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THE COMPLETENESS OF THE NONCOMMUTATIVE $H_E(\mathrm{A}, \ell_\infty)$ SPACE

Abstract. Considering the commutative case, we know that a maximal function $f = \sup_n |f_n|$ belongs to $L_p(\mu)$ if and only if there is a factorization $f_n = cz_n = z_n c$ for all $n \in \mathbb{N}$, where $c \in L_p(\mu)$ and $\sup_n \|z_n\|_\infty < \infty$. The theory of vector-valued noncommutative L_p-spaces are introduced first time by Pisier in 1998. Pisier considered the case M is hyperfinite. This theory solved maximal function's problem in noncommutative case. Later in 2002 Junge and Xu introduced general case. By using these noncommutative vector valued L_p-spaces Junge solved noncommutative version of Doob's maximal inequality problem in general case.

The noncommutative vector-valued Hardy spaces were introdused in [2]. In this paper, we consider maximal function's problems on noncommutative Hardy spaces. For this reason we introduce a noncommutative vector-valued symmetric Hardy space.

Our aim is discover their properties. It is presented another useful proof of completeness of this space. We also obtain factorization theorem like Saito's theorem. The work is mostly theoretical. The results can be used to further develop of noncommuative martingale theory, noncommutative ergodic theory, and operator valued Hardy spaces theory.

Key words: von Neumann algebra, τ -measurable operator, subdiagonal algebra, noncommutative symmetric space, noncommutative Hardy space.

1 Preliminaries and Introduction

Let $S(0,1)$ be the space of all measurable real-valued functions on $(0,1)$ equipped with Lebesgue measure m (functions which coincide almost everywhere are considered identical).

For $x \in S(0,1)$ we denote by $\mu(x)$ the decreasing rearrangement of the function $|x|$. That is,

$$\mu(t,x) = \inf\{s \geq 0 : m(\{|x| > s\}) \leq t\}, \quad t > 0.$$

Definition 1 We say that $(E, \|\cdot\|_E)$ is a symmetric Banach function space if the following holds.

(a) E is a subset of $S(0,1)$.

(b) $(E, \|\cdot\|_E)$ is a Banach space.

(c) If $x \in E$ and if $y \in S(0,1)$ are such that $|y| \leq |x|$, then $y \in E$ and $\|y\|_E \leq \|x\|_E$.

(d) If $x \in E$ and if $y \in S(0,1)$ are such that $\mu(y) = \mu(x)$, then $y \in E$ and $\|y\|_E = \|x\|_E$.

Furthermore we recall that the norm in E is said to be order continuous if, for every sequence $\{x_n\}_{n\geq 0} \subset E$ such that $x_n \downarrow 0$ in $S(0,1)$, we have that $\|x_n\|_E \to 0$. Order continuity of the norm is equivalent to separability of the space E (see [10]).