N E W S
OF THE NATIONAL ACADEMY OF SCIENCES OF THE REPUBLIC OF KAZAKHSTAN
PHYSICO-MATHEMATICAL SERIES
ISSN 1991-346X
Volume 2, Number 312 (2017), 39 – 53

UDC 530.12:531.51

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THE ORBITAL STABILITY OF THE MOTION OF A TEST PARTICLE IN A FIELD OF TWO MASSIVE ROTATING BODIES

Annotation. The paper considers the orbital stability of the circular motion of a test body in the restricted three-body problem, where all bodies have their own rotation. The position of the central body coincides with the reference point of coordinates, the second body is moving in circular orbit around a central body (first body), and without disturbance. The test body moves in a perturbed circular orbit. This task belongs to the class of quasi-Keplerian problem and based on the adiabatic theory of motion of bodies in the General relativity (GR) mechanics. The adiabatic theory of motion of bodies is the approach for study the evolutionary motion of bodies in the mechanics of the GR and developed by M.M. Abdildin. The corresponding theory based on the vector elements to describe the motion in the asymptotic methods of nonlinear oscillations and in the method of adiabatic invariants.

We derived the Lagrangian of a system up to the terms of second order. This accuracy is sufficient for the problems of relativistic celestial mechanics, the influence of the internal structure of bodies can be neglected and omitted all the members which associated with that influence. We are limited to zero terms of the expansion in powers of the relationship of body size to their mutual distances.

Key words: General relativity, rotational motion, translational motion, stability of motion, three-body problem.

Introduction

The main task of celestial mechanics is the problem of the motion of a system of bodies, special cases of which are the tasks of two, three, four, etc. bodies, to which the problems of the motion of various concrete celestial bodies are indicated [1, 2].

The classical method of investigating the motion of celestial bodies involves representation of the solution of the corresponding equations of perturbed motion in the form of segments of series. However, Henri Poincaré showed that the series, used to describe the motion of celestial bodies, diverge. Consequently, they can not be used to analyze the behavior of the solar system on an infinite time interval [3]. According to the KAM-theory, if the masses of the planets are small enough, the eccentricities and slopes of the orbits are small, then for the majority of the initial conditions (excluding the resonant and close to them) the motion will be conditionally periodic, the eccentricities and slopes will remain small, and the major semiaxes will always oscillate near their original values, that is, the Solar system will be Lyapunov-stable over an infinite time interval. Unfortunately, resonances play a very important role in the real Solar system. Therefore, the conclusions of KAM-theory can not be applied to the Solar system as a whole over the entire range of its existence [4].

Currently, the stability of the Solar system is mainly considered within the framework of classical mechanics, although the modern theory describing the motion and interaction of the Solar system is the general relativity theory (GRT). Therefore, the study of the problem of the stability of motion in the framework of GRT is very relevant.

Qualitative and approximate methods of studying the motion of bodies in general relativity play an extremely important role, since even to obtain the most exact equations of motion (except for test bodies - the equation of geodesics) is practically impossible to achieve.

A rigorous and correct statement of the problem of the stability of the motion of bodies in GRT is more complicated than in classical mechanics. This is due to the complexity of the GRT mathematical

apparatus and, as a result, the impossibility of obtaining important physical corollaries of the theory with the help of exact solutions of basic and other equations of general relativity.

It should also be noted that in classical mechanics, the exact differential equations describing the class of motions under study are usually known. In GRT, as mentioned above, the exact equations of motion are known only for test bodies. In the case of bodies of comparable masses, the strict formulation of the problem of motion stability is still clear, since in this case the exact equations of motion are unknown. They are inferred only in a certain approximation. Moreover, most of the problems considered in the GRT mechanics are quasi-Keplerian because of the smallness of the relativistic perturbations in comparison with the Newtonian force. This circumstance makes it possible to search for special, optimal, simple methods for studying problems and other questions of the problem of the motion of bodies in general relativity [5-8].

In works [9-12], the orbital stability of the circular motion of a test body in a restricted three-body problem in GRT is investigated, when all bodies have no proper rotation. Perturbations from a second body moving in a circular orbit in the motion area of the test body (in the plane of the second body motion) are of the order of relativistic corrections to the motion of the test body from the central body

$$U_1 << c^2, U_2 << U_1 \tag{1}$$

where U_1, U_2 – potentials of the central and second body, respectively. The position of the resting central body coincides with the reference point of coordinates, the second body moves around the central (first) body in a circle and is not subjected to disturbance. The test body moves along a perturbed circular orbit. The problem belongs to the class of quasi-Keplerian ones and is considered on the basis of the adiabatic theory of the motion of bodies in the GRT mechanics. Under the name of adiabatic theory of motion of bodies it is meant the approach developed by M.M. Abdildin for the study of evolutionary motion in the GRT mechanics. It is based on the use of vector elements for the description of motion, on the asymptotic methods of the theory of nonlinear oscillations, and on the method of adiabatic invariants [13-19]. As a result, [9] it is shown that the motion of the test body in the plane of the orbit of the second body is stable.

Methods of research

In this paper, we consider the orbital stability problem for the circular motion of the test body in a restricted three-body problem, when all bodies have their own rotation. We derived the Lagrangian functions of a system of bodies to within second-order terms. With accuracy that is completely sufficient for the problems of relativistic celestial mechanics, the influence of the internal structure of bodies can generally be neglected and omitted from all terms associated with taking this influence into account. We confine ourselves to the zero terms of the expansion in powers of the ratio of the sizes of the bodies to their mutual distances.

The Lagrange function of translational and rotational motion for three rotating bodies can be represented as:

$$L = L^{(0)} + L^{(*)}. \tag{2}$$

where $L^{(0)}$ – the Lagrange function for three point masses, and the second term $L^{(*)}$ is responsible for corrections containing rotational terms.

The Hamiltonian of the problem can be represented as

$$H = H^{(0)} + H^{(*)} \tag{3}$$

where $H^{(0)}$ – the Hamiltonian for three point masses, and the second term $H^{(*)}$ is responsible for corrections containing rotational terms.

We write down the equations of motion of the problem under consideration in the representation of the vector elements $\vec{M}$ and $\vec{A}$, which is convenient for the application of asymptotic methods of nonlinear mechanics, since, in this case, the separation of the variables into fast and slow ones is evident in the equations of motion. The latter circumstance is precisely the characteristic feature of those problems for the analysis of which the asymptotic methods of investigation are applied.

$\vec{M}$ and $\vec{A}$ – the vector elements of the orbit (angular momentum and the Laplace vector) and are equal to:

$$\vec{M} = [\vec{r}, \vec{p}], \tag{4}$$

$$\vec{A} = \left[\frac{\vec{p}}{m}, \vec{M}\right] - \gamma \frac{mm_0}{r}\vec{r}, \tag{5}$$

where

$$A = \gamma m m_0 e = \alpha e,$$

where e – eccentricity of orbit.

So, following the procedure, we get the derivatives

$$\dot{M}_i = \left[\dot{\vec{r}}_i, \vec{p}_i\right] + \left[\vec{r}_i, \dot{\vec{p}}_i\right], \tag{6}$$

where

$$\dot{\vec{r}}_i = \frac{\partial H}{\partial p_i} = \frac{\partial H_0}{\partial p_i} + \frac{\partial H^*}{\partial p_i} = \dot{\vec{r}}_i^{(0)} + \dot{\vec{r}}_i^{(*)}; \quad \dot{\vec{p}}_i = -\frac{\partial H}{\partial r_i} = -\frac{\partial H_0}{\partial r_i} - \frac{\partial H^*}{\partial r_i} = \dot{\vec{p}}_i^{(0)} + \dot{\vec{p}}_i^{(*)}; \tag{7}$$

We can also write the equation (6) in the following form:

$$\dot{M} = \dot{M}^{(0)} + \dot{M}^{(*)} \tag{8}$$

where $\dot{M}^{(0)}$ – change in the moment for three point masses, and $\dot{M}^{(*)}$- change in the moment due to body rotation.

To obtain the equation of evolutionary motion, it is necessary to integrate the equation (8) over the period of the repetition of the configurations of the system T (the synodic period of the test body):

$$\bar{\dot{M}} = \frac{1}{T}\int_0^T \left(\bar{\dot{M}}^{(0)} + \bar{\dot{M}}^{(*)}\right) dt. \tag{9}$$

where

$$T = \frac{2\pi}{\omega_2 - \omega_3}. \tag{10}$$

The first component on the right-hand side of equation (9) describes the average change in the orbital angular momentum for three point masses. It was shown in [9] that this quantity is zero. Thus, in this case, the motion of the test body in the plane of the orbit is stable.

Let us now consider the case when all bodies have their own rotation. In equation (2) $L^{(*)}$, containing rotational terms, takes the form [20]

$$L^* = \sum_{i=1}^{3} \omega_i^2 I_i + \frac{1}{c^2}\left[\sum_{i=1}^{3} \omega_i^2 I_i - \frac{1}{2}\left(I_2\left(\vec{\omega}_2 \cdot \dot{\vec{r}}_2\right)^2 + I_3\left(\vec{\omega}_3 \cdot \dot{\vec{r}}_3\right)^2\right)\right]$$

$$-\frac{\gamma}{c^2}\cdot\left\{\frac{1}{\left|\vec{r}_2\right|^3}\left(\vec{r}_2 \cdot \dot{\vec{r}}_2\right)\cdot\left[3m_1 I_2.\omega_2 + 4m_2 I_1.\omega_1\right] - \frac{1}{\left|\vec{r}_3\right|^3}\left(\vec{r}_3 \cdot \dot{\vec{r}}_3\right)\cdot\left[3m_1 I_3.\omega_3 + 4m_3 I_1.\omega_1\right] + \right.$$

$$\left. + \frac{\vec{r}_2 - \vec{r}_3}{\left|\vec{r}_2 - \vec{r}_3\right|^3}\cdot\left[\left(3\dot{\vec{r}}_3 - 4\dot{\vec{r}}_2\right)\cdot m_2 I_3.\omega_3 - \left(3\dot{\vec{r}}_2 - 4\dot{\vec{r}}_3\right)\cdot m_3 I_2.\omega_2\right]\right\} + \frac{\gamma}{2c^2}\cdot\frac{m_2 I_2 + m_3 I_3}{\left|\vec{r}_2 - \vec{r}_3\right|^3}\cdot$$