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M.I. Akylbayev[1], A. Beysebayeva[2], A. Sh. Shaldanbayev[2]

[1] Regional Social-Innovational University,Shymkent;
[2]M.Auezov South Kazakhstan State University, Shymkent.
musabek_kz@mail.ru akbope_a@mail.ru shaldanbaev51@mail.ru

ON THE PERIODIC SOLUTION OF THE GOURSAT PROBLEM FOR A WAVE EQUATION OF A SPECIAL FORM WITH VARIABLE COEFFICIENTS

Abstract. In this work the task of Goursat in a characteristic quadrangle for a wave equation of an express view with variable coefficients is solved. The spectral impression of the decision not traditional for such Voltaire tasks is gained. For this purpose as a vvvspomogoalny task the spectral task for the equation is used with we otklonyashchitsya by an argument. It is shown that the oprator of a type of Su(x)=u(1-x) plays a role of the operator Schmidt встречающиееся in decomposition of Voltaire operators.

Keywords: Volterra operators, indefinite metric, Goursat problem, similarity operators, spectrum, spectral decomposition, Fourier method, orthogonal basis, the Hulbert-Schmidt theorem.

1. Introduction. The investigations of the Dirichlet problem for the string vibration equation in a bounded region go back to J. Hadamard (Filler) who first noted the uniqueness of the solution in the rectangle. D.Burgin and Duffin [2] considered the Dirichlet problem for the equation $u_{xx} = u_{tt}$in the rectangle { 0 < x <X ; 0<t<T}. It is shown that the un uniqueness of a solution in a certain space appears if and only if X / T is rational. The existence theorems for a solution in classical spaces are established, and the smoothness of the solution is as greater as the smoothness is larger of the boundary function and as worse the number X / T is approximated to the rational numbers. Also the Neumann problem considered. Later these results were refined and generalized by various authors (see, for example, [3], [4], [5], [6]). Sobolev [7] constructed an example of a well-posed boundary value problem in a rectangle for a hyperbolic system of equations, Yu.M. Berezanskii [8] constructed a class of regions with angles, a change in the domain inside which leads to a continuous changing of solution of the Dirichlet problem. For regions with a smooth boundary in smooth spaces, only the question of the uniqueness of solution of the Dirichlet problem was studied (see, for example, the work of RA Aleksandryan [9]). In work [3], Arnol'd, applying his results on the maps of the circle into itself, refines the results of [2], indicating that the proof of theorems on the existence of classical solutions of the Dirichlet problem can be carried over to the case of an ellipse. Row of investigations T.Sh. Kalmenov and M.A. Sadybekov's are also devoted to boundary value problems of hyperbolic equations [10] - [12], the results of these researches are summarized in the monograph [13].

In [14], using the new general method, the properties of solutions of the Cauchy problem, are researched as well as of the first, second and third boundary value problems in the disk for a second-order

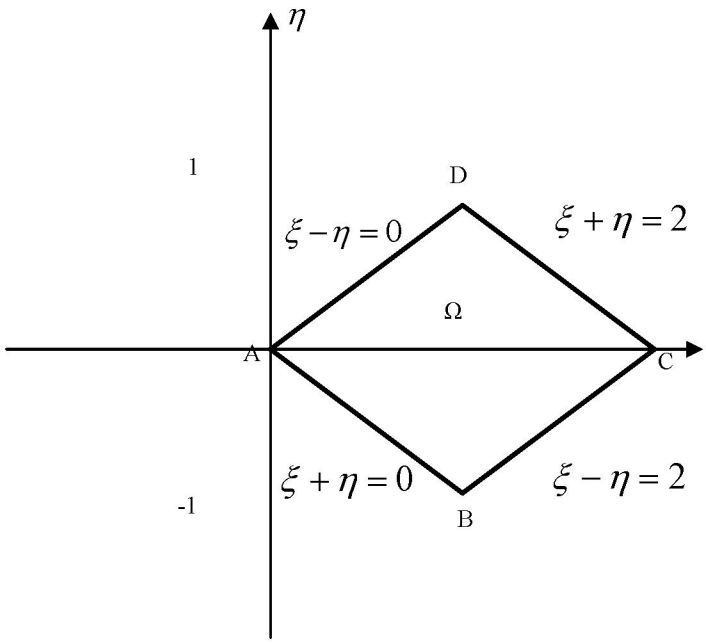
hyperbolic equation with constant coefficients are investigated. The application of this method to higher-order equations can be found in [15]. A new and relatively simple method for constructing a system of polynomial solutions of the Dirichlet problem for second-order hyperbolic equations with constant coefficients in the disk is proposed in [16], and it is also proposed to construct a complete set of eigenfunctions for the Dirichlet problem for the string oscillation equation. The eigenfunctions constructed in this paper coincide with the eigenfunctions constructed earlier by RA Aleksandaryan [9].

The analysis of the contents of these studies has shown that the spectral properties of these boundary-value problems depend on the geometry of the region, in particular, on the group of motion of the region. A non-equilateral triangle does not have a symmetry group, so we abandoned the characteristic triangle and began to consider boundary value problems inside the characteristic quadrangle. In this case, equations with deviating arguments appear naturally, which deserve a separate investigation [17] - [24].

1. Let- Ω be the characteristic quadrangle of the wave equation,

$$u_{\xi\xi} - u_{\eta\eta} + q\left(\frac{\xi-\eta}{2}\right)(u_\xi + u_\eta) + p\left(\frac{\xi+\eta}{2}\right)(u_\xi - u_\eta) + p\left(\frac{\xi+\eta}{2}\right) q\left(\frac{\xi-\eta}{2}\right) u = f(\xi,\eta) \qquad (1)$$

with the sides $AB: \xi + \eta = 0, BC: \xi - \eta = 2, CD: \xi + \eta = 2, DA: \xi - \eta = 0$
(see Pic.1).



Pic. 1

Suppose that the right-hand side of equation (1) is a periodic function with some periods. The question is whether equation (1) can have a periodic solution for the corresponding behavior of the coefficients. It is known that a periodic problem is poorly posed for the wave equation because of the presence of an infinite eigenvalue at the point λ = 0. Therefore, we consider the Goursat problem for equation (1) and study the possibilities of periodic continuability of the solution of this problem to the whole (ξ,η)plane.