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AXISMETRIC PROBLEMS OF ELASTIC LAYER OSCILLATION LIMITED BY RIGID OR DEFORMED BOUNDRIES

Abstract. In the study of wave processes of plane and circular elements in deformable bodies, the concept of phase velocity is introduced as the rate of change of the phase medium. In the case of harmonic vibrations of cylindrical shell, the phase velocity is expressed in terms of the frequency of natural oscillations freely supported along the edges of the shell, and therefore, the investigation of waves in plane and circular elements has the most direct relation to the problem of determining the proper forms and vibration frequencies of shells of finite length. If the research is carried out in view of the rheological properties of the system or material, has environmental management systems, in general, exhibiting the rheological properties, the use of these methods is significantly hampered. In such cases, the influence of rheological parameters on the components of the complex phase velocity is studied for certain values of the vibration frequencies. Some axismetric problems of oscillation of elastic layer are limited by rigid or deformable boundaries under the influence of normal or rotational shearing stress. The solutions of the problems under consideration are obtained using integral transformations in coordinate or time.

Key words: elastic, axismetry, wave, oscillations, deformed, longitudinal, anisotropic medium, Laplace transform, anisotropic layer.

First we consider the problem for half-space under the assumption that the half-space $z > 0$ is an anisotropic medium with the axis of symmetry of the mechanical properties (axis) (axis z), and the surface of which is subjected to an impulse voltage at the time $\xi_{z0} = -f(r,t)$.

Because of the symmetry of the mechanical properties of the medium relative to the axis z of the unique nonzero component of the displacement vector, $U_0(r,z,t)$ only the stresses and ξ_{r0} and ξ_{z0} the ones determined by formulas

$$\xi_{r0} = C_{44}\left(\frac{\partial U_0}{\partial r} - \frac{U_0}{r}\right), \qquad \xi_{z0} = C_{66}\frac{\partial U_0}{\partial z} \tag{1}$$

The equation of motion reduces to one

$$\frac{\partial \xi_{r0}}{\partial r} + \frac{\partial \xi_{r0}}{\partial z} + \frac{2\xi_{r0}}{r} = \rho\frac{\partial^2 U_0}{\partial t^2} \tag{2}$$

Substituting the expressions for ξ_{r0} and ξ_{z0} from (1) into equation (2), we bring it to the form:

$$\frac{\partial^2 U_0}{\partial r^2} + \frac{1}{r}\frac{\partial U_0}{\partial r} - \frac{U_0}{r^2} + \gamma^2\frac{\partial^2 U_0}{\partial z^2} = \frac{1}{b^2}\frac{\partial^2 U_0}{\partial t^2} \tag{3}$$

where

$$b^2 = \frac{C_{44}}{\rho}; \ \gamma^2 = \frac{C_{66}}{C_{44}}$$

If the half-space is isotropic, then $\gamma = 1$ and $b = \sqrt{\frac{\mu}{\rho}}$.

The boundary conditions for U_0 has:

$$\xi_{z0} = -f(r,t) \quad \text{under } z = 0, \ t \geq 0 \tag{4}$$

$$U_0 \to 0 \quad \text{under } z \to \infty \tag{5}$$

The initial conditions of the problem are zero

$$U_0 = \frac{\partial U_0}{\partial t} = 0 \quad \text{under } t = 0 \tag{6}$$

The solution of equation (3) for the boundary (4) - (5) and the initial conditions (6) will be sought, by applying the Laplace transform. Suppose,

$$U(r,z,p) = \int_0^\infty U_0(r,z,t)e^{-pt}dt, \ \operatorname{Re} p > 0 \tag{7}$$

Definitely, for the function $U(r,z,p)$ we obtain the equation

$$\frac{\partial^2 U}{\partial r^2} + \frac{1}{r}\frac{\partial U}{\partial r} - \left(\frac{1}{r^2} + \frac{p^2}{b^2}\right)U + \gamma^2 \frac{\partial^2 U}{\partial z^2} = 0 \tag{8}$$

And U must satisfy the boundary conditions:

$$\frac{\partial U}{\partial z} = -\frac{f_0(r,p)}{C_{66}} \quad \text{under } z = 0, \ t > 0 \tag{9}$$

$$U_0 \to 0 \quad \text{under } z \to \infty \tag{10}$$

where

$$f_0(r,p) = \int_0^\infty (r,t)e^{-pt}dt.$$

The general solution of equation (5.6.8) is sought by the method of separation of variables (the Fourier method) and has the form:

$$U(r,z,p) = \int_0^\infty \alpha \left[A(\alpha,p)e^{-\frac{z}{\gamma}\sqrt{\alpha^2+\frac{p^2}{b^2}}} + B(\alpha,p)e^{\frac{z}{\gamma}\sqrt{\alpha^2+\frac{p^2}{b^2}}} \right] J_1(\alpha r)d\alpha. \tag{11}$$

where $A(\alpha,p)$ and $B(\alpha,p)$ are defined from boundary conditions (9)-(10) from the condition (10) follows that

$$B(\alpha,p) = 0 \tag{12}$$

Using boundary condition (9), for defining $A(\alpha,p)$ we'll take integral equation:

$$\int_0^\infty \alpha A(\alpha,p)\sqrt{\alpha^2 + \frac{p^2}{b^2}} J_1(\alpha r)d\alpha = \frac{\gamma}{C_{66}} f_0(r,p) \tag{13}$$

Let

$$f_0(r,p)=\int_0^\infty \alpha f_1(\alpha,p)J_1(\alpha r)d\alpha \tag{13}$$

Then

$$A(\alpha,p)=\frac{\gamma f_1(\alpha,p)}{C_{66}\sqrt{\alpha^2+\frac{p^2}{b^2}}} \tag{14}$$

Substituting expression (12) and (14) into formula (11), we obtain the following expression:

$$U(r,z,p)=\frac{\gamma}{C_{66}}\int_0^\infty \frac{\alpha f_1(\alpha,p)}{\sqrt{\alpha^2+\frac{p^2}{b^2}}}J_1(\alpha r)e^{-\frac{z}{\gamma}\sqrt{\alpha^2+\frac{p^2}{b^2}}}d\alpha \tag{15}$$

We consider the special case when

$$f_0(r,p)=\frac{\varphi_0(p)}{r} \tag{16}$$

In case (16) function

$$f_1(\alpha,p)=\frac{\varphi_0(p)}{\alpha}$$

and (15) has

$$U(r,z,p)=\frac{\gamma}{C_{66}}\int_0^\infty \frac{\varphi_0(p)}{\sqrt{\alpha^2+\frac{p^2}{b^2}}}e^{-\frac{z}{\gamma}\sqrt{\alpha^2+\frac{p^2}{b^2}}}J_1(\alpha r)d\alpha=\frac{1}{C_{66}}\varphi_0(p)I_{\frac{1}{2}}\left[\frac{p}{2b}\left(\sqrt{\frac{z^2}{\gamma^2}+r^2}-\frac{z}{\gamma}\right)\right]K_{\frac{1}{2}}\left[\frac{p}{2b}\left(\sqrt{\frac{z^2}{\gamma^2}+r^2}+\frac{z}{\gamma}\right)\right]$$

where $K_{\frac{1}{2}}, I_{\frac{1}{2}}$ Bessel functions of imaginary argument. Using the representations of the functions $I_{\frac{1}{2}}(\zeta)$ and $K_{\frac{1}{2}}(\zeta)$, for $U(r,z,p)$ we find that

$$U(r,z,p)=\frac{b\varphi_0(p)}{C_{66}rp}\left[e^{-\frac{pz}{\gamma a}}-e^{-\frac{p}{a}\sqrt{\frac{z^2}{\gamma^2}+r^2}}\right] \tag{17}$$

Paying attention to the expression (17) in p, for the required value $U_0(r,z,t)$, we'll take the expression

$$U_0(r,z,t)=\frac{b}{\gamma C_{66}r}\int_0^t f_1(t-\xi)\left[H\left(\xi-\frac{z}{\gamma b}\right)-H\left(\xi-\frac{1}{b}\sqrt{\frac{z^2}{\gamma^2}+r^2}\right)\right]d\xi \tag{18}$$

where

$$f(r,t)=\frac{f_1(t)}{r}.$$

The resulting expression $U_0(r,z,t)$ consists of two terms, the first term corresponding to plane wave propagating in half-space with velocity γb and parallel to the plane $z=0$, and the second term to the diffracted wave, which has the form of half-ellipsoid of revolution (hemisphere at $\gamma=1$) and in contact with plane wave on the rotation axis at $z=b\gamma t$.

Besides, from (18) follows, that $U_0(r,z,t)$ fades out from r as $1/r$.

If the acting function $f(r,t)$ is arbitrary, then we represent it in the form of Schlemmich series:

$$f(r,t)=\frac{1}{r}\sum_{j=0}^{\infty}a_j(t)J_0(jr)=\frac{f_r(t)}{r} \qquad (19)$$

where

$$a_0(t)=\frac{1}{\pi}\int_0^{\pi}\left\{f_r(0,t)+U\int_0^1\frac{\frac{\partial f_r(\xi U,t)}{\partial r}}{\sqrt{1-\xi^2}}d\xi\right\}dU;$$

$$a_j(t)=\frac{2}{\pi}\int H\cos(jU)\left\{\int_0^1\frac{\frac{\partial f_r(\xi U,t)}{\partial r}}{\sqrt{1-\xi^2}}d\xi\right\}dU;\ j=1,2,... \qquad (20)$$

For $f(r,t)$ type (19) function $f_1(\alpha,p)$, entering to the formula (15), is

$$f_1(\alpha,p)=\frac{1}{\alpha}\sum_{j=0}^{\infty}a_{j0}(p)H(\alpha-j) \qquad (21)$$

where

$$a_{j0}(p)=\int_0^{\infty}a_j(p)e^{-pt}dt. \qquad (22)$$

Following,

$$U(r,z,p)=\frac{\gamma}{C_{66}}\sum_{j=0}^{\infty}a_{j0}(p)\int_0^{\infty}\frac{e^{-\frac{z}{\gamma}\sqrt{\alpha^2+\frac{p^2}{b^2}}}}{\sqrt{\alpha^2+\frac{p^2}{b^2}}}J_1(\alpha r)d\alpha \qquad (23)$$

Paying attention (23) on p and using theory about convolution, we'll take

$$U_0(r,z,p)=\frac{\gamma}{2C_{66}}\sum_{j=0}^{\infty}\int_g^t a_j(t-\xi)T_j(r,z,\xi)d\xi \qquad (24)$$

where

$$T_j(r,z,\xi)=b\int_j^{\infty}J_1(\alpha r)J_0\left[\alpha\sqrt{b^2t^2-\frac{z^2}{\gamma^2}}\right]d\alpha \qquad (25)$$

We generalize the problem for anisotropic layer of thickness h. where $z=h$ there can be two kinds of boundary conditions

1) $\tau_{z0}=-F(r,t)$ under $z=h$

2) $U_0=0$ under $z=h$

(26)

If $F(r,t)=0$, then condition (26) means, that the substance $z=h$ is free from stresses.

First of all, let's look at the problem, when $z=h$ there is boundary condition (26).

General decision has the type (11),