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# CRITERIA FOR STRONG CONVERGENCE OF SOLUTIONS SINGULARLY OF THE PERTURBED CAUCHY PROBLEM

**Abstract**: The first-order integral equation with a non-smooth right-hand side is solved by the deviating argument method. It is shown that the solution of the corresponding singularly perturbed Cauchy problem converges to the solution of a non-perturbed problem. The criteria of strong convergence and the spectral properties of the auxiliary problem are used. The square root of a class of Sturm-Liouville operators is found.

**Keywords**: strong convergence, spectrum, spectral decomposition, square root of the operator, equation with deviating argument, Hilbert-Schmidt theorem.

## 1. Introduction

We consider a singularly perturbed Cauchy problem

$$\begin{aligned} &L_\varepsilon y = \varepsilon y'(\mathrm{x}) + a\,\mathrm{y}(\mathrm{x}) = \mathrm{f}(\mathrm{x}),\ \mathrm{x} \in (0,1] \\ &y(0) = 0 \end{aligned} \qquad (1.1)\text{-}(1.2)$$

in space $L^2(0,1)$, where $a$ is the complex constant, $\varepsilon > 0$ is a small parameter, and $f(\mathrm{x}) \in L^2(0,1)$.

For what values of the constant $a$ the solution of problem (1.1) - (1.2) converges strongly to the solution of the problem $L_0 y_0 = a y_0(x) = f(x)$, for $\varepsilon \to +0$ ?

The solution of problem (1.1) - (1.2) has the form $y(x,\varepsilon,f) = L_\varepsilon^{-1} f(x)$, hence, we are talking about the strong convergence of the sequence of operators $L_\varepsilon^{-1}$ to the operator $L_0^{-1} = I/a$, where is the $I$ unit operator. Since the right-hand side of (1.1) is not assumed to be smooth, we mean the strong solutions of problem (1.1) - (1.2), and (1.1) is understood almost everywhere in (0.1).

To clarify the formulation of the problem, we give the corresponding definitions [1, c.11].

**Definition 1.1**. Let's $\{A_n\}$ a sequence of bounded operators. It is said that this sequence converges uniformly to the operator $A$, if $\|A_n - A\| \to 0$ for $n \to \infty$.

**Definition 1.2.** A sequence $\{A_n\}$ of linear operators (generally speaking, unbounded) with a common domain of definition is called strongly convergent on $D$ (to the operator) if for any $u \in D$

$$\|A_n u - Au\| \to 0 \text{ at } n \to \infty.$$

**Definition 1.3** A sequence $\{A_n\}$ is called weakly convergent (to A) if for any $u \in D$ sequence the $\{A_n u\}$ sequence converges weakly to $Au$, in other words, this means that $(A_n u, \psi) \to (Au, \psi)$ for any $u \in D$ and any $\psi \in H = L^2(0,1)$.