L. KH. ZHUNUSSOVA

THE SOLUTIONS OF A SUM OF CONTROL OPTIMIZATION IN THE CLASS OF BILINEAR SYSTEMS

It has being researched a task of optimal control of the biological type which can be interpreted as a model of optimal control.

The description of the system state and its movement is carried out by various ways. It sometimes turns out well to find the summarized mathematical expression describing the regularity of the system movement in the analytical form.

The fundamental result is the principle of L. S.Pontryagin's maximum, he indicated for a classic task of optimal control in usual dynamic systems a form in which it is necessary to search necessary conditions of the first order in various classes of optimization sums of control [1].

In practice models are often met which are described by bilinear systems of differential equations, especially biomodels, ecomodels and etc. The research of them for optimality is a very laborious task..

In the given article it has been considered the sum of optimal control by bilinear systems. The method of solution of optimization sum is different from a classic approach of optimization tasks solution in that the first integral /or some first integrals/ is used here of the regulated part of the modeled system [2]. It is else important to note here optimal controls are defined without using a principle of L.S.Pontryagin's maximum and R.Bellman's dynamic programming [3].

The last years quite much work is devoted to problems of modeling of immune reaction. Among them more distinctive ones are: a model reflecting functional connection between concentration of antibodies and antibody-synthesizing cells; a model describing interaction of "inertia start" and some "immune agents"; a model reflecting functional proportions binding one dose of antigen and total quantity of anti-synthesizing cells which are formed during initial immune answer [4].

At present mathematical methods and computer modeling find a wider application in biological researches. The development of methods of a system analisys of inlinear models promotes to that.

We remind the key theorem before to give a definition and a solution of our task. Let a system is given which is described by equations

$$\dot{x} = f(x,t) + B(x,t)u(t), \qquad t \in [t,T]. \quad (1)$$

Where X – n – measured vector of state; U – m – measured vector of control; f (X, t) – n – measured vector of function; B – (nXm) – measured matrix consequently.

It is given the initial state of the system

$$x(t_0) = x_0. \qquad (2)$$

On the function U (t), before $t \in [t,T]$ the following restrictions are imposed:

$$\text{I u (t) I} < M, \qquad (3)$$

where M – const, M > 0.

Positing of a problem.

It is necessary to find such a managing vector-function u (t), satisfying (3), which would minimize Bolts' functional

$$F_0(p) \to \min, \qquad (4)$$

where through p – it was noted a biological process, and for this process Bolts' functional is defined as follows:

$$F(p) = V(x_i,T) + \sum_{j=1}^{m} \int_{t_0}^{T} M_j \left| \sum_{i=1}^{n} \frac{\partial V(t,x)}{\partial x_i} B_{ij}(x,t) \right| dt. \qquad (5)$$

Here V (x,t) – the first integral of the system

$$\dot{x} = f(x,t). \qquad (6)$$

The following theorem is right.

Theorem [2]. Let function V (x,t) is given for the system (1) which is the first integral for the unregulated system (6). Then the control as

$$u_i^0(x,t) = -M_i sign\left(\frac{\partial V(x,t)}{\partial x_i} B_{ii}(x,t) \right), j = \overline{1.3} \quad (7)$$

affords, absolute minimum to Bolts' functional (5) and it is equal

$$F(p^0) = \min_{\substack{|u_j| \le M_j \\ j=\overline{1.m}}} F(p) = V(x(t_0),t_0) = V(x_0,t_0).$$

Demonstration. Because of V (x,t) is the first integral of the system (6), we have

$$\frac{\partial V(x,t)}{\partial t} + \sum_{i=1}^{n} \frac{\partial V(x,t)}{\partial x} f_i(x,t) = 0, \qquad (8)$$

and the full derivative by t from function V (x,t) by force of the system (1) is equal

$$\dot{V}(x,t) = \frac{\partial V(x,t)}{\partial t} + \sum_{i=1}^{n} \frac{\partial V(x,t)}{\partial x_i} \left(f_i(x,t) + \right. \qquad (9)$$

$$\left. + \sum_{j=1}^{m} B_{ij}(x,t)u_j \right) = \sum_{i=1}^{n} \frac{\partial V(x,t)}{\partial x_i} \sum_{j=1}^{m} B_{ij}(x,t)u_j.$$

Let's integrate proportions (9) by t within the limits from t_0 до T:

$$\dot{V}[x(T),T] = V(x(t_0),t_0) +$$

$$+ \int_{t_0}^{T} \sum_{j=1}^{m} u_j(x,t) \sum_{i=1}^{n} \frac{\partial V(x,t)}{\partial x_i} B_{ij}(x,t)dt. \quad (10)$$

Moving up expression *V(x(T),T)* from (10) into proportion (5), we'll get

$$F(p) = V[x(t_0),t_0] + \int_{t_0}^{T} \left[\sum_{j=1}^{m} u(x,t) \sum_{i=1}^{n} \frac{\partial v(x,t)}{\partial x_i} \times \right.$$

$$\left. \times B_{ij}(x,t)dt + \sum_{j=1}^{m} M_j \left\| \sum_{i=1}^{n} \frac{\partial V(x,t)}{\partial x_i} B_{ij}(x,t)dt \right\| dt = \right.$$

$$=V(x(t_0),t_0)+\int_{t_0}^{T}\sum_{j=1}^{m}\sum_{i=1}^{n}\frac{\partial V(x,t)}{\partial x_i}B_{ij}(x,t)\times$$

$$\times\left[u_j(x,t)+M_j sign\left(\sum_{i=1}^{n}\frac{\partial V(x,t)}{\partial x_i}B_{ij}(x,t)\right)\right]dt,$$

on

$$u_j^0(x,t)=-M_j sign\left(\sum_{i=1}^{n}\frac{\partial V(x,t)}{\partial x_i}B_{ij}(x,t)\right), j=\overline{1,m}$$

hence,

$$F(p^0)=\min_{\substack{|u_j|\le M_j\\ j=\overline{1,m}}}F(p)=V(x(t_0),t_0)=V(x_0,t_0).$$

The theorem is proved.

The solution of a problem.

Thus, we'll consider p-process. The dynamic of the given managed object is described by the following ordinary differential equations

$$\frac{dx_1}{dt}=\kappa_1(x_2-x_1)+b_{11}u_1,$$

$$\frac{dx_2}{dt}=\beta x_2-x_2x_3+b_{22}u_2, \quad (11)$$

$$\frac{dx_3}{dt}=\kappa_3(\sigma x_1-x_2x_3-x_3)+b_{33}u_3,$$

$$x(t_0)=x_0. \quad (12)$$

Limits are imposed on managements, $|u_i|\le M_i,\ i=\overline{1,3}$; где M_i=*const*.

This model describes periodical course of illness where they are denoted through x_1 – concentration of mature plasmazits; x_2 – concentration of antigens; x_3 – concentration of antibodies; κ_1 – permanent defining duration of immunity; κ_3 – time of natural death of antibodies; β-к coefficient of reproduction of carriers of antigen determinat; s – coefficient of ratio of speed of growth of antibodies by mature plasmazits to concentration of immature plasmazits. Let's define Bolts' functional for the given model, thus

$$F(p)=V[x(T),T]+\int_0^T M\left|\sum_{i=1}^{3}\frac{\partial V(x,t)}{\partial x_i}b_{ii}(x,t)dt\right|dt, \quad (13)$$

where

$$V(x,t)=x_2+\ln x_2-\frac{1}{\kappa_3}[x_3-\beta\ln x_3]-$$

$$-\frac{1}{\kappa_1}\sigma x_1+\sigma\int_0^t x_2(\lambda)d\lambda-\beta\sigma\int_0^t\frac{x_1(\lambda)}{x_3(\lambda)}d\lambda \quad (14)$$

is the first integral of the irregulated system.

And the optimal value of the management will have the following form:

$$u_1^0(x,t)=-M_1 sign(-\frac{\sigma b_{11}}{k_1}),$$

$$u_2^0(x,t)=-M_2 sign[(1+\frac{1}{x_2})b_{22}],$$

$$u_3^0(x,t)=-M_3 sign[(\frac{\beta}{x_3}-\frac{1}{k_3})b_{33}].$$

We want to note that the built model of the immune answer is based on some universally adopted hypotheses and theories at present. And at the same time quite a number of factors were not taken into consideration by us. In this connection, we want to underline that maximum simplification of a model, the reduction of a number of independent variable leads, in our opinion, to deeper understanding of the modeled biological phenomenon.

Thus, the solution of the formulated problem consists in defining the value of function of management analytically.

The given put task of optimization of management of bilinear systems, from one side, generalizes the way a problem is put about optimal management of dynamic of models in the class of ordinary differential equations [5], and from the other side, spreads the conditions of optimization in the class of bilinear systems.

After all, it is necessary to emphasize that the given optimizational method is not connected with the choice of limit conditions which describe the behavior of an object, though in the equations of a problem one can always distinguish a group of dependent variables describing the state of an object and a group of managing functions which are available directly to changing from outside.

LITERATURE

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Резюме

Аталған жұмыста басқару ісін оңтайландырудың дәстүрлі әдістерінен өзгеше әдісі қарастырылған. Ескерте кететін жайт, бұл жерде модельденетін құбылыс дифференциалды теңдеулер не дифференциалды теңдеулер жүйесі арқылы беріліп,оның алғашқы интгралдарының болуы шарт.

Резюме

В работе рассматривается задача оптимального управления билинейных биологических систем при наличии первых интегралов нерегулируемой части системы и с помощью выбора функционала Больца. Преимуществом данного оптимизационного подхода является то, что оптимальное управление находится в аналитическом виде.

Kazakh akademy of transport
and communication named
after M. Tynychpaev

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