thus, the knowledge of coefficient values of transfer of heavy, charged particles in the plasma will allow obtaining more accurate calculation of the construction of a thermonuclear target.

This work was supported by the Ministry of Education and Science of the Republic of Kazakhstan under the grant №0115RK03029 (2016).

**REFERENCES**


Т.С. Рамазанов¹, С.К. Коданова¹, М.К. Исанова¹, А.Тихонов², М. Кайканов²

¹ ЭТФЖИ, ал-Фараби ат. Қазақ Үлттық университеті, Алматы, Қазақстан;
² Астана Үлттық Лабораториясы, Назарбаев Университеті, Астана, Қазақстан
isanova_moldir@mail.ru

ИНЕРЦИЯЛЫҚ ТЕРМОЯДРОЛЬЫҚ СИНТЕЗ ПЛАЗМАСЫҢЫҢ ТРАНСПОРТТЫҚ ҚАСИЕТЕРІ

Аннотация. Бұл жұмыста ізотермиялық емес, тығыз дейтерий-триций плазмасының транспорттық қасиеттері зерттелді. Болшектердің есерлерінің ефективті потенциалы негізінде екі температуралық, ізотермиялық емес, тығыз плазма үшін Кулон логарифмі алынды. Бұл потенциал екі температуралық плазмада кіші ара-кашықтықта қвантов-механикалық дифракция есерін, ұлан ара-қауіпсіздікқа әрекет етуінің есері ескереді. Кулон логарифмі комегімен ізотермиялық емес тығыз плазманың тасымалдау коэффициенттері зерттелді. Алынған ғатысшылық база авторлардың теориялық жұмыстарымен, молекулярдың динамикасына бейімделген, ізотермиялық қасиеттері салыстырылған.

Түйін сөз: тығыз плазма, Кулон логарифмі, інерциялық термоядрольық синтез, транспорттық қасиеттер.

УДК 533.93

Т.С. Рамазанов¹, С.К. Коданова¹, М.К. Исанова¹, А.Тихонов², М. Кайканов²

¹НИИЭТФ, Қазақстан национальның университет им. Аль-Фараби, Алматы, Қазақстан;
²Национальная Лаборатория Астана, Назарбаев Университет, Астана, Казахстан
isanova_moldir@mail.ru

ТРАНСПОРТНЫЕ СВОЙСТВА ПЛАЗМЫ
ИНЕРЦИОНАЛЬНОГО ТЕРМОЯДРОЕРНОГО СИНТЕЗА

Аннотация. В данной работе исследованы транспортные свойства неизотермической, плотной дейтерий-триций плазмы. На основе эффективного потенциала взаимодействия частиц для двухтемпературной, неизотермической, плотной плазмы был получен кулоновский логарифм. Данный потенциал учитывает квантово-механические эффекты дифракции на малых расстояниях и эффекты экранировки - на больших в двухтемпературной плазме. С помощью кулоновского логарифма исследованы коэффициенты переноса в неизотермической плотной плазме. Полученные результаты сравнены с теоретическими работами других авторов и результатами моделирования молекулярной динамики.

Ключевые слова: плотная плазма, кулоновский логарифм, инерционный термоядерный синтез, транспортные свойства

RAMAZANOVA T.S., chief researcher, Doctor of science, Professor, Member - correspondent NAN RK, IETP, Al-Farabi Kazakhstan National University, Almaty, Kazakhstan
KODANOVA S.K. leading researcher, Ph.D., Professor, IETP, Al-Farabi Kazakhstan National University, Almaty, Kazakhstan
ISSANOVA M.K. researcher, IETP, Al-Farabi Kazakhstan National University, Almaty, Kazakhstan
TIKHONOVA A. leading researcher, Ph.D., National Laboratory Astana, Nazarbayev University, Astana, Kazakhstan
KAIKANOVA M. leading researcher, Ph.D., National Laboratory Astana, Nazarbayev University, Astana, Kazakhstan