BULLETIN OF NATIONAL ACADEMY OF SCIENCES
OF THE REPUBLIC OF KAZAKHSTAN
ISSN 1991-3494
Volume 1, Number 365 (2017), 24 – 28

UDC 625.855.3

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THE CALCULATION OF FREE OSCILLATIONS OF AN ANISOTROPIC THREE-DIMENSIONAL ARRAY OF UNDERGROUND STRUCTURES

Abstract. This work is a theoretical research aimed at studying the amplitude-frequency characteristics of the system "lining-soil". It was found that fluctuations in the deformation occur, not only the soil mass, but in an underground structure in the course of a numerical experiment to study the free oscillations of an anisotropic three-dimensional array with the station tunnel.

We investigate on the basis of the variational formulation of the finite element method of amplitude-frequency characteristics of the system "lining-soil". A generalized problem of eigen values is solved iteratively in the subspace based on the scheme of the Jacobi algorithm.

Keywords: free oscillation, stress-strain state, lining, stress, displacement, algorithm.

Creating a reliable method of calculating vehicle stability of underground structures of finite size in difficult ground conditions under the influence of static and dynamic loads is very challenging. In Kazakhstan a developed mining industry, with increasing depth of mining operations and the deterioration of the conditions of development of mineral deposits, the requirements to ensure the sustainability of developments rise sharply. In addition, with the construction of the Almaty Metro in the zone of possible 9-10-magnitude earthquake, it needs reliable recommendations for earthquake resistance.

All this calls for fundamental research involving modern apparatus of mathematics and mechanics of solids, unconventional analytical and numerical methods for solving tasks and creation on their basis of software tools for the analysis of dynamic stability of various vehicles designed and constructed underground structures for various purposes [1, 2].

Study of free oscillations of transport facilities is important to determine the effect of physical and mechanical properties and geometric parameters of the structural elements and the surrounding massif complex structure at their resonant frequency response. On the other hand, the study of the dynamic response of a spatial reference system "underground structure array rocks" lower frequencies is necessary for the formation and basic solutions allowing movement of the matrix equations [3-5].

We study the free oscillations of the soil mass with a three-dimensional transport of underground structures on the basis of the numerical method – finite element method (FEM) – in conjunction with an iterative method in the subspace.

Object of research is the lower half-space with underground facilities shallow emplacement. Rock mass consists of a non-uniform layers with different physical and mechanical properties. Elastic status of each layer is described by the generalized Hooke's law:

$$\{\sigma\}=[D]\{\varepsilon\}, \tag{1}$$

where $\{\sigma\}=\{\sigma_x,\sigma_y,\sigma_z......,\tau_{xz}\}^T,\{\varepsilon\}=\{\varepsilon_x,\varepsilon_y,\varepsilon_z,.....\lambda_{xz}\}^T$, $[D]=[d_{ij}],(i,j=1,2,...,6)$ - elastic solid matrix; elastic moduli d_{ij} represented by the elastic constants transtropic array $E_k,\nu_k,G_2,\quad(k=1,2)$ of angles of inclination of the plane of isotropy φ and inclination to the longitudinal axis of the horizontal three-dimensional structures of the transport line stretch isotropic plane ψ [6,7].

Boundary conditions: the lateral faces and the base of the calculation region with the construction of non-deformable - u = v = w = 0; internal the breed contour lining and free from external loads - Xn = Yn = = Zn = 0. Spatial computational domain is divided into 1606 prismatic elements with 2875 nodes.

Differential equations oscillation system for the array to transport underground structures can be represented as:

$$[M]\{\ddot{U}(t)\}+[C]\{\dot{U}(t)\}+[K]\{U(t)\}=\{R(t)\}, \tag{2}$$

where $\{R(t)\}$ - vector external nodal forces, $\{\ddot{U}(t)\}, \{\dot{U}(t)\}, \{U(t)\}$ - Vectors of nodal accelerations, velocities and displacements, $[M]$, $[C]$, $[K]$ - accordingly, the mass matrix, rigidity and damping system. The matrix equation of free oscillations "lining-ground" of the system obtained from (2), when the effect of damping and the impact of external forces that are missing [C]=0, {R}=0

$$[M]\{\ddot{U}\}+[K]\{U\}=0. \tag{3}$$

Stiffener matrix is computed using the integral [3,4]:

$$[k]=\int_V [B]^T[D][B]dV=\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1}[B]^T[D][B]\det[J]d\xi\, d\eta\, d\zeta. \tag{4}$$

Integral expression (4) after the application of Gauss-Legendre quadrature to the form

$$[k]=\sum_{i=1}^{p}\sum_{j=1}^{p}\sum_{k=1}^{p}H_i H_j H_k [B]_{ijk}^T [D][B]_{ijk} \det[J]. \tag{5}$$

System stiffness matrix [K] is produced by summing all elements of the stiffness matrix

$$[K]=\sum_{i=1}^{K}[k_i]. \tag{6}$$

System mass matrix is formed from the matrix elements of the masses is similar to the system stiffness matrix. Mass Matrix prismatic element has the form [8,9]:

$$[m]=\sum_{i=1}^{m}\sum_{j=1}^{m}\sum_{k=1}^{m}H_i H_j H_k \rho [P_{ijk}]^T [P_{ijk}]\det[J], \tag{7}$$

where $\lfloor P_{ijk} \rfloor$ - matrix interpolating movements. System mass matrix is obtained by summing all elements of the mass matrix

$$[M]=\sum_{i=1}^{K}[m_i]. \tag{8}$$

The solution of ordinary differential equations of second order (3) can be written as:

$$\{U\}=\{\varphi\}\sin(\omega(t-\alpha_0)). \tag{9}$$

Substituting (9) into (3) gives a common problem eigen values

$$[K]\{\varphi\}=\omega^2[M]\{\varphi\}. \tag{10}$$

We introduce the notation $\lambda=\omega^2$, then (10) takes the form:

$$[K]\{\varphi\}=\lambda[M]\{\varphi\}. \tag{11}$$

For the solution of the generalized problem of eigen values used in the subspace iterative method based on the algorithm of Jacobi method and the properties of the Sturm sequence [10].

When iterative methods are necessary at each step to analyze the convergence of the obtained approximations. Let a (k-1) and (k) - iteration step approximate calculated eigenvalues $\lambda_i^{(k)}$ and $\lambda_i^{(k+1)}$ then the convergence is achieved at

$$\frac{\lambda_i^{(k+1)} - \lambda_i^{(k)}}{\lambda_i^{(k+1)}} \le \varepsilon, \quad (i = 1,2,\dots,n). \tag{12}$$

The effectiveness of the chosen method is explained, first, the initial choice of the subspace, sufficiently close to the desired lowest eigen values; secondly, the transition from the convenience of the algorithm to another subspace, which ensures "best" approximation of eigen values vectors. Furthermore, the use of translations and other accelerating processes also increases the efficiency of the method [10].

In the study of stress-strain state of the system "lining-ground" on the seismic action the first and necessary step in the calculation is to determine the frequencies and modes of vibration of the system. Calculation of amplitude-frequency characteristics of the system “lining-soil” is made in the subspace iteration method above.

Received 100 first frequencies and forms of oscillations in the frequency range up to 22.2 Hz. Values "lining-soil" system of the lower frequencies of free oscillations are shown in Table. As you can see, the range of "lining-soil" system of natural frequencies is quite dense.

The values of the frequency of free oscillations "lining-soil" system

Frequency rooms	1	2	3	4	5	6	7	8
ω_i (Hz)	1,78	3,09	3,61	3,84	4,31	4,65	5,87	6,13

Figures 1-3 show three-dimensional forms (fashion) free oscillations of the system "lining-ground."

Modes 1 and 2 are horizontal oscillations soil layer, wherein the first mode is a skew-symmetric and the second mode – symmetric. In modes 3-5 are more pronounced vertical oscillations. The third mode is a skew-symmetric, and 4-5 fashion – symmetrical. Higher forms shown in Figures 1-8, are rather complex

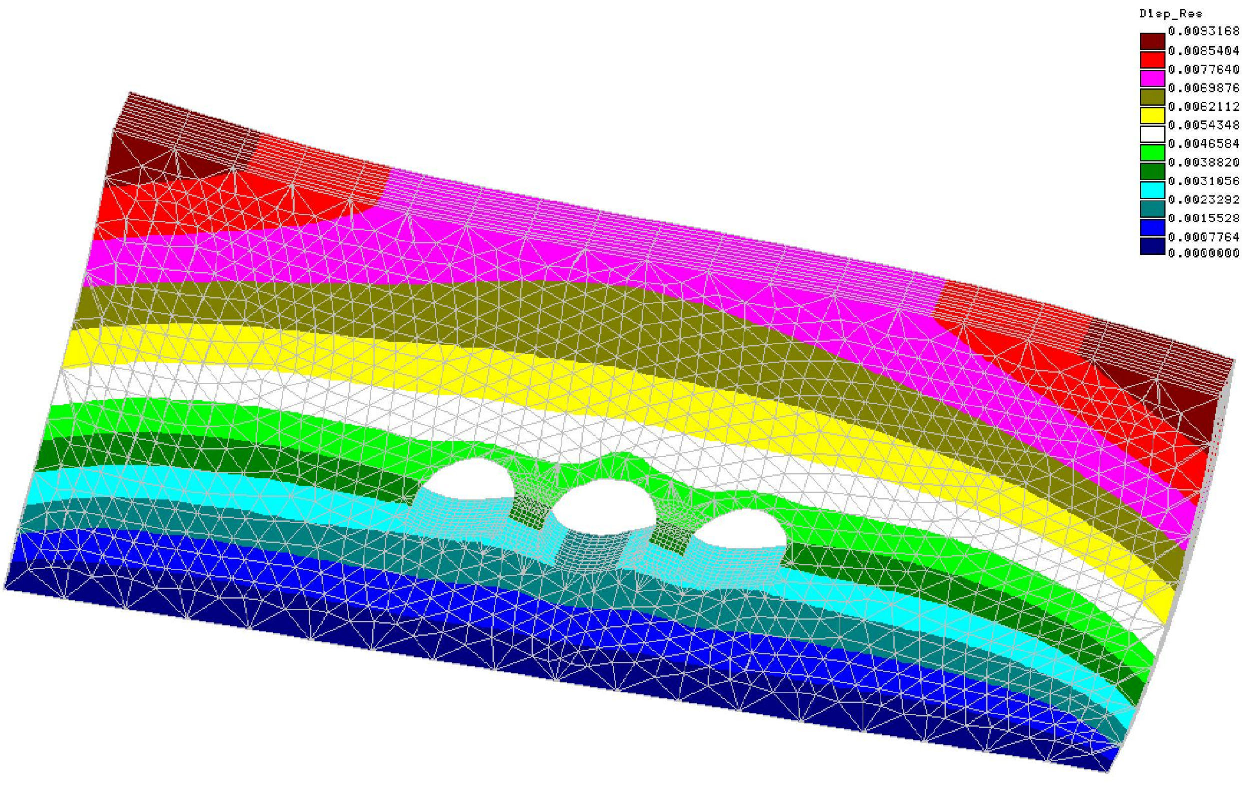

Figure 1 – The first mode of free oscillations "lining-soil" system

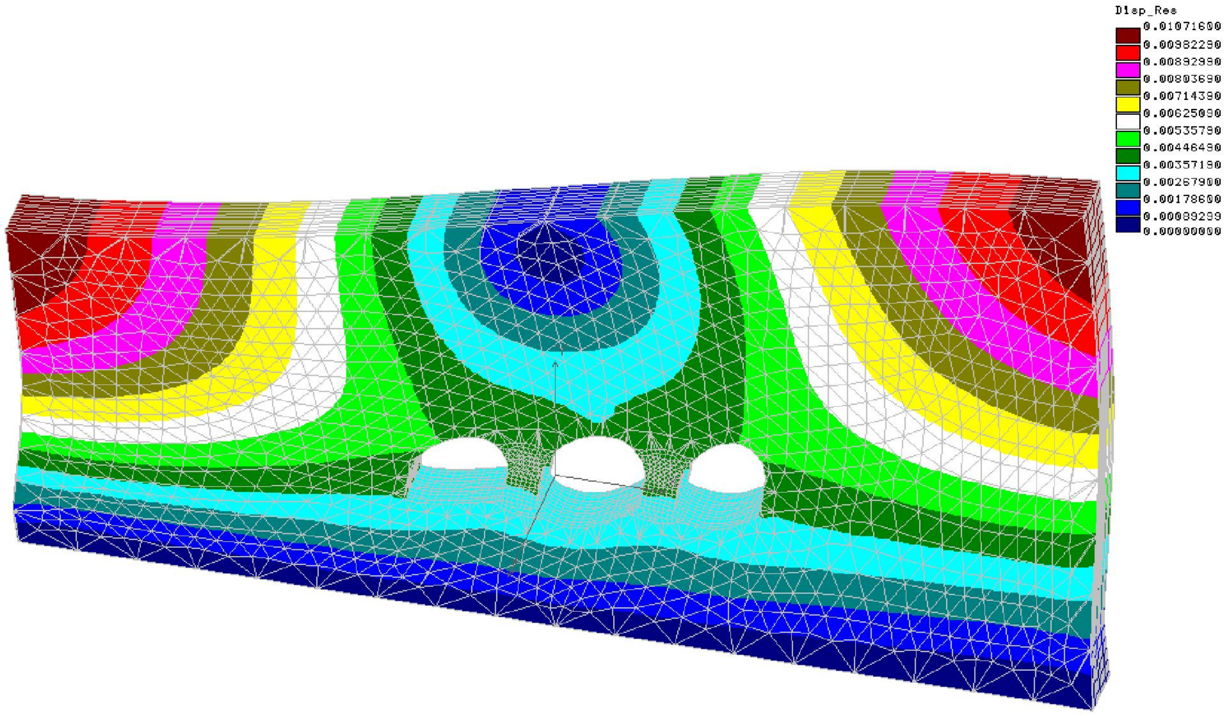

Figure 2 – The third mode of free oscillations "lining-soil" system

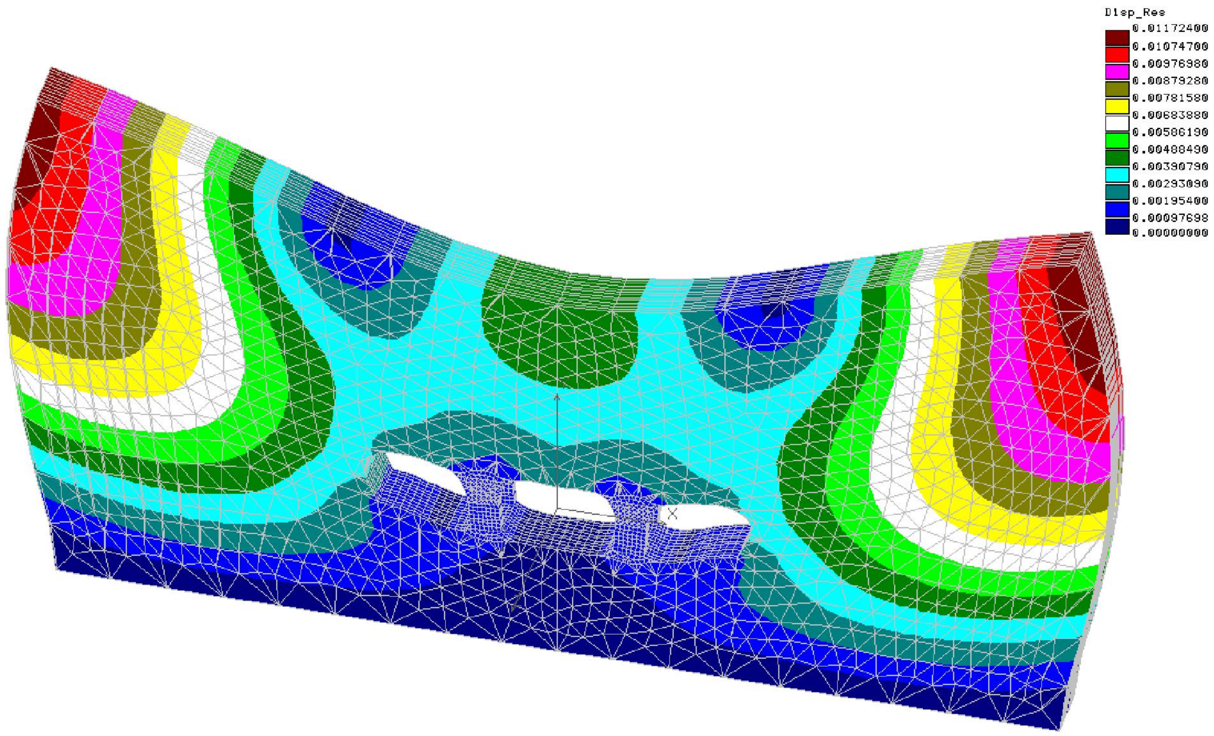

Figure 3 – The fifth mode of free vibrations "lining-soil" system

oscillatory motion of the ground and, apparently, do not make a significant contribution in determining the seismic movements, but can have a significant impact in finding the acceleration system and the seismic stresses in the construction of tunnels lining.

Conclusions. Multivariate numerical experiments to study free "lining-soil" oscillations of the system revealed that there is a complex pattern of deformation of anisotropic array and being in it three-dimensional underground structure in the course of oscillation, comprising a tensile elements, compression, bending and torsion (see Figures 1-3).