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APPROXIMATE SOLUTION OF THE AN ELASTIC LAYER VIBRATION TASK BEING EXPOSED OF MOVING LOAD

Annotation: The present work is devoted to the dynamics stability study of wave processes of plane and circular elements, and it is also being considered a class of plane tasks about moving loads effect on the surface of a layered elastic half-plane under the nonlinear law of stress versus deformation. While studying the wave processes of planar and circular elements in deformable bodies, the concept of phase velocity is introduced as the rate of phase medium change. In the case of harmonic vibrations of a cylindrical shell, the phase speed is expressed through the frequency of natural oscillations of freely supported along the edges of the shell, and therefore, the investigation of waves in plane and circular elements has the most direct relation to the problem of determining the own forms and vibration frequencies of finite length shells. If the studies are carried out taking into account the material rheological properties of considered system, there is a surrounding system environment, also generally displaying rheological properties, the use of these methods is considerably difficult. In such cases, the influence of rheological parameters on the components of the complex phase velocity is studied in definite values of the vibration frequencies. At present article, it is being considered an approximate solution of the nonlinear task about the effect of a moving load on an elastic layer lying on an undeformable base under the nonlinear stress law of depending strains from deformations.

Key words: stability, dynamics, mobile loading, harmonic oscillations, deformation, rheology, oscillation, purity, wave processes.

While solving the problems of wave processes of plane and circular elements in deformable bodies, the concept of phase velocity is introduced as the rate of change of the phase medium. In this case of harmonic vibrations of a cylindrical shell, the phase velocity is expressed through the frequency of natural oscillations freely supported along the edges of the shell, and therefore, the investigation of waves in plane and circular elements has the most direct relation to the problem of determine the proper forms and vibration frequencies of finite length shells.

In many studies, two methods are usually used to determine the characteristics of waves.

It is studied the instantaneous state of the medium, corresponding to a certain fixed time moment.

It is studied the change in time of the body constant state at some fixed point.

If the studies are carried out taking into account the rheological properties of the considered system material or, there is a surrounding medium, also generally displaying rheological properties, the use of these methods is considerably difficult. In such cases, the influence of rheological parameters on the components of the complex phase velocity is studied for certain values of the vibration frequencies.

The physical law of stress-strain relation in a plane formulation is adopted in the form

$$\sigma_{jj} = 3K\varepsilon_0\left[1+\alpha x_0\varepsilon_0^2\right] + 2\mu\left(\varepsilon_{jj}-\varepsilon_0\right)\left[1+\alpha\gamma_0\psi_0^2\right],$$

$$\sigma_{ij} = \mu\varepsilon_{ij}\left[1+\alpha\gamma_0\psi_0^2\right], \quad \left(i \neq j; i,j = x,y\right) \tag{1}$$

где $\varepsilon_0;\psi_0$ - average volumetric strain and intensity of strain rate α - small parameter.

Oscillations in a layer are boundary conditions:

$$\sigma_{yy} = f_y(x,t), \quad \sigma_{xy} = 0, \quad y = h,$$
$$v = 0, \quad \sigma_{xy} = 0, \quad y = 0. \tag{2}$$

Due to the boundary conditions, the longitudinal displacement u is an even function of the coordinate and a transverse movement v odd in x.

Therefore, we will continue these functions on y from $-h$ to 0.

Then, instead of the problem for an elastic layer lying on an undeformable base, we obtain the task for a thickness layer $2h(-h \le y \le h)$, on the surface of which $y = \pm h$ external forces are set

$$\sigma_{yy} = f(x,t), \quad \sigma_{xy} = 0. \tag{3}$$

However, in such a formulation the problem remains very complicated in the mathematical formulation and obtaining its solution presents great difficulties. This problem can be solved in an approximate formulation, considering the elastic layer as an elastic thickness plate $2h$.

Approximate equation of the longitudinal vibration of a plate in a nonlinear setting for the main part of the longitudinal displacement U has the following form:

$$\frac{\partial^2 U}{\partial t^2} - C_{пл}^2 \frac{\partial^2 U}{\partial x^2} - \frac{2}{9}\alpha \frac{x_0 k}{\rho}\left(\frac{4\mu - \rho C_{пл}^2}{4\mu}\right)^4 \frac{\partial}{\partial x}\left[\left(\frac{\partial U}{\partial x} + \right.\right.$$
$$\left.\left. + \frac{1}{\mu} f_y\right)^3\right] = \frac{1}{\rho}\left(\frac{\rho C_{пл}^2}{2\mu} - 1\right)\frac{\partial f_y}{\partial x} \tag{4}$$

Let us consider a special case of an external rolling effort, when:

$$f_y(x,t) = f_y(x - Dt), \tag{5}$$

where D - the speed of movement of the external load on the surface layer is considered constant.

We introduce the moving coordinate

$$x' = x - Dt, \tag{6}$$

and for simplicity x, y we omit the primes.

In moving coordinates, the longitudinal vibration of elastic layer or plate with the physical nonlinearity of the law $\sigma(\varepsilon)$ with allowance for the external mobile load is described by the following nonlinear ordinary equation:

$$\left(D^2 - \tilde{N}_{ïë}^2\right)\frac{d^2U}{dx^2} - \frac{2}{9}\alpha \frac{x_0 k}{\rho}\left(\frac{4\mu - \rho c_{ïë}^2}{4\mu}\right)^4 \frac{d}{dx}\left[\left(2\frac{dU}{dx} + \right.\right.$$
$$\left.\left. + \frac{f_y}{\mu}\right)^3\right] = \frac{1}{\rho}\left(\frac{\rho \tilde{N}_{ïë}^2}{2\mu} - 1\right)\frac{\partial f_y}{\partial x} \tag{7}$$

obtained from the (4) when replacing (6).

Integrating (7) on x, taking into account the damping condition at infinity

Relatively $\frac{dU}{dx}$ we obtain a cubic equation:

$$\left(\frac{dU}{dx}\right)^3 + \frac{3}{2\mu} f_y \left(\frac{dU}{dx}\right)^2 + \left[\frac{3}{4\mu^2} f_y^2 - \frac{9\rho\left(D^2 - C_{пл}^2\right)}{16\alpha x_0 k}\left(\frac{4\mu}{4\mu - \rho C_{пл}^2}\right)^4\right] \times$$

$$\times \frac{dU}{dx} + \left[\frac{1}{8\mu^3} f_y^3 - \frac{9\left(2\mu - \rho C_{пл}^2\right)}{32\alpha\mu x_0 k}\left(\frac{4\mu}{4\mu - \rho C_{пл}^2}\right)^4 f_y\right] = 0 \tag{8}$$

Substitution

$$\frac{dU}{dx} = S - \frac{a}{3};$$

the equation(8) is reduced to the form:

$$S^3 + PS + q = 0, \tag{9}$$

where: $a = \frac{3}{2\mu} f_y$,

$$P = -\frac{9\rho\left(D^2 - C_{іё}^2\right)}{16\alpha x_0 k}\left(\frac{4\mu}{4\mu - \rho C_{іё}^2}\right)^4,$$

i.e. P does not depend on external load a

$$q = -\frac{9\left(2\mu - \rho C_{іё}^2\right)}{32\alpha\mu x_0 k}\left(\frac{4\mu}{4\mu - \rho C_{іё}^2}\right)^4 f_y.$$

Let us consider the case when $\alpha < 0$ (similarly, the problem is solved in the case $\alpha > 0$).

If we consider the excess sound mode

$$D > C_{іё}$$

i.e. when the perturbation in the layer before the load is absent, it is clear that $P > 0$ and the equation(9) (respectively (8)) has one real solution.

If the speed of movement of an external load D satisfies the inequality

$$D < C_{іё}$$

i.e subsonic mode,then $P < 0$ and the equation (9) (respectively(8)) can have another solution.

One real solution of equation (8) in the subsonic mode has the form:

$$\frac{dU}{dx}=\sqrt[3]{-\frac{q(x)}{2}+\sqrt{\left(\frac{p}{3}\right)^3+\left(\frac{q(x)}{2}\right)^2}}+\sqrt[3]{-\frac{q(x)}{2}-\sqrt{\left(\frac{p}{3}\right)^3+\left(\frac{q(x)}{2}\right)^2}}-\frac{a(x)}{3}. \qquad (10)$$

Integrating (10) X with respect to the damping condition of the displacement at infinity, and the transition to the fixed coordinates, we obtain:

$$U=\int_{\infty}^{x-Dt}\left[\sqrt[3]{-\frac{q(\xi)}{2}+\sqrt{\left(\frac{p}{3}\right)^3+\left(\frac{q(\xi)}{2}\right)^2}}+\sqrt[3]{-\frac{q(\xi)}{2}-\sqrt{\left(\frac{p}{3}\right)^3+\left(\frac{q(\xi)}{2}\right)^2}}-\frac{a(\xi)}{3}\right]d\xi \qquad (11)$$

The magnitude of the voltage σ_{xx} is approximately calculated by the formula:

$$\sigma_{xx}=\rho C_{ïë}^2\left[\sqrt[3]{-\frac{q(x-Dt)}{2}+\sqrt{\left(\frac{p}{3}\right)^3+\left(\frac{q(x-Dt)}{2}\right)^2}}+\sqrt[3]{-\frac{q(x-Dt)}{2}-\sqrt{\left(\frac{p}{3}\right)^3+\left(\frac{q(x-Dt)}{2}\right)^2}}\right]-$$
$$-f_y(x-Dt)-\alpha\left[\left(\frac{\rho C_{ïë}^2}{2\mu}-1\right)f_2(U)-f_1(U)\right], \qquad (12)$$

where,

$$f_2(U)=\frac{1}{9}x_0k\left(\frac{4\mu-\rho C_{пл}^2}{4\mu}\right)^3\left(2\frac{\partial U}{\partial x}+\frac{1}{\mu}f_y\right)^3;$$

$$f_1(U)=\frac{1}{9}x_0k\left[2\left(1-\frac{\rho C_{пл}^2}{4\mu}\right)\frac{\partial U}{\partial x}+\frac{1}{\lambda+2\mu}f_y-\frac{\alpha}{\lambda+2\mu}f_2(U)\right]^3.$$

From the last expressions it follows that when $\alpha\to 0$ we obtain a solution for the linear problem.

If the non-linearity parameter α is assumed sufficiently small, then instead of (11) we can approximately use the formula for displacement:

$$U=\frac{1}{\rho(D^2-C_{ïë}^2)}\left(\frac{\rho C_{ïë}^2}{2\mu}-1\right)\int_{\infty}^{x-Dt}f_y(\xi)d\xi+\alpha\left\{\frac{2}{9}\frac{x_0k}{\rho(D^2-C_{ïë}^2)}\times\right.$$
$$\left.\times\left(\frac{4\mu-\rho C_{пл}^2}{4\mu}\right)^4\left[\frac{\rho D^2-2\mu}{\mu\rho(D^2-C_{пл}^2)}\right]^3\int_{\infty}^{x-Dt}f_y^3(\xi)d\xi\right\} \qquad (13)$$

At small α values for the voltage $\sigma_{xx},\sigma_{yy},\sigma_{xy}$ formulas are obtained:

$$\sigma_{xx}=\frac{D^2}{D^2-C_{ïë}^2}\left(\frac{\rho C_{ïë}^2}{2\mu}-1\right)f_y+\alpha\left\{\frac{2}{9}\frac{x\,kC_{ïë}^2}{D^2-C_{ïë}^2}\times\left(\frac{4\mu-\rho C_{ïë}^2}{4\mu}\right)^4\left[\frac{\rho D^2-2\mu}{\mu\rho(D^2-C_{ïë}^2)}\right]^3f_y^3+2\left(1-\frac{\rho C_{ïë}^2}{4\mu}\right)f_2(U)\right\}+$$

$$+\frac{y^2}{2}\left\{\left[\rho D^2\left(\frac{\rho C_{пл}^2}{2\mu}+1\right)-\rho D^2\left(\frac{\rho C_{пл}^2}{2\mu}-1\right)^2+3(\lambda+2\mu)\left(\frac{\rho C_{пл}^2}{2\mu}-2\right)-2\left(\rho C_{пл}^2-3\mu\right)\right]\frac{\partial^3 U}{\partial x^3}+\right.$$

$$+\frac{1}{\lambda+2\mu}\left[\rho D^2\left(\frac{\rho C_{пл}^2}{2\mu}-1\right)+4\mu-3(\lambda+2\mu)\right]\frac{\partial^2 f_y}{\partial x^2}-$$

$$\left.-\frac{\alpha}{\lambda+2\mu}\left[\rho D^2\left(\frac{\rho C_{пл}^2}{2\mu}-1\right)+4\mu-3(\lambda+2\mu)\right]\frac{\partial^2 f_2(U)}{\partial x^2}-\alpha\frac{4\mu}{4\mu-\rho C_{пл}^2}\frac{\partial^2 f_2(U)}{\partial x^2}\right\}; \qquad (14)$$

$$\sigma_{zz}=f_y+\frac{y^2-h^2}{2}\left\{\frac{\rho C_{пл}^2}{2\mu}\left(\rho D^2-2\mu\right)\frac{\partial^3 U}{\partial x^3}-\left(\frac{\rho D^2}{\lambda+2\mu}+\frac{\rho C_{пл}^2}{2\mu}-1\right)\frac{\partial^2 f_y}{\partial x^2}+\right.$$

$$\left.+\alpha\left(\frac{\rho D^2}{\lambda+2\mu}+\frac{\rho C_{пл}^2}{2\mu}-2\right)\frac{\partial^2 f_2(U)}{\partial x^2}\right\};$$

$$\sigma_{xz}=\alpha+\left(y-\frac{1}{h^2}y^3\right)\frac{x_0 k}{192}\left(\frac{4\mu-\rho C_{їё}^2}{\mu}\right)^4\left[\frac{\rho D^2-2\mu}{\mu\rho\left(D^2-C_{їё}^2\right)}\right]^3 f_z^2 f_z^1 .$$

If $f_z(x-Dt)$ given as

$$f_y(x-Dt)=\sigma_0\beta(x-Dt)l^{-\beta(x-Dt)}, \qquad (15)$$

then, for example, with respect to the dimensionless relation $\frac{\sigma_{xx}}{\sigma_0}$ approximately obtain:

$$\frac{\sigma_{xx}}{\sigma_0}=l_1\xi l^{-\xi}+\alpha l_2\sigma_0^2\xi^3 l^{-3\xi},\ (\alpha<0), \qquad (16)$$

where,

$$l_1=\frac{D^2}{D^2-C_{пл}^2}\left(\frac{\rho C_{пл}^2}{2\mu}-1\right)>0,$$

$$l_2=\frac{D^2}{D^2-C_{пл}^2}\cdot\frac{x_0 k\left(4\mu-\rho C_{пл}^2\right)^4\left(\rho D^2-2\mu\right)}{1152\mu^7\rho^3\left(D^2-C_{пл}^2\right)^3}>0$$

The present work is devoted to the dynamics stability study of wave processes of plane and circular elements, and it is also being considered a class of plane tasks about moving loads effect on the surface of a layered elastic half-plane under the nonlinear law of stress versus deformation. A class of plane problems on the effect of moving loads on the surface of a layered elastic half-plane is solved. The