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APPROXIMATION OF PROBLEM FOR FINDING THE BOUNDED SOLUTION TO SYSTEM OF NONLINEAR LOADED DIFFERENTIAL EQUATIONS

Abstract. On the whole axis the system of nonlinear loaded differential equations is considered. The questions of existence and approximation bounded solution to the system are studied. The definition of «limit as $t \to \pm\infty$» solution to the system of nonlinear loaded differential equations is introduced. Sufficient conditions for the existence of bounded solution to the system of nonlinear loaded differential equations and convergence of the function sequence composed by the bounded solutions to the linearized system of loaded differential equations are obtained. Regular nonlinear two-point boundary value problem for the system of nonlinear loaded differential equations on the finite interval is constructed, which approximate the problem of finding bounded solutions to the original system of loaded differential equations. It is given an estimate of the difference between the solution to initial singular problem and the solution to the approximating regular two-point boundary value problem.

Keywords: singular problem, nonlinear loaded differential equation, bounded solution, approximation.

Questions of existence and construction of approximate methods for finding of nonlinear ordinary differential equations, restricted on the whole axis, are considered by many authors [1-11]. Various problems for loaded differential equations and methods for their solutions are studied in [12-17].

In this article, nonlinear loaded differential equations is considered on $R = (-\infty, \infty)$

$$\frac{dx}{dt} = f(t,x) + f_0(t, x(\theta_{-m}), x(\theta_{-m+1}), \ldots, x(\theta_m)), \quad x \in R^n, \quad \| x \| = \max | x_i |, \tag{1}$$

where $f : R^{n+1} \to R^n$, $f_0 : R^{2n+2} \to R^n$ are continuous, $\theta_{-m} < \theta_{-m+1} < \ldots < \theta_0 = 0 < \theta_1 < \ldots < \theta_m$.

The aim of this research is to find conditions for the existence of system solutions of nonlinear loaded differential equations restricted on the whole axis (1) and the construction of the regular two-point boundary value problems on a finite interval, which allows determining the narrowing of the decision on the final interval with given accuracy.

In work [11] there was introduced the definition of "limit at $t \to \infty$" solution of nonlinear ordinary differential equations, and proved that, if the system is linearized along such solution is exponentially dichotomous on semi-axis, the "limit at $t \to \infty$" decision has the attractive property. This result allowed building approximate two-point boundary value problems on a finite interval for the singular boundary value problems for nonlinear ordinary differential equations on the whole axis. Methods and results [11] are used to find conditions for the existence of the equation solutions restricted on the whole axis (1) and for the construction of approximating regular boundary value problems on a finite interval.

The following symbols are used:

$\widetilde{C}(J, R^n)$ – space of continuous and restricted on $J \subseteq R$ functions $x : J \to R^n$ with norm $\|x\|_1 = \sup_{t \in R} \|x(t)\|$;