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ON EXPANDING COUNTABLY CATEGORICAL WEAKLY O-MINIMAL THEORIES BY BINARY PREDICATES

Abstract. In the present work, questions of preservation of model-theoretical properties at expanding a model of a 1-indiscernible countably categorical weakly o-minimal theory by an arbitrary binary predicate are studied. Questions of preservation of model-theoretical properties at expanding of countably categorical weakly o-minimal theories by unary predicates had been before studied. Here thenotionofanequivalence-genera table formula hasbeenintroduced: if $R(x, y)$ is a p -stable formula for some non-algebraic 1-type p, then $R(x, y)$ is called an equivalence-generatable formula if every p -stable convex to the right or convex to the left formula formed from maximal convex subsets of the set $R(M, a)$ for some element $a \in p(M)$ is equivalence-generating. In terms of the introduced notion of an equivalence–generatable formula, a criterion for preserving the countable categoricity of a 1-indiscernible weakly o-minimal expansion by a binary predicate of 1-indiscernible countably categorical weakly o-minimal structures having the convexity rank 1has been obtained.

Keywords: weak o-minimality, countable categoricity, 1-indiscernibility, expansion of models, equivalence-generating formula, equivalence relation.

Let L be a countable first-order language. Throughout this paper we consider L-structuresandsupposethat L contains the binary relation symbol $<$, which is interpreted as a linear order in these structures. The present paper deals with the concept of *weak o-minimality*, originally deeply studied by D. Macpherson, D. Marker, and C. Steinhorn in [1]. A subset A ofalinearlyorderedstructure M is called *convex*if for any $a, b \in A$ and $c \in M$ whenever $a < c < b$ we have $c \in A$. A *weakly o-minimal* structure is a linearly ordered structure $M = \langle M, =, <, \ldots \rangle$ such that any definable(with parameters) subset of the structure M is a union of finitely many convex sets in M . Recall that such a structure M is called *o-minimal* if every definable (with parameters) subset of the structure M is a union of a finite number of intervals and points in M . Thus, weak o-minimality is a generalization of o-minimality. Real closed fields with a proper convex valuation ring provide an important example of weakly o-minimal (not o-minimal) structures.

Let A, B be arbitrary subsets of a linearly ordered structure M . Then the expression $A < B$ means that $a < b$ whenever $a \in A$ and $b \in B$. The expression $A < b$ means that $A < \{b\}$. We denote by A^+ (and, respectively, A^-) the set of elements b of the considered structure M with the condition $A < b$ ($b < A$).

Definition 1[2] Let T beaweaklyo-minimaltheory, M be a sufficiently saturated model of the theory T, and let $\phi(x)$ be an arbitrary M -definable formula with one free variable.

The convexity rank of the formula $\phi(x)$ $(RC(\phi(x)))$ is defined as follows:

1) $RC(\phi(x)) \geq 1$ if $\phi(M)$ infinite.

2) $RC(\phi(x)) \geq \alpha + 1$ if there exist a parametrically definable equivalence relation $E(x, y)$ and an infinite number of elements $b_i, i \in \omega$, such that: