N E W S
OF THE NATIONAL ACADEMY OF SCIENCES OF THE REPUBLIC OF KAZAKHSTAN
PHYSICO-MATHEMATICAL SERIES
ISSN 1991-346X
Volume 6, Number 316 (2017), 5 – 13

UDK 517.956

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APPLICATION OF POLYGONAL METHOD TO SOLVE OF PERIODIC PROBLEM FOR LOADED AND INTEGRO-DIFFERENTIAL PARABOLIC EQUATIONS

Abstract. In the first section, we investigate the periodic boundary value problem for a loaded parabolic equations in a rectangular domain. Using the polygonal method we construct of an algorithms for finding solutions of the periodic boundary value problem for loaded parabolic equations. And the convergence of algorithms is proved. Conditions of unique solvability of the investigated problem are establiched in the terms of initial data. In the second section, we investigate the periodic boundary value problem for parabolic integro-differential equation in a rectangular domain. The polygonal method develops on parabolic integro-differential equation. Algorithms for finding solutions of the periodic boundary value problem for parabolic integro-differential equations are constructed, and their convergence is proved. Conditions of unique solvability of the investigated problem are establiched in the terms of initial data.

Key words: periodic problem, loaded parabolic equations, integro-differential parabolic equations, polygonal method, algorithm, unique solvability.

This results are partially supported by grant of the Ministry education and science of Republic Kazakhstan No 0822 / ГФ4.

Loaded partial differential equations parabolic type arise in the study of various processes of physics, chemistry, biology, ecology and others [1-5]. Another important class of problems closely related to evolutionary integro-differential equations in partial derivatives are the parabolic integro-differential equations and boundary value problems for them [6-12]. The conditions for unique solvability, the assessment of solutions and their derivatives in terms of the geometric characteristics of the coefficients, the right-hand side, boundary values, and the region where linear boundary value problems for loaded partial differential equations are given, find in numerous applications in the qualitative theory of boundary value problems.

In [13] by using the polygonal method on the spatial variable the boundary value problem for a parabolic equation has been reduced to the solving of family of the Cauchy problem for a system of ordinary differential equations. Using the parametrization method [14] there were established the effective estimates of solutions through the initial data [15]. This approach will be developed on the parabolic loaded and integro-differential equations. There will be developed a constructive method for the solving of periodic boundary value problems for parabolic loaded and integro-differential equations.

By polygonal method on spatial variable the periodic and nonlocal boundary value problems for parabolic integro-differential equations will be reduced to a family of Cauchy problems for systems of ordinary integro-differential equations. On the basis of parameterization method the algorithms for finding the solution will be built and the conditions of unique solvability of the considered problem will be established.

1. Periodic boundary value problem for loaded parabolic equations

We consider a periodic boundary value problem for a loaded parabolic equation

$$\frac{\partial u}{\partial t}=a(t,x)\frac{\partial^2 u}{\partial x^2}+c(t,x)u(t,x)+\alpha(t)u(\theta,x)+f(t,x),\ (t,x)\in\Omega=(0,T)\times(0,\omega),\quad(1.1)$$

$$u(0,x)=u(T,x),\qquad x\in[0,\omega],\quad(1.2)$$

$$u(t,0)=\psi_0(t),\qquad u(t,\omega)=\psi_1(t),\qquad t\in[0,T],\quad(1.3)$$

where $a(t,x)\geq a_0>0$, $c(t,x)\leq 0$ $c(t,x)\leq 0$, $f(t,x)$ are continuous with respect to t, Holder continuous with respect to x, $\alpha(t)$ is continuous function on $[0,T]$. It is assumed that the functions $\psi_0(t)$, $\psi_1(t)$ are sufficiently smooth and satisfy the matching conditions $\psi_0(0)=\psi_0(T)$, $\psi_1(0)=\psi_1(T)$.

The parametrization method is applied to the periodic boundary value problem (1.1)-(1.3). Let $\lambda(x)=u(0,x)$ and in a problem (1.1)-(1.3) we will carry out replacement $u(t,x)=\lambda(x)+\tilde{u}(t,x)$, where $\tilde{u}(t,x)$ is a new unknown function. Then the periodic boundary value problem (1.1)-(1.3) reduce to the following problem

$$\frac{\partial\tilde{u}}{\partial t}=a(t,x)\frac{\partial^2\tilde{u}}{\partial x^2}+c(t,x)\tilde{u}(t,x)+\alpha(t)\tilde{u}(\theta,x)+$$

$$+a(t,x)\ddot{\lambda}(x)+c(t,x)\lambda(x)+\alpha(t)\lambda(x)+f(t,x),\ (t,x)\in\Omega,\quad(1.4)$$

$$\tilde{u}(0,x)=0,\qquad x\in[0,\omega],\quad(1.5)$$

$$\tilde{u}(t,0)+\lambda(0)=\psi_0(t),\qquad \tilde{u}(t,\omega)+\lambda(\omega)=\psi_1(t),\qquad t\in[0,T],\quad(1.6)$$

$$\tilde{u}(T,x)=0,\qquad x\in[0,\omega].\quad(1.7)$$

From conditions (1.5) and (1.6) follows

$$\lambda(0)=\psi_0(0),\ \lambda(\omega)=\psi_1(0).$$

The problem (1.4)-(1.7) is an initial-boundary value problem for a loaded parabolic equation with a parameter. An algorithm for finding the solution of the problem (1.4)-(1.7) is constructed, which consists of two stages: 1) the solving of the initial-boundary value problem for the loaded parabolic equation (1.4)-(1.6) at the fixed parameter by means of a justification polygonal method [42-43] ; 2) determination of the parameter from the relation (1.7).

The first stage of the algorithm. We consider an auxiliary initial-boundary value problem for the loaded parabolic equation

$$\frac{\partial\tilde{u}}{\partial t}=a(t,x)\frac{\partial^2\tilde{u}}{\partial x^2}+c(t,x)\tilde{u}(t,x)+\alpha(t)\tilde{u}(\theta,x)+\tilde{f}(t,x),\ (t,x)\in\Omega,\quad(1.8)$$

$$\tilde{u}(0,x)=0,\qquad x\in[0,\omega],\quad(1.9)$$

$$\tilde{u}(t,0)=\psi_0(t)-\psi_0(0),\qquad \tilde{u}(t,\omega)=\psi_1(t)-\psi_1(0),\ t\in[0,T],\quad(1.10)$$

where function $\tilde{f}(t,x)$ is continuous with respect to t, and is Holder continuous with respect to x.

The scheme of the polygonal method with respect to the problem (1.8)-(1.10). We take $h>0$ and make a discretization by x: $x_i=ih$, $i=\overline{0,N}$, $Nh=\omega$, $\tilde{u}_i(t)=\tilde{u}(t,ih)$. The problem (1.8)-(1.10) is replaced by the following form

$$\frac{\partial\tilde{u}_i}{\partial t}=a_i(t)\frac{\tilde{u}_{i+1}-2\tilde{u}_i+\tilde{u}_{i-1}}{h^2}+c_i(t)\tilde{u}_i+\alpha(t)\tilde{u}_i(\theta)+\tilde{f}_i(t),\ \tilde{u}_i(0)=0,\ i=\overline{0,N},\quad(1.11)$$

$$\tilde{u}_0(t)=\psi_0(t)-\psi_0(0),\qquad \tilde{u}_N(t)=\psi_1(t)-\psi_1(0),\ t\in[0,T].\quad(1.12)$$

Owing to linearity of system for $\forall h>0$, there exists a unique solution of problem (1.11): $\{\tilde{u}_1(t),...,\tilde{u}_{N-1}(t)\}$ defined on $[0,T]$.