МАТЕМАТИКА

UDC 681.3

S. N. BORANBAEV

METHODS OF DECISION OF OPTIMIZATION PROBLEMS OF DISTRIBUTION OF RESOURCES

Article is devoted to development mathematical and the software of the decision optimized tasks of distribution of resources. Problems of a choice of an effective method of the decision are investigated.

The problems of distribution of resources can be reduced to problems of conditional or unconditional optimization [1-2]. Further, to these problems it is possible to apply methods of optimization. The basic methods of optimization are resulted, in particular, in works [3-11].

Let's consider the following problem:

$$F(x) \to \min$$
$$x \in E^n,$$
$$c_s \le x_s \le d_s,\ s = 1,\dots n;$$
$$q_v(x) \ge 0;\ v = 1,\dots t_1;$$
$$r_k(x) = 0;\ k = 1,\dots,t_2.$$

Where functions $F(x)$, $q_v(x)$, $r_k(x)$ are convex.

At the decision of problems with the unexplored criterion function it is necessary to face a problem of a choice of a concrete method of the decision. The method chosen at random can converge very slowly to a point of a minimum of criterion function or not give at all result. To receive characteristics of criterion function (camber, gullied, etc.) is not always possible. Therefore at a choice of a method it is difficult to apply the heuristic reasons based on such characteristics. Thus, there is a problem of a choice of an effective method of the deci-sion of a problem. This process can be automated.

For the decision of the considered problem the program system which enables, depending on the information received during search of the decision is developed, to change not only methods of optimization, but to replace and values of parameters of a method on which also in many respects efficiency of methods depends. Thus, it is possible to apply to the decision of a problem some methods, not stopping process of the decision. At each stage of the decision of a problem automatic the most effective algorithm gets out.

In a basis of system the following approach for the decision оптимизационных problems lays. We shall designate P_1, P_2, …, P_n – methods of the optimization, included in system. Various methods are included in system under characteristics as criterion function can have complex structure.

From set of methods included in system, the problem-oriented list of methods $P_{k_1}, P_{k_2}, \dots, P_{k_s}$ if about criterion function any information is known (for example if it is known, that it convex, gullied, square-law, etc.). The decision of a problem will consist of steps. On each step there is a revealing the most effective method from the list, and then the decision of a problem with the help of this method. The certain time intervals are allocated for revealing and the decision. Time allocated for revealing of an effective method, is used for promotion to a point of a minimum as at the decision of a problem the current point is used. It allows to save time of the decision of a problem.

At a stage of revealing of an effective method to all methods from the list $P_{k_1}, P_{k_2}, \dots, P_{k_s}$ it is enabled to solve a problem during the allocated time interval Dt. After all methods had an opportunity to solve a problem for allocated time, search of an effective method stops, gets out the most effective method of the list $P_{k_1}, P_{k_2}, \dots, P_{k_s}$. The size D$t$ depends on time, spent by a computer on one calculation of criterion function and amount calculation of criterion function on one iteration, and is calculated as follows:

$$\mathrm{D}t = am,$$

Where m – time spent for one calculation of criterion function,

$$\alpha = \max_r \alpha_{k_r},$$

Where α_{k_r} – amount of calculations of criterion function on one iteration by a method P_{k_r}.

Thus, at this stage comes out the most effective method.

At the following stage the most effective method is used for the decision of a problem. Time allocated for the decision of a problem, is calculated as follows.

$$\tau_0 = \nu\Delta t, \quad \nu > 1,$$

$$\tau_s = q_s\tau_{s-1} + \tau_0.$$

The size n gets out a priori, and depends on complexity and dimension of criterion function. The size $q_s = 1$ if on two consecutive steps the most effective appeared the same method, differently gets out $q_s = 0$. It means, that the decision of a problem if any method appeared effective on several consecutive steps proceeds. Efficiency E_i of a method P_i is calculated under the formula:

$$E_i = \frac{\left|F(x^k) - F(x^{k+1})\right|}{\left|F(x^k)\right| + \delta},$$

Where x^k, x^{k+1} – initial and final points at use of a method P_i; $F(x^k)$, $F(x^{k+1})$ – values of criterion function in these points; d – small positive number. If value of criterion function for allocated time has not decreased, efficiency of a method is equal to zero.

If for all methods P_i from the list of methods $P_{k_1}, P_{k_2}, ..., P_{k_s}$ it is carried out $E_i = 0$, it means, that the considered list of methods cannot effectively solve the given problem. In this case the further search of an effective method by the given list of methods stops. For avoidance of such situation, it is necessary to include various methods that the list has been focused on the decision of a wide class of problems in the list.

Criterion of end of work is performance of a condition of the termination of search of the decision for all methods included in system. The system gives out the saved up information on a course of search of the decision of a problem (carried out methods, values of criterion function, time of search, etc.).

Due to self-training, the system enables an automatic choice of an effective method of optimization from the available list $P_{k_1}, P_{k_2}, ..., P_{k_s}$ for the decision of specific targets.

Let's consider now the approach incorporated in system to the decision of a problem(task) of search of a global minimum of function $P(x)$. Let functions $F(x)$ $q_v(x)$, $r_k(x)$ not necessarily are convex.

In practical problems usually does not set the purpose of search of a global minimum with high accuracy. With methods of global optimization it is possible to find good initial approach, and then to use effective methods of local descent. Generally it is difficult to solve the problem on, whether last found local minimum is global. Except for some narrow classes of problems.

Search of a global minimum can be stopped, if in the current point the value of criterion function which is not necessarily being value of a global minimum, but meeting the requirements of practice is received. On another, if time which has been released on the decision of a problem is exhausted. Thus the best value of func-tion achieved as a result of search makes for the final decision of a problem.

At the decision of practical problems of global optimization the large value has a choice of good initial approach. In this case it is possible to find the minima nearest to them.

In very rare cases probably analytical research of criterion function and reception of the information on value of a global optimum or its site.

But generally a problem complex enough though there is a big number of numerical methods and algo-rithms of global optimization. At the same time, there is no established classification, both methods of global optimization, and corresponding problems. We shall note, that the majority of authors adheres to classifi-cation of methods depending on the used information on criterion function.

Process of search of a global minimum differs from process of local optimization by that, that all methods of global optimization are not relaxation. Also, by global optimization, each method should make the certain amount of steps, only then it is possible to make a decision on transition to other method. Thus the information received as a result of the previous search, further not always it is possible to use.

However, despite of these difficulties, process of search of a global minimum can be operated.

First, at work of one any method of global search it is possible to change some parameters of this method.

Second, from some current points of process of global search, it is possible to do transition to processes of local descent.

Thirdly, at search of a global minimum of the function having the big number of variables, it is possible to fix values of some variables and to carry out global minimization on the staying variables in view of their influence on criterion function.

These moments allow to operate process of global search.

In system, process of the decision of a problem on search of a global minimum will consist of stages. Methods of global optimization use methods of conditional local optimization for localization of an optimum point. Methods of conditional local optimization in turn use methods unconditional local optimization.

LITERATURE

1. *Boranbayev S.N.* Method of the first and second orders for the decision of a problem of network structure // Yalichanov readings 4. Kokshetau, 1998.

2. *Boranbayev S.N.* Optimality condition in optimization network tasks // Program 989 American Methematical Society Meeting. University of Colorado. Boulder. USA. 2003.

3. *Vasyilev F.P.* Numerical methods of the decision of extreme problems. M.: Science, 1980.

4. *Evtushenko Ju.G.* Method of the decision of extreme problems and their application in systems of optimization. M.: Science, 1982.

5. *Zhilinskas A.* Global optimization. Axiomatics of statistical models, algorithms, applications. Vilnius, 1986.

6. *Cea Z.* Optimization. The theory and algorithms. M.: the World, 1979.

7. *Strongin R.G.* Numerical methods in multiextreme problems. M.: Science, 1978.

8. *Fiakko A., G. Mack–Kormik G.* Nonlinear programming. Methods of consecutive unconditional minimization. M.: the World, 1979.

9. *Himmelblau D.* Applied nonlinear programming. M.: the World, 1975.

10. *Bellavia S., Macconi M., Morini B.* On the Numerical Solution of Large-Scale Constrained Nonlinear Systems // High performance algorithms and software for nonlinear optimization: status and perspectives. Italy, 2004.

11. *Zhilinskas A., Zilinskas J.* Global Optimization by Means of Stochastic Interval Arithmetic // High performance algorithms and software for nonlinear optimization: status and perspectives. Italy, 2004.

Резюме

Құнарларды үлестіру оңтайлылық есептердің математикалық және бағдарламалық қамтамасыздығына арналған. Үнемді әдістердің таңдалу проблемасы зерттеледі.

Резюме

Разработано математическое и программное обеспечение решения оптимизационных задач распределения ресурсов. Исследованы проблемы выбора эффективного метода решения.

L. N. Gumilyov Eurasian
National University

Поступила 3.04.06г.